United States Department of Agriculture

Natural Resources Conservation Service

Part 634 Hydraulics
National Engineering Handbook

Chapter 3 Hydrokinetics

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Chapter 3 Hydrokinetics

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Part 634 – Hydraulics

Chapter 3 - Hydrokinetics

634.0300 Introduction

A. Hydrokinetics is the study of fluids in motion. Motion in fluids is produced by the action of forces. For example, in pressurized pipelines pressure forces act as the driving forces in the flow, whereas, in open channel flow, it is the weight of water mass (gravity forces) that produces the motion. In both cases, friction between the water and the walls of the pipe or channel act as opposing forces. In steady flows, driving and opposing forces are in equilibrium.

B. The analysis of fluids in motion typically requires the determination of flow quantities such as discharge or velocity, or the determination of a linear quantity such as pipe diameter or flow depth in open channel flow. The determination of energy or head losses is also an important aspect of the analysis of fluids in motion. These analyses have practical applications in the operation of devices or systems through which fluids (water) flow, for example, irrigation pipes, outlet works from dams, and irrigation canals.

C. The principles of flow continuity and of conservation of energy are used to analyze fluids in motion.

634.0301 Flow Continuity

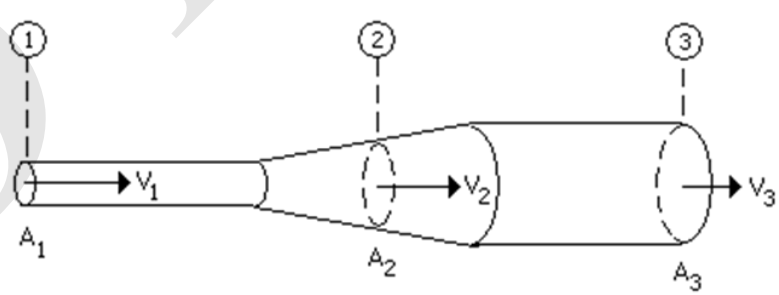
A. The equation of continuity represents a statement of conservation of mass in fluid flow.

(1) Consider, for example, a steady flow in a closed conduit of varying cross-section as shown in figure 3-1 below. Let Q be the discharge through the conduit. The equation of continuity for steady flow (i.e., Q = constant) states that:

$$Q = V_1 A_1 = V_2 A_2 = V_3 A_3 \qquad \text{(eq. 3-1)}$$

where V_1, V_2, *and* V_3 represent the flow velocities at sections 1, 2, and 3 in figure 3-1 respectively. The areas of the corresponding cross-sections are A_1, A_2, and A_3.

Figure 3-1: Flow continuity in a pipe expansion.



(2) The cross-sectional area of a circular pipe of diameter D is given by:

$$A = \pi r^2 \text{ or } A = \frac{\pi D^2}{4} \qquad \text{(eq. 3-2)}$$

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(3) The discharge can be written as:

$$Q = VA = V\frac{\pi D^2}{4} \qquad \text{(eq. 3-3)}$$

(3) Knowing the discharge Q, the velocity V in a circular conduit of diameter D can be calculated as:

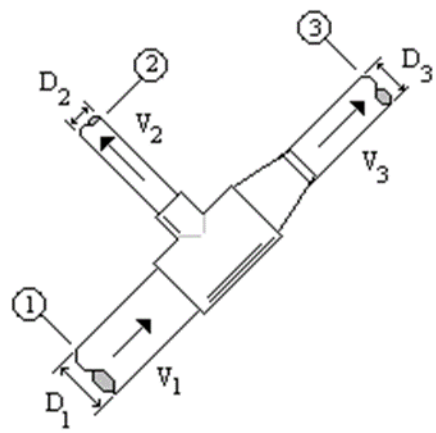
$$V = \frac{4\,Q}{\pi D^2} \qquad \text{(eq. 3-4)}$$

B. In units of the English System, the velocity V is given in *ft/s* (or *fps*), while the discharge Q is given in ft^3/s or *cfs*. Using the International System, the velocity V is given in *m/s* and the discharge Q is given in m^3/s. Other units of discharge in the English System include *gal/min* (or *gpm*) and *gal/day* (or *gpd*), which are commonly used in water supply applications. In the operation of large reservoirs, for example, the use of *acre-foot/day* as a unit of discharge is not uncommon. In the International System one could also use m^3/min or m^3/day for large discharge operations, while the units of *liters/s* or *liters/min* are used for small discharges.

C. Example – Equation of continuity in a pipe - discharge and velocity calculation

(1) Suppose that in the pipeline of figure 3-2 the diameters are $D_1 = 0.25\ ft$ (*3 in*), $D_2 = 0.50\ ft$ (*6 in*), and $D_3 = 0.75\ ft$ (*9 in*), and that the velocity at section 3 is measured to be $V_3 = 0.5\ fps$. Find the velocities in sections 1 and 2 and the discharge.

Figure 3-2: Schematic of flow in branching pipelines.



(2) Solution:

(i) The discharge is calculated as:

$$Q = V_3\frac{\pi D_3^2}{4} = 0.5\,\frac{ft}{s}\,\frac{(3.1416)(0.75ft)^2}{4} = 0.22\ cfs$$

(ii) The velocities at sections 1 and 2 are, therefore, calculated as:

$$V = \frac{4\,Q}{\pi D_1^2} = \frac{4\,(0.22\ cfs)}{3.1416\,(0.25\ ft)^2} = 4.48\ ft/sec$$

and

$$V = \frac{4\,Q}{\pi D_2^2} = \frac{4\,(0.22\ cfs)}{3.1416\,(0.5\ ft)^2} = 1.12\ ft/sec$$

(iii) A branching pipeline in which a pipe of diameter D_1 splits into two pipelines with diameters D_2 and D_3 is shown in figure 3-2. Continuity requires that the discharges through sections 2 and 3 add to equal the discharge through section 1:

$$Q_1 = Q_2 + Q_3 \qquad \text{(eq. 3-5)}$$

(iv) In terms of areas and velocities, the continuity equation for the branching pipe case is written as:

$$V_1 A_1 = V_2 A_2 + V_3 A_3 \qquad \text{(eq. 3-6)}$$

(v) Replacing the areas in terms of the diameters:

$$V_1 \frac{\pi D_1^2}{4} = V_2 \frac{\pi D_2^2}{4} + V_3 \frac{\pi D_3^2}{4} \qquad \text{(eq. 3-7)}$$

(vi) Simplifying:

$$V_1 {D_1}^2 = V_2 {D_2}^2 = V_3 {D_3}^2 \qquad \text{(eq. 3-8)}$$

D. Example – Equation of continuity – velocity and discharge calculation in branching pipeline

(1) A *12-in*-diameter pipeline carrying water at a velocity of 3.5 *fps* branches into a *6-in*-diameter pipeline and a *9-in*-diameter pipeline. If the velocity in the *6-in* pipeline is measured to be *4.0 fps*, what is the velocity in the *9-in* pipeline, and what is the total discharge through the *12-in* pipeline?

(2) Solution:

(i) Using D_1 = 12 in = 1 ft, V_1 = 3.5 fps, D_2 = 6 in = 0.5 ft, V_2 = 4.0 fps, and D_3 = 9 in =0.75 ft, find V_3 and Q_1. From equation 35, i.e., $V_1D_1^2 = V_2D_2^2 + V_3D_3^2$, it follows that:

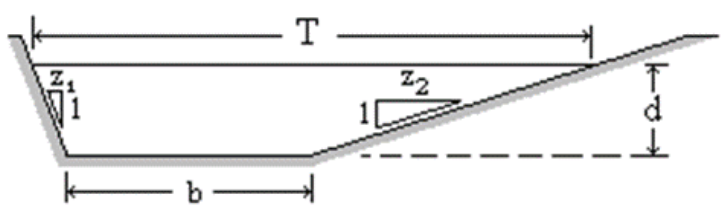
$$V_3 = \frac{V_1 D_1^2 - V_2 D_2^2}{D_3^2} = \frac{(3.5\,fps)(1ft)^2 - (4\,fps)\,(0.5\,ft)^2}{(0.75ft)^2} = 4.44\,fps$$

(ii) The discharge through section 1 is:

$$Q_1 = V_1 \frac{\pi D_1^2}{4} = (3.5\,fps)\frac{3.1416\,(1\,ft)^2}{4} = 2.75\,cfs$$

E. The equation of continuity can also be applied to flow in open channels. Consider the flow in an open channel with a non-symmetric trapezoidal cross-section as illustrated in figure 3-3.

Figure 3-3: Non-symmetric trapezoidal cross-section in an open channel flow.



F. This cross-section is characterized by the *bottom width b*, the *side slopes* z_1 and z_2, and the *flow depth d*. The side slopes are interpreted as z_1 H:1V, i.e., z_1 units horizontal to 1 unit vertical. The *top width T* is the length of the free surface at the cross-section. For the non-symmetric trapezoidal cross-section shown above, the top width is given by:

$$T = b + (z_1 + z_2)d \qquad \text{(eq. 3-9)}$$

and the area is calculated as the average of trapezoid's bases, *b* and *T*, multiplied by the height of the trapezoid, *d* :

$$A = \left(\frac{T+b}{2}\right) d = \left(b + \frac{z_1 + z_2}{2} d\right) d \qquad \text{(eq. 3-10)}$$

G. Example – Equation of continuity – velocity in a trapezoidal open-channel cross-section

(1) Determine the flow velocity in a non-symmetric trapezoidal cross section of figure 3-3 has $b = 3\,ft$, $d = 1.3\,ft$, $z_1 = 0.5$, and $z_2 = 1$, and carries a discharge $Q = 15.2\,cfs$.

(2) Solution:

(i) The area is calculated as:

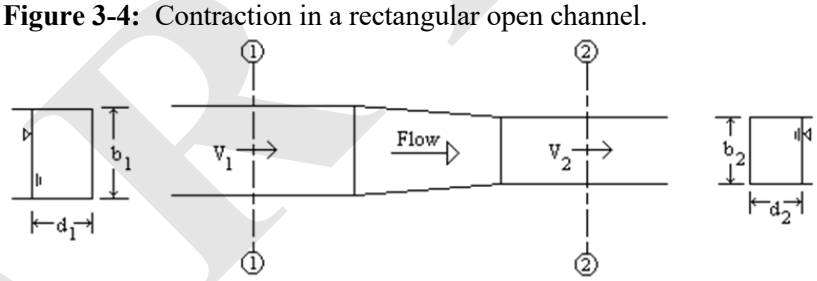
$$A = \left(b + \frac{z_1 + z_2}{2} d\right) d = \left(3ft + \frac{0.5 + 1}{2} 1.3ft\right) 1.3\,ft = 5.17\,ft^2$$

(ii) And the flow velocity is: V = Q/A = 15.2 cfs/5.17 ft^2 = 2.94 fps

H. Example – Equation of continuity – channel width reduction

(1) Figure 3-4 below shows a rectangular open channel that reduces in channel width from b_1 to b_2. In figure 3-4 suppose that $b_1 = 7.5\,ft$, and $b_2 = 5.0\,ft$. The depth of flow and velocity in section (1) are $d_1 = 2.5\,ft$ and $V_1 = 4.5\,fps$. What depth of flow is required in section (2) to maintain the same flow velocity (i.e., $V_2 = V_1 = 4.5\,fps$)? What is the flow discharge through the channel?

Figure 3-4: Contraction in a rectangular open channel.



(2) Solution:

(i) With the cross-sectional shape being rectangular in both sections (1) and (2), the area of the cross-section is given by $A = bd$. Thus, the continuity equation (equation 3-1) can be written as:

$$Q = V_1 b_1 d_1 = V_2 b_2 d_2$$

(ii) The depth at section (2) is given by:

$$d_2 = \frac{V_1 b_1 d_1}{V_2 b_2} = \frac{(4.5fps)(7.5\,ft)(2.5\,ft)}{(4.5\,fps)(5.0\,ft)} = 3.75\,ft$$

(iii) The discharge is calculated as:

$$Q = V_1 b_1 d_1 = 4.5\,fps \times 7.5\,ft \times 2.5\,ft = 84.38\,cfs$$

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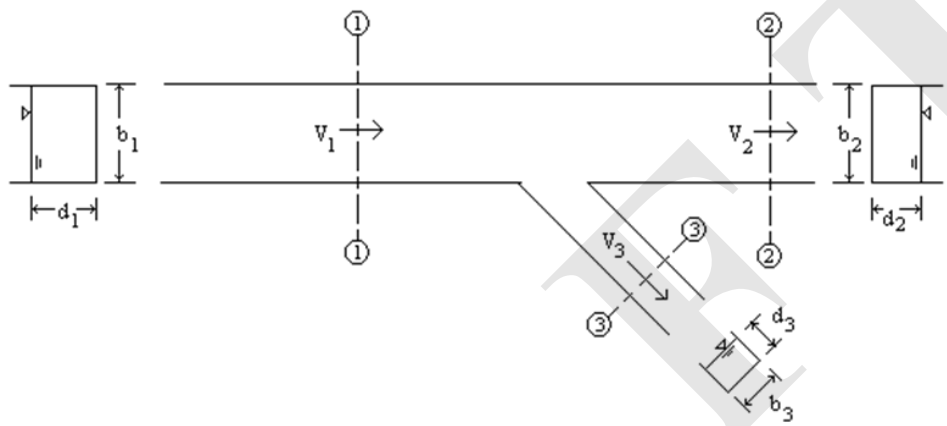
I. Example – Equation of continuity – rectangular open channel diversion

(1) Figure 3-5 shows a rectangular channel of width b_1 from which water is diverted through a lateral rectangular channel of width b_3. The section of the main channel downstream from the diversion has a width $b_2 = b_1$. Using b_1=b_2=10 *ft*, d_1=4.0 *ft*, V_1=1.2 *fps*, d_2=3.5 *ft*, b_3=5.0 *ft*, d_3=2.5 *ft*, and V_3=0.6 *fps*, determine the flow velocity at section 2, V_2, as well as, the discharges through sections 1, 2, and 3.

Figure 3-5: Rectangular open channel diversion.



(2) Solution:

(i) For a branching channel as shown in figure 3-5, continuity requires that

$$Q_1 = Q_2 + Q_3, i.e., V_1 b_1 d_1 = V_2 b_2 d_2 + V_3 b_3 d_3$$

Thus, the velocity at section 2 is:

$$V_2 = \frac{V_1 b_1 d_1 - V_3 b_3 d_3}{b_2 d_2}$$

$$= \frac{1.2\ fps \times 10\ ft \times 4\ ft - 0.6\ fps \times 5\ ft \times 2.5\ ft}{10\ ft \times 3.5\ ft}$$

$$= 1.157\ fps$$

(ii) The discharges through the three sections are calculated as follows:

$$Q_1 = V_1 b_1 d_1 = 1.2\ fps \times 10\ ft \times 4\ ft = 48\ cfs$$

$$Q_2 = V_2 b_2 d_2 = 1.157\ fps \times 10\ ft \times 3.5\ ft = 40.5\ cfs$$

$$Q_3 = V_3 b_3 d_3 = 0.6\ fps \times 5\ ft \times 2.5\ ft = 7.5\ cfs$$

634.0302 Conservation of Energy

A. In the analysis of fluid flow, three types of energy are typically considered: potential or elevation energy, pressure energy, and kinetic energy. Figure 3-6 illustrates these concepts using a simple reservoir-sprinkler system.

B. The elevation of the water in the reservoir represents potential energy since it is this elevation that provides the available energy for the system. As the water flows through the pipe and is discharged through the sprinkler, it acquires motion which converts some of the potential energy into kinetic energy (energy of motion). Also, at the sprinkler, a measurable pressure exists that is related to pressure energy at that point.

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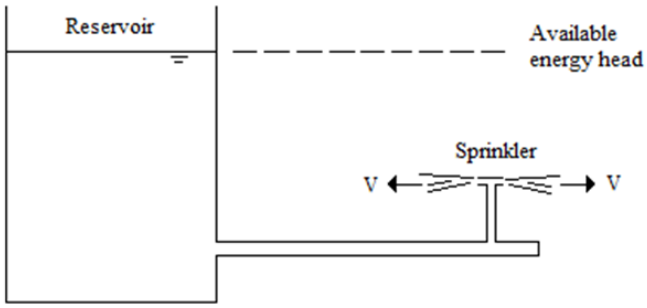
C. In a flowing fluid system as the one illustrated above, energy is conserved as it is converted from one type of energy (potential) into another (pressure or kinetic). In fluid flow these energies are typically converted to energy heads that can be easily visualized as in the potential energy head of Figure 3-6. The description of these energy heads follows in the next sections.

Figure 3-6: Flow energies illustrated with a simple reservoir-sprinkler system



D. Potential Energy - Potential energy is the ability of a water mass to perform work because of the elevation of that mass of water with respect to an arbitrary datum line or reference level. A mass of weight *W*, at an elevation of *z* feet, has a potential energy equal to *Wz* (ft-lb) with respect to the datum. The elevation head, *z*, expresses not only a linear quantity (ft), but also energy per unit weight, i.e., *ft-lb/lb = ft*.

E. Pressure Energy - Pressure energy at a point of a fluid flow is produced by the local pressure at the point. This pressure could be the result of a pumping action in a pipeline or of the weight of water above a certain point in open channel flow. The pressure head (pressure energy per unit weight) in pipeline flow is calculated as p/ω, pressure divided by the specific weight of water. In open channel flow applications, the pressure head is equal to the flow depth, *d,* and the pressure distribution is assumed to be hydrostatic.

F. Kinetic Energy - The kinetic energy (K) of a mass (M) of fluid moving at a velocity (V) is given by $K = 1/2\ MV^2$. Since its weight is W = Mg, the kinetic energy per unit weight is calculated as:

$$h_v = \frac{K}{W} = \frac{\frac{1}{2}MV^2}{Mg} = \frac{V^2}{2g} \qquad \text{(eq. 3-11)}$$

where *g* is the acceleration of gravity ($g = 32.2\ ft/s^2$). The term h_v in equation 3-11 is referred to as the *velocity head*.

G. The *total energy head*, *H*, is the sum of the potential energy head (*z*), the pressure head (p/ω or *d*), and the velocity head ($V^2/2g$). All three forms of energy head may be expressed as a linear quantity (ft), or as energy per unit weight (ft-lb/lb).

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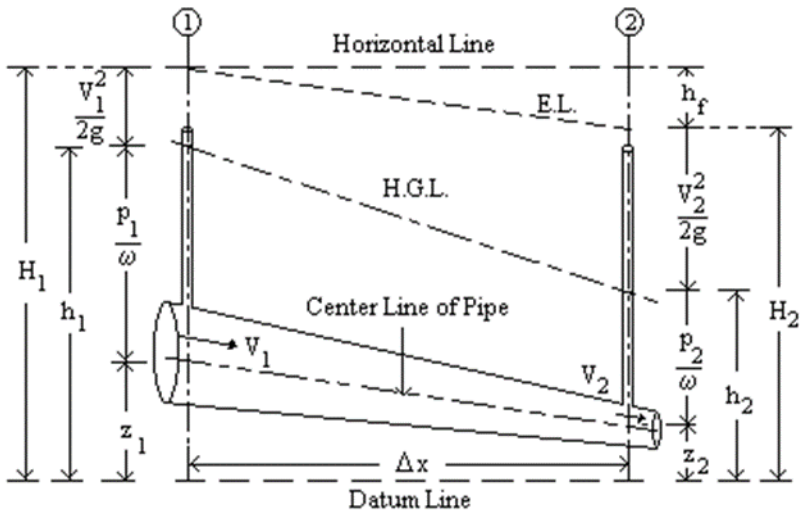
H. *Energy heads in pipe flow.* Figure 3-7 illustrates the three different energy heads applied to pipe flow. Notice that, typically, the potential energy head (elevation) in a pipe flow is referenced at the centerline of the pipe. The pressure head is measured from the pipe centerline, and the distance from the datum to the top of the pressure head represents the piezometric head:

$$h = z + \frac{p}{\omega}$$

Figure 3-7: Energy heads in pipe flow.



I. The difference between the total energy head and the piezometric represents the kinetic energy. The total energy in a pipe flow is given by:

$$H = h + \frac{V^2}{2g} = z + \frac{p}{\omega} + \frac{V^2}{2g} \qquad \textbf{(eq. 3-12)}$$

J. The line representing the values of the total head, *H*, as function of position *x* along the pipeline is referred to as the *Energy Line* (E.L.), while that representing the values of the piezometric head, *h*, is referred to as the *Hydraulic Grade Line* (HGL). Energy losses, h_f, are the losses due to friction between sections (1) and (2).

K. Example – Calculation of pressure, velocity, piezometric and total head

(1) A manometer located at a point of a 6-*in*-diameter pipe shows a pressure reading of 6 *psi*. The point of pressure measurement is located at an elevation of 1500 *ft* above mean sea level and the pipe is carrying a discharge of 0.6 *cfs*. Calculate the pressure head, the velocity head, the piezometric head, and the total head.

(2) Solution:

(i) The data given are $D = 6\ in = 0.5\ ft$, $p = 6\ psi = 6\times 144\ psf = 864\ psf$, $z = 1500\ ft$, and $Q = 0.6\ cfs$. Using $\omega = 62.4\ lb/ft^3$ for the specific weight of water, the pressure head is calculated as:

$$p/\omega = (864\ lb/ft^2)(62.4\ lb/ft^3) = 13.85\ ft$$

(ii) The velocity can be calculated from the continuity principle using equation 3-4:

$$V = \frac{4\,Q}{\pi D^2} = \frac{(4)(0.6\,\frac{ft^3}{s})}{3.1416\,(0.5\,ft^2)} = 3.06\,fps$$

(iii) The velocity head is:

$$h_v = \frac{V^2}{2g} = \frac{(3.06\,\frac{ft^3}{s})^2}{(2)(32.2\,\frac{ft}{s^2})} = 0.15\,ft$$

(iv) The piezometric head is:

$$h = z + p/\omega = 1500\,ft + 13.85\,ft = 1513.85\,ft$$

(v) The total head is

$$H = h + h_v = 1513.85\,ft + 0.15\,ft = 1514.0\,ft$$

(v) Notice that in this example mean sea level is used as the datum line or reference level. If the pressure measurement point were located at 10 ft above the floor and the floor used as the datum line, then z = 10 *ft*, and the piezometric head and total head would be calculated as

$$h = z + {}^{p}/_{\omega} = 10\,ft + 13.85\text{ ft} = 23.85\text{ ft}$$

and

$$H = h + h_v = 23.85\,ft + 0.15\,ft = 24.0\,ft$$

L. In the previous example, the discharge Q was used to calculate the velocity V in a circular pipeline of diameter D. In terms of the discharge (see equations 3-4 and 11), the velocity head can be calculated as:

$$h_v = \frac{1}{2g}\left(\frac{4Q}{\pi D^2}\right)^2 = \frac{8Q^2}{\pi^2 g D^4} \qquad \text{(eq. 3-13)}$$

M. Using this result the total energy head in a pipeline of diameter D carrying a discharge Q is given by:

$$H = z + \frac{p}{\omega} + \frac{8Q^2}{\pi^2 g D^4} \qquad \text{(eq. 3-14)}$$

N. Example – Calculation of total head

(1) A 2-*in* diameter pipeline carries a flow discharge of 0.3 *cfs*. At a point of the flow, located at an elevation of 2.5 *ft* above the floor, the pressure is measured to be 2 *psi*. What is the total energy head at that point?

(2) Solution - Given D = 2 *in* = 2 *in*/12 *ft* = 0.166 *ft*, Q = 0.3 *cfs*, z = 2.5 *ft*, p = 2 *psi* = 2×144 *psf* = 288 *psf*, and, with ω = 62.4 *lb/ft*^{3}, the total head is:

$$H = z + \frac{p}{\omega} + \frac{8Q^2}{\pi^2 g D^4} = 2.5\,ft + \frac{288\,\frac{lb}{ft^2}}{62.4\,\frac{lb}{ft^3}} + \frac{8\,(0.3\,\frac{ft^3}{2})^2}{3.1416^2(32.2\,\frac{ft}{s^2})} = 10.1\,ft$$

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O. Energy heads in open channel flow.

(1) The energy heads for open channel flow are illustrated in figure 3-8. The elevation head, *z,* refers to the location of the channel bed, while the pressure head is represented by the flow depth *d*.

(2) The sum of the potential energy head (*z*) and the flow depth (*d*) in open channel flow represents the water surface elevation or stage, $WS = z + d$. The difference between the total energy and the water surface elevation is the velocity head. The total energy in a channel flow is given by:

$$H = WS + V^2/2g = z + d + V^2/2g \qquad \text{(eq. 3-15)}$$

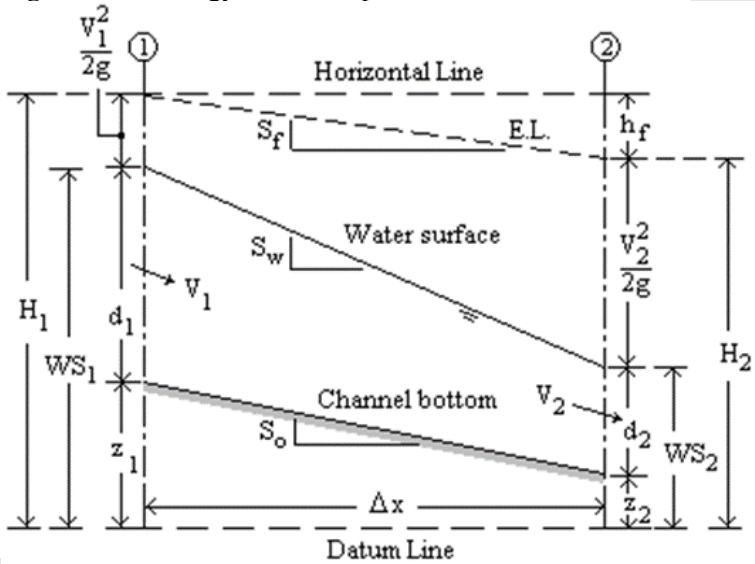
(3) Of the total energy, the quantity called the *specific energy* is:

$$E = d + V^2/2g \qquad \text{(eq. 3-16)}$$

(4) This quantity represents the flow energy measured with respect to the channel bed at a given cross-section.

(5) In open channel flow, as illustrated in figure 3-8, the hydraulic grade line is represented by the water surface. As in figure 3-7, the term *hf* represents the energy losses due to friction between section (1) and section (2).

Figure 3-8: Energy heads in open channel flow.



(6) In both the pipe flow and the open channel flow illustrated in figure 3-7 and figure 3-8, respectively, the horizontal distance between cross-sections 1 and 2 is referred to as *Δx*. The slope length of pipeline measured along its centerline, or the slope length of the channel between the two cross-sections, will be referred to as *L*.

P. Example – Velocity head, specific energy, and total head in open-channel flow

(1) A symmetric trapezoidal channel with bottom width $b = 8.5$ *ft* and side slope $z = 0.5$, carries a discharge of $Q = 30$ *cfs* at a depth $d = 2.3$ *ft*. Calculate the velocity head, and specific energy for this channel. If the channel bed is located at an elevation of 1255.32 *ft* above mean sea level, calculate the water surface elevation at that point, as well as the total energy head.

(2) Solution:

(i) Equation 3-10, or methods in the following Section 634.0303, Open Channel Flow, can be used to calculate the cross-sectional area for a symmetric trapezoidal cross-section by taking $z_1 = z_2 = z$, i.e., $A = (b+z \cdot d) \cdot d$. For this example, the data given are $b = 8.5\ ft$, $z = 0.5$, and $d = 2.3\ ft$. Thus,

$$A = (8.5\ ft + 0.5 \times 2.3\ ft) \times 2.3 ft = 22.20\ ft^2$$

(ii) The flow velocity for this case is given by

$$V = Q/A = \frac{30\ ft^3/s}{22.20\ ft^2} = 1.35\ ft/s$$

and the velocity head is

$$h_v = \frac{V^2}{2g} = \frac{(1.35\ ft/s)^2}{(2 \times 32.2\, ft/s^2)} = 0.028\ ft$$

Thus, the specific energy is:

$$E = d + h_v = d + V^2/2g = 2.3\ ft + 0.028\ ft = 2.328\ ft$$

(iii) The elevation of the channel bed is $z = 1255.32\ ft$ (notice that this is a different z than the side slope z for the cross-sectional geometry), and the water surface elevation is

$$WS = z + d = 1255.32\ ft + 2.3\ ft = 1257.62\ ft$$

(iv) The total energy head is calculated as

$$H = z + E = 1255.32\ ft + 2.328\ ft = 1257.65\ ft$$

Q. Equation of Energy and Bernoulli's Principle

(1) In the diagrams shown in figure 3-7 and figure 3-8, water flows from section (1) to section (2). The diagrams indicate that the total energy head at the upstream section (1) is larger than the total energy head at the downstream section (2), i.e., $H_1 > H_2$. The difference represents the *energy losses* h_f due to friction as the water moves from section (1) to section (2). The law of conservation of energy for both pipe flow and open channel flow can be written as:

$$H_1 = H_2 + h_f \qquad \text{(eq. 3-17)}$$

where h_f represents the energy losses due to friction between sections (1) and (2). Specifically, for the pipe flow case illustrated in figure 3-7:

$$h_1 + \frac{V^2}{2g} = h_2 + \frac{V^2}{2g} + h_f \qquad \text{(eq. 3-18)}$$

$$z_1 + \frac{p_1}{\omega} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\omega} + \frac{V_2^2}{2g} + h_f \qquad \text{(eq. 3-19)}$$

(2) The law of conservation of energy for open channel flow, as illustrated in figure 3-8, is written as:

$$WS_1 + \frac{V_1^2}{2g} = WS_2 + \frac{V_2^2}{2g} + h_f \qquad \text{(eq. 3-20)}$$

or

$$z_1 + d_1 + \frac{V_1^2}{2g} = z_2 + d_2 + \frac{V_2^2}{2g} + h_f \qquad \text{(eq. 3-21)}$$

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(3) In some instances of pipe flow (or other enclosed flow), the fluid can be assumed to be ideal and the friction losses are zero. This is an assumption of the *Bernoulli's principle*, and the equation of energy (Bernoulli's equation) can be written as:

$$z_1 + \frac{p_1}{\omega} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\omega} + \frac{V_2^2}{2g} \qquad \text{(eq. 3-22)}$$

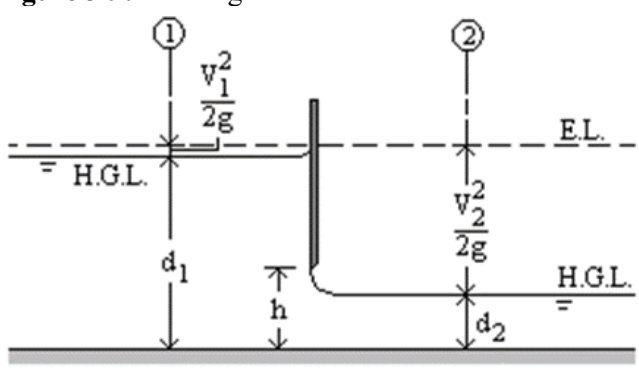
(4) In the most general case of pipe flow, however, friction losses, h_f, and local losses, h_L, (due to the presence of appurtenances in the pipe) must be included. Examples of appurtenances include elbows and valves. Taking into account both friction and local losses, the energy equation for pipelines can be written as:

$$z_1 + \frac{p_1}{\omega} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\omega} + \frac{V_2^2}{2g} + h_f + h_L \qquad \text{(eq. 3-23)}$$

R. Example – Bernoulli's principle applied to sluice gate flow

(1) Figure 3-9 shows a schematic of a sluice gate in a rectangular channel of width *b*. The energy line (E.L.) and the hydraulic grade line (H.G.L.) are also depicted. The horizontal energy line suggests no energy losses as the flow passes under the gate. Determine the discharge *Q* if the upstream and downstream depths are measured as 3.5 ft and 1.0 ft and the channel width, *b,* is 10 ft.

Figure 3-9: Sluice gate flow.



(2) Solution:

(i) Since there are no energy losses and a horizontal bed, Bernoulli's equation reduces to:

$$d_1 + \frac{V_1^2}{2g} = d_2 + \frac{V_2^2}{2g}$$

(ii) Writing the velocity head in terms of the discharge, $V_1^2/2g = Q^2/(2gA_1^2)$ and $V_2^2/2g = Q^2/(2gA_2^2)$, and using $A_1 = bd_1$, $A_2 = bd_2$, Bernoulli's equation becomes:

$$d_1 + \frac{Q^2}{2gb^2d_1^2} = d_1 + \frac{Q^2}{2gb^2d_2^2}$$

(iii) Solving for *Q* and simplifying the result produces:

$$Q = bd_1d_2\sqrt{\frac{2g}{d_1 + d_2}} = (10ft)(3.5ft)(1.0ft)\sqrt{\frac{2(32.2\frac{ft}{s^2}}{3.5ft + 1.0ft}}$$

$$= 132.4\ cfs$$

S. Example – Energy equation in pipelines

(1) Let the diagram in figure 3-7 represent a pipeline of constant diameter $D = 6\ in$ carrying a discharge $Q = 0.5\ cfs$. Let the elevations of points 1 and 2 be given by $z_1 = 7\ ft$ and $z_2 = 12.5\ ft$ with respect to an arbitrary horizontal reference level (datum line). The pressure at point 2 is measured to be $p_2 = 15.2\ psi$. Calculate the velocity head for this pipe flow. If the energy loss in the pipeline is estimated to be $h_f = 15\ ft$, what is the pressure at point 1, p_1 (psi)? Use $\omega = 62.4\ lb/ft^3$ for the specific weight of water.

(2) Solution:

(i) Since the pipeline has a constant diameter, $D = 6\ in = 6/12\ ft = 0.5\ ft$, the velocities at points 1 and 2 are the same (equation 3-4):

$$V_1 = V_2 = V = \frac{4Q}{(\pi D^2)} = \frac{4 \times (0.5\ ft^3/s)}{\pi(0.5\ ft)^2} = 2.55\ fps$$

(ii) The velocity head throughout the pipeline is the same and equal to (equation 3-11):

$$h_v = V^2/2g = (2.55\ \text{ft/s})^2/(2\ \times 32.2\ ft/s^2) = 0.10\ ft$$

(iii) Also, the velocity head can be calculated in terms of the discharge as (equation 3-13):

$$h_v = \frac{8Q^2}{\pi^2 g D^4} = \frac{8\left(0.5\frac{ft^3}{s}\right)^2}{(3.1416)^2(32.2\frac{ft}{s^2})(0.5ft)^4} = 0.10\ ft$$

(iv) The equation of energy for the case of Figure 3-7 is given by (equation 3-19):

$$z_1 + \frac{p_1}{\omega} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\omega} + \frac{V_2^2}{2g} + h_f$$

(v) Since the velocities are the same ($V_1 = V_2$), the equation simplifies to:

$$z_1 + \frac{p_1}{\omega} + = z_2 + \frac{p_2}{\omega} + h_f$$

(vi) From which the pressure head at point 1 is solved for, as:

$$p_1/\omega = z_2 - z_1 + p_2/\omega + h_f$$
$$= 12.5\ ft - 7\ ft + (15.2\ x\ 144\ lb/ft^2)/(62.4\ lb/ft^3) + 15\ ft$$
$$= 55.58\ ft$$

(vii) Thus, the pressure at point 1 is:

$$p_1 = 55.58ft\ \times 62.4\ lb/ft^3 = 3{,}468.19\frac{lb}{ft^2} = \frac{3{,}468.19}{144}psi$$
$$= 24.08\ psi$$

T. Example - Energy equation in open channel flow

(1) Suppose that figure 3-8 represents a reach in an open channel flow with a rectangular cross-section of width $b = 12.5\ ft$. The depth of flow at sections 1 and 2 are measured to be $d_1 = 4.7\ ft$ and $d_2 = 3.8\ ft$, respectively, and the velocity at section 1 is $V_1 = 4.2\ fps$. The channel bed elevations at sections 1 and 2 are given as $z_1 = 25\ ft$ and $z_2 = 22.5\ ft$. Calculate the flow discharge Q and the energy loss, h_f, between sections 1 and 2.

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(2) Solution:

(i) The area at section 1 is:

$$A_1 = bd_1 = 12.5\ ft\ \times 4.7\ ft = 58.75\ ft^2$$

and the flow discharge is

$$Q = V_1 A_1 = 4.2\ ft/s \times 58.75\ ft^2 = 246.75\ cfs$$

The area at section 2 is

$$A_2 = bd_2 = 12.5\ ft\ \times 3.8\ ft = 47.50\ ft^2$$

and the flow velocity at that location is

$$V_2 = Q/A_2 = (246.75\ ft^3/s)/(47.50\ ft^2) = 5.19\ fps$$

(ii) The energy equation for the case of figure 3-8 is written as (equation 3-21):

$$z_1 + d_1 + \frac{V_1^2}{2g} = z_2 + d_2 + \frac{V_2^2}{2g} + h_f$$

(iii) From which:

$$h_f = \left(z_1 + d_1 + \frac{{V_1}^2}{2g}\right) - \left(z_2 + d_2 + \frac{{V_2}^2}{2g}\right)$$

$$= \left(25\ ft + 4.7\ ft + \frac{(4.2\ ft/s)^2}{2(32.2\ ft/s^2)}\right) - \left(22.5\ ft + 3.8\ ft + \frac{(5.19\ ft/s)^2}{2(32.2\ ft/s^2)}\right)$$

$$= 3.26\ ft$$

U. Hydraulic and Energy Gradients

(1) Figure 3-7 and figure 3-8 represent the variation of the different energy heads in pipe flow and open channel flow, respectively. In both types of flow the line joining the total energy heads along the pipeline or open channel is the *energy line* (E.L.). In pipe flow, the line joining the piezometric heads along the pipeline ($h = z + p/\omega$) is the *hydraulic grade line* (H.G.L.), whereas, in open-channel flow the water surface represents the hydraulic grade line.

(2) For both pipe flow and open channel flow the *energy gradient*, or *slope of the energy line*, S_f, is defined as the rate of friction head loss, h_f, per unit length, L, of the pipeline or open channel:

$$S_f = \frac{h_f}{L} = \frac{H_1 - H_2}{L} \qquad \text{(eq. 3-24)}$$

(3) However, in open channel flow, the channel bed slope, S_o, is typically very small such that the length of the channel, L, is approximately equal to the horizontal distance Δx, i.e., $L \approx \Delta x$. The *energy gradient for open channels*, therefore, can be defined as:

$$S_f \approx \frac{h_f}{\Delta x} = \frac{H_1 - H_2}{\Delta x} \qquad \text{(eq. 3-25)}$$

(4) The *hydraulic gradient* is the *slope of the hydraulic grade line*. For pipe flow, such gradient is defined as the change in piezometric head per unit length of pipe:

$$S_h = \frac{h_1 - h_2}{L} \qquad \text{(eq. 3-26)}$$

(5) Whereas, for an open channel flow the hydraulic gradient is the *water surface slope*. This slope is the change in water surface elevation, *WS*, per unit of horizontal distance along the channel path:

$$S_w = \frac{WS_1 - WS_2}{\Delta x} \qquad \text{(eq. 3-27)}$$

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V. Example – Hydraulic and energy gradients in pipe flow

(1) The diameter of a 10-*ft*-long pipeline tapers from 1-*ft*-diameter at section 1 to 0.5-*ft*-diameter at section 2 of figure 3-7. Let the pressure at section 1 be p_1 = 6.0 psi and that at section 2 be p_2 = 5.5 *psi*. The pipeline is laid so that point 1 is at elevation z_1 = 12.5 *ft* and z_2 = 6.2 *ft*. If the pipeline is carrying a flow Q = 1.5 *cfs*, determine the friction loss, *hf*, the hydraulic gradient, S_h, and the energy gradient, S_f, for the flow.

(2) Solution:

(i) For $L = 10\,ft$, $D_1 = 1\,ft$, $D_2 = 0.5\,ft$, $p_1 = 6.0\,psi$, $p_2 = 5.5\,psi$, $z_1 = 12.5\,ft$, and $z_2 = 6.2\,ft$, the velocities are:

$$V1 = 4Q/(\pi D_1^2) = (4\times1.5\,ft^3/s)/(3.1416\times(1ft)^2) = 1.91\,fps$$

and,

$$V2 = 4Q/(\pi D_2^2) = (4\times1.5\,ft^3/s)/(3.1416\times(0.5\,ft)^2) = 7.64\,fps$$

(ii) The piezometric or pressure heads at sections 1 and 2 are:

$$h_1 = z_1 + \frac{p_1}{\omega} = 12.5ft + \frac{6.0(144\frac{lb}{ft^2})}{62.4\frac{lb}{ft^3}} = 26.35ft$$

and,

$$h_2 = z_2 + \frac{2}{\omega} = 6.2ft + \frac{5.5(144\frac{lb}{ft^2})}{62.4\frac{lb}{ft^3}} = 18.89ft$$

(iii) The difference in piezometric heads is:

$$\Delta h = h_1 - h_2 = 26.35\,ft - 18.89\,ft = 7.46\,ft$$

(iv) The total energy heads at sections 1 and 2 are:

$$H = h_1 + \frac{V_1^2}{2g} = 26.35ft + \frac{\left(1.91\frac{ft}{s}\right)^2}{2\left(32.2\frac{ft}{s^2}\right)} = 26.41\,ft$$

and,

$$H = h_2 + \frac{V_2^2}{2g} = 18.89ft + \frac{\left(7.64\frac{ft}{s}\right)^2}{2\left(32.2\frac{ft}{s^2}\right)} = 19.79\,ft$$

(v) Thus, the energy loss is:

$$h_f = h_1 - h_2 = 26.41\,ft - 19.79\,ft = 6.62\,ft$$

(vi) The hydraulic gradient is calculated as:

$$S_h = \frac{\Delta h}{L} = \frac{7.46ft}{10ft} = 0.746$$

(vii) The energy gradient is:

$$S_f = \frac{h_f}{L} = \frac{6.62ft}{10ft} = 0.662$$

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W. Example - Hydraulic and energy gradients in open channel flow

(1) In the example, Energy equation in open channel flow, the following flow parameters were given or calculated: rectangular cross-section of width $b = 12.5\ ft$. Depths of flow: $d_1 = 4.7\ ft$ and $d_2 = 3.8\ ft$. Flow velocities: $V_1 = 4.2\ fps$ and $V_2 = 5.19\ fps$. Bed elevations: $z_1 = 25\ ft$ and $z_2 = 22.5\ ft$. Energy head loss, $h_f = 3.26\ ft$. If the distance between the two sections is $\Delta x = 2500$ ft, calculate the energy gradient and the hydraulic gradient for this flow.

(2) Solution:

(i) The energy gradient:

$$S_f = {}^{h_f}/_{\Delta X} = {}^{3.26\ ft}/_{2{,}500\ ft} = 0.0013$$

(ii) The water surface elevations at sections 1 and 2 are, respectively, $WS_1 = z_1 + d_1 = 25\ ft + 4.7\ ft = 29.7\ ft$, and $WS_2 = z_2 + d_2 = 22.5\ ft + 3.8\ ft = 26.3\ ft$. Thus, the hydraulic gradient, or water surface slope, is:

$$S_W = \frac{(WS_2 - WS_1)}{\Delta X} = \frac{29.7\ ft - 26.3\ ft}{2{,}500\ ft} = \frac{3.4 ft}{2{,}500\ ft} = 0.00136$$

634.0303 Open channel Flow

A. Open channel flow occurs when water is conveyed to a lower elevation through a conduit or channel open to the atmosphere or when a pipe flows without being full. Open channel flow is also referred to as free-surface flow. Flows in creeks, rivers, aqueducts, flumes, irrigation canals, gutters, and culverts are examples of open channel flows.

B. Open channels occur on a slope. If the slope is in the direction of the flow it is referred to as a favorable slope. If the slope is opposite to the direction of the flow, then the slope is referred to as an adverse slope. An open channel could also have a horizontal bed, in which case the slope is zero. A channel of constant slope and constant cross-section which does not change its alignment is referred to as a *prismatic channel*. Such is often the case for constructed channels. Natural channels, on the other hand, are often highly irregular showing varying alignment, curves, and changing cross-sectional geometry.

634.0304 Uniform Open Channel Flow

A. Uniform flow in a prismatic open channel occurs when the flow depth remains constant for a constant discharge. Uniform flow typically develops in long prismatic channels, away from head or tail sections. Natural channels rarely maintain uniform flow for long reaches.

B. Figure 3-10 shows the forces acting on a section of length L of uniform flow on a channel laid on a slope So = tan(□o). The slope is sufficiently small so that the distribution of pressure with depth in the flow is hydrostatic, and so that $S_o = tan(\theta_o) \approx sin(\theta_o) \approx \theta_o$ (measured in radians).

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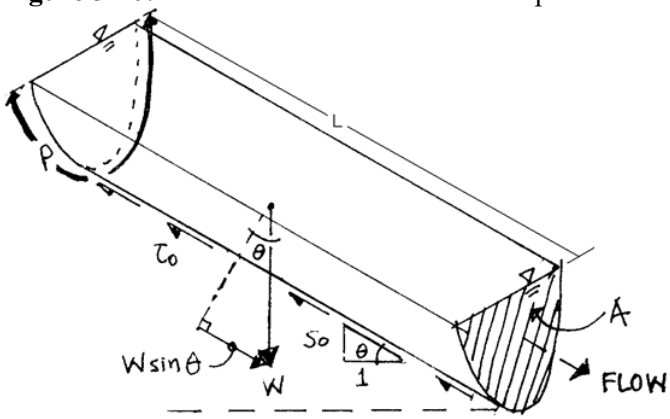
Figure 3-10: Schematic of uniform flow in open channels.



C. Because the flow depth and the cross-sectional shapes are the same at both ends of the flow element shown, the pressure forces acting at the upstream and downstream cross-sections of the flow element will cancel each other. The remaining driving force will be the component of the weight of the flow element parallel to the channel bed. This force is given by:

$$F_D = Wsin(\theta_0) = \omega ALsin(\theta_0) = \omega ALS_0$$

where A is the cross-sectional area of the flow.

D. Opposing this driving force is an opposite force due to friction on the channel walls, also known as the shear force. In figure 3-10, τ_o represents the bed shear stress (assumed uniform through the channel), and the total shear force is given by:

$$F_0 = \tau_0 LP$$

where P, the *wetted perimeter*, is the length of the channel cross-section in contact with the water.

E. For a uniform flow to occur, the driving force (weight component) and the shear force must be in equilibrium, i.e., $F_S = F_D$, or:

$$\tau_0 LP = \omega ALS_0$$

F. The bed shear stress is, therefore, given by:

$$\tau_o = \omega \frac{A}{P} S_o = \omega RS \qquad \text{(eq. 3-28)}$$

where R, the *hydraulic radius*, has been introduced:

$$R = A/P \qquad \text{(eq. 3-29)}$$

G. Example – Shear stress in uniform open-channel flow

(1) Calculate the shear stress on the bed of an open channel with a hydraulic radius, $R = 0.75\ ft$, laid on a slope, $S_o = 0.001$. Use $\omega = 62.4$ lb/ft^3 for the specific weight of water.

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(2) Solution:

(i) The shear stress is calculated as:

$$\tau_o = \omega RS = \left(62.4\frac{lb}{ft^3}\right)(0.75ft)(0.001) = 0.0468\frac{lb}{ft^2}$$

$$= \frac{0.0468\frac{lb}{ft^2}}{144\frac{in^2}{ft^2}} = 0.000325\,psi$$

(ii) The shear stress τ_o can be written in terms of the mean flow velocity V, the density of water ρ, and a dimensionless "drag" coefficient C_D:

$$\tau_o = \frac{1}{2}C_D\rho V^2$$

(iii) Substituting this result into equation 3-28 for τ_o, and using $\omega = \rho \cdot g$:

$$\frac{1}{2}C_D\rho V^2 = \rho g RS$$

(iv) From which it follows that the velocity can be calculated as:

$$V = \sqrt{\frac{2g}{C_D}}\sqrt{RS_o} = C\sqrt{RS_o} \qquad \text{(eq. 3-30)}$$

(3) This result is known as Chezy's equation, and the coefficient C is referred to as Chezy's coefficient. Typical values of the Chezy coefficient range between 80 and 140. For example, the value C = 120 is typically used for concrete.

H. Example – Velocity calculation in open-channel flow using Chezy's equation

(1) Using the Chezy equation with C = 120, calculate the flow velocity in an open channel with a hydraulic radius $R = 0.75\,ft$, laid on a slope $S_o = 0.001$.

(2) Solution - The velocity in the open channel flow is calculated as:

$$V = C\sqrt{RS_o} = 120\sqrt{(0.75)(0.001)} = 3.29fps$$

I. While Chezy's equation is dimensionally sound, a different equation, Manning's equation, has been used for more than a century for solving practical problems of uniform flow in open channels. Manning's equation has become the most widely used uniform flow equation, with many references available for the selection of a Manning's coefficient. For historical notes on the development of both the Chezy's and Manning's equations, refer to Chow (1959). Manning's equation is presented in section 634.0306.

634.0305 Geometric Characteristics of Prismatic Channels

A. In general, we are interested in calculating the following geometric characteristics of open channel cross-sections:

(1) The cross-sectional area, A (ft^2)
(2) The wetted perimeter, P (ft)
(3) The top width of the section, T (ft)
(4) The hydraulic radius, R = A/P (ft)
(5) The hydraulic depth, D_h = A/T (ft)

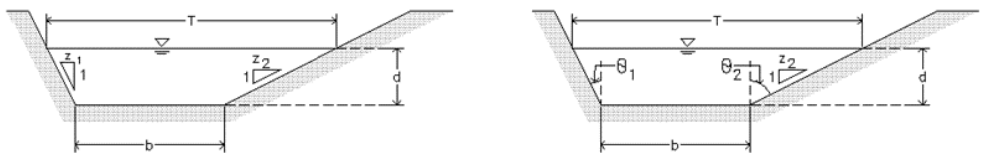
B. The cross-sectional area, wetted perimeter, and hydraulic radius have been defined in section 634.0301. The top-width of the section (T) is the length of the water surface in the cross-section, whereas the hydraulic depth (D_h) is the depth of an equivalent rectangular cross-section of the same area and width equal to the top-width (T).

C. The calculation of geometric characteristics for open channels of regular-shaped cross-sections is straightforward as indicated next.

D. Non-symmetric trapezoidal channel

(1) Figure 3-11 shows a non-symmetric trapezoidal channel cross-section.

Figure 3-11: Non-symmetric trapezoidal channel



(2) Given the bottom width of the cross-section, b, side slopes z_1 and z_2, and the flow depth, d, the geometric characteristics of the non-symmetric trapezoidal cross-section are:

$$A = \frac{1}{2}(2b + (z_1 + z_2)d)b, P$$

$$= b + d(\sqrt{1 + z_1^2} + \sqrt{1 + z_2^2}, T$$

$$= b + (z_1 + z_2)d \qquad \text{(eq. 3-31)}$$

(3) In this case, z_1 and z_2 represent the dimensionless side slope as horizontal displacement per unit vertical rise. For example, on the left-hand side the channel bank slopes z_1 ft horizontal per each vertical foot. This is also represented as z_1H:*1*V (z_1 ft horizontal to 1 ft vertical). The right-hand-side slope would be represented as z_2H:*1*V. If the angles θ_1 and θ_2 are given, the side slopes can be calculated as:

$$z_1 = \tan(\theta_1) \qquad z_2 = \tan(\theta_2) \qquad \text{(eq. 3-32)}$$

(4) Example – Geometric characteristics of a non-symmetric trapezoidal cross-section

(i) A non-symmetric trapezoidal channel flowing at a depth d = 1.5 ft, has a bottom width b = 6.5 ft, and side slopes laid on angles, $\theta_1 = 30^o$ and $\theta_2 = 60^o$, with respect to a vertical line. Calculate the side slopes and the geometric characteristics for this cross-section.

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(ii) Solution:

- First, the side slopes, z_1 and z_2 are calculated as:

$$z_1 = \tan(\theta_1) = \tan(30°) = 0.5773$$

$$z_2 = \tan(\theta_2) = \tan(60°) = 1.7320$$

- The geometric characteristics of this cross-section are calculated as follows:

$$A = (1/2)[2b + (z_1 + z_d)d]$$

$$= (1/2)[2 \times 6.5ft + (0.5773 + 1.7321)1.5ft]1.5ft = 12.35ft^2$$

$$P = b + d(\sqrt{1 + z_1^2} + \sqrt{1 + z_2^2}$$

$$= 6.5ft + 1.5\sqrt{1 + 0.5773^2} + \sqrt{1 + 1.7320^2} = 11.23ft$$

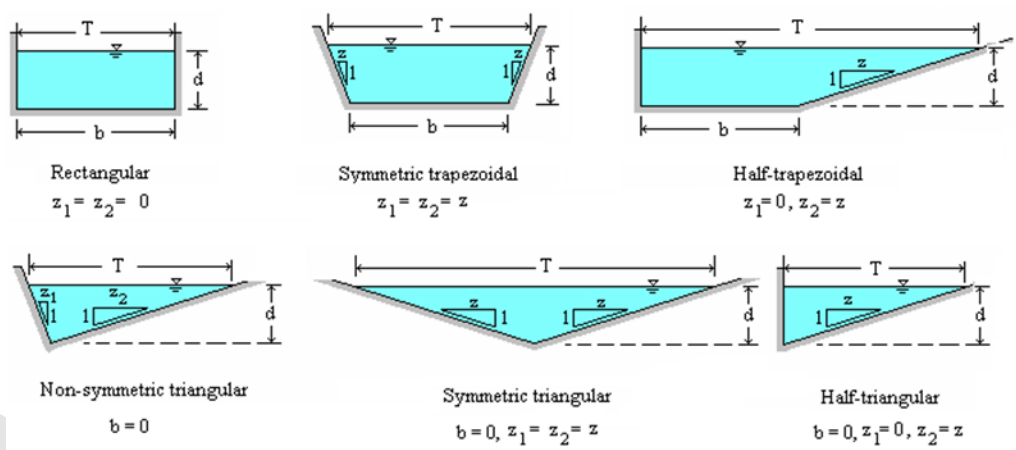
$$R = A/P = (12.35\ ft^2)/(11.23ft) = 1.10\ ft$$

$$T = b + (z_1 + z_2)d = 6.5\ ft + (0.5773 + 1.7320) \times 1.5ft = 9.96\ ft$$

$$D_h = A/T = \frac{12.35\ ft^2}{9.96\ ft} = 1.24\ ft$$

(5) Figure 3-12 shows different cross-sections that can be derived from the non-symmetric trapezoidal cross-section. The geometric elements of some of them are presented in detail in figure 3-12.

Figure 3-12: Channel cross-sections that can be derived from a non-symmetric trapezoidal cross- section.



E. Wide channels

(1) Channels of approximately rectangular shape that are much wider than they are deep, say, b/d > 10, are referred to as wide channels. For such channels, the hydraulic radius is approximately equal to the channel depth and to the hydraulic depth:

$$R \approx d \approx D_h \qquad \text{(eq. 3-33)}$$

(2) Example – Hydraulic radius for wide rectangular channel

(i) For a rectangular channel with b = 20 ft and d = 1.5 ft, calculate the hydraulic radius using the full formula and the approximation for a wide channel.

(ii) The hydraulic radius using the full definition for a rectangular channel is:

$$R_R = \frac{bd}{b + 2d} = \frac{20ft \times 1.5ft}{20ft + 2 \times 1.5ft} = 1.3\ ft$$

(iii) Using the wide-channel approximation, RW = d = 1.5 ft, results in about a 15% difference.

F. Symmetric trapezoidal channel

(1) Many constructed trapezoidal channels are symmetric, thus, $z_1 = z_2 = z$, and the geometric characteristics are calculated as:

$$A = (b + zd)d, \quad P = b + 2d\sqrt{1 + z^2}, \qquad T = b + 2xd$$

(2) Example – Geometric characteristics of a symmetric trapezoidal cross-section in open channels

(i) A symmetric trapezoidal open channel with a bottom width b = 2.5 ft and side slope 1.5 H:1 V (z = 1.5) flows with a water depth d = 0.75 ft. Determine the geometric characteristics for this open channel. Also, determine the angle θ that the channel banks make with a vertical line.

(ii) The solution is given by:

$$A = (b + zd)d = (2.5ft + 1.5 \times 0.75ft) \times 0.75ft = 2.72ft^2$$

$$P = b + 2d\sqrt{1 + z^2} = 2.5ft + 2(0.75)\sqrt{1 + 1.5^2} = 5.20ft$$

$$R = A/P = \frac{2.72\ ft^2}{5.20\ ft} = 0.52ft$$

$$T = b + 2zd = (2.5\ ft) + (2)(1.5)(0.75\ ft) = 4.75\ ft\ D_h = A/T$$

$$= (2.75ft^2)/(4.75\ ft) = 0.57\ ft$$

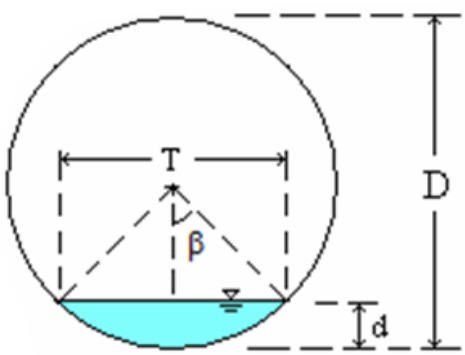
(iii) To calculate the angle θ that the channel banks make with a vertical line, use z = tan (θ), or:

$$\theta = tan^{-1}(z) = tan^{-1}(1.5) = 56.31°$$

G. Circular channel

(1) A circular open channel results when water flows in a pipe or circular conduit with a free surface, as shown in figure 3-13.

Figure 3-13: Circular cross-section in open channel flow.



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(2) For such a cross-section, the diameter D and the flow depth d (with d<D) are typically known. To calculate the geometric characteristics, the central angle β (in radians) is calculated as:

$$\beta = \cos^{-1}(1 - 2\frac{d}{D}) \qquad \text{(eq. 3-34)}$$

(3) The geometric characteristics are calculated as:

$$A = \frac{D^2}{4}(\beta - \sin(\beta)\cos(\beta) \qquad \text{(eq. 3-35)}$$

$$P = \beta D \qquad \text{(eq. 3-36)}$$

$$T = D \cdot \sin(\beta) \qquad \text{(eq. 3-37)}$$

Note: A full circle is composed of 360° or 2π (= 6.2832) radians. The relationship between angles in radians (θ^r) and angles in degrees (θ^o) is the same as the ratio of 2π:360 = π:180, or:

$$\frac{\theta^r}{\theta^o} = \frac{\pi}{180} = 0.01745 \qquad \text{(eq. 3-38)}$$

$$\frac{\theta^o}{\theta^r} = \frac{180}{\pi} = 57.29 \qquad \text{(eq. 3-39)}$$

(4) Example – Geometric characteristics of a circular cross-section in open channels

(i) A pipeline 2.5-ft in diameter is flowing at a depth of 6 inches. Determine the geometric characteristics of this cross-section.

(ii) Solution:

- For D = 2.5 ft and d = 6 in = 6/12 = 0.5 ft, the central angle β is calculated as follows:

$$\beta = \cos^{-1}(1 - 2\frac{d}{D}) = \cos^{-1}(1 - 2\frac{0.5}{2.5}) = \cos^{-1}(0.6) = 0.9273^r$$

- The corresponding geometric characteristics are as follows:

$$A = \frac{D^2}{4}(\beta - \sin(\beta)\cos(\beta))$$

$$= \frac{2.5^2}{4}(0.9273 - (\sin(0.9723)\cos(0.9723)) = 0.70\ ft$$

$$P = \beta D = 0.9723 \times 2.5\ ft = 2.32\ ft$$

$$R = A/P = (0.70\ ft^2)/(2.32\ t) = 0.30\ ft$$

$$T = D\sin(\beta) = 2.5\ ft\ (\sin(0.9273)) = 2\ ft$$

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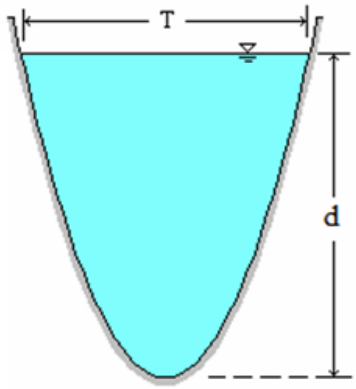
H. Parabolic channel

(1) The parabolic cross-section is used to approximate some natural cross-sections. Some waterways are also constructed of parabolic shape. A parabolic cross-section is characterized by its top width T and its depth d, as shown in the figure below.

Figure 3-14: Parabolic cross-section



(2) The geometric characteristics of this cross section are calculated as follows:

$$A = \frac{2}{3} dT \qquad \text{(eq. 3-40)}$$

$$P = \frac{T}{2}\sqrt{1 + \left(\frac{4d}{T}\right)^2} + \frac{T^2}{8d} * ln\left(\frac{4d}{T} + \sqrt{1 + \left(\frac{4d}{T}\right)^2}\right) \qquad \text{(eq. 3-41)}$$

(3) Example – Geometric characteristics of a parabolic cross-section

(i) A parabolic waterway has a depth d = 2 ft and a top width T = 50 ft. Determine the geometric characteristics of the cross-section.

$$A = \frac{2}{3} dT = \frac{2}{3}(2ft)(50ft) = 66.67ft^2$$

$$P = \frac{T}{2}\sqrt{1 + \left(\frac{4d}{T}\right)^2} + \frac{T^2}{8d} * ln\left(\frac{4d}{T} + \sqrt{1 + \left(\frac{4d}{T}\right)^2}\right)$$

$$= \frac{50ft}{2}(1.013) + \frac{50ft^2}{8(2ft)} * \ln(0.16 + 1.013)$$

$$P = 50.26ft$$

$$R = \frac{A}{P} = \frac{66.67ft^2}{50.26ft} = 1.33ft$$

$$D_h = \frac{A}{T} = \frac{66.67ft^2}{50ft} = 1.33ft$$

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634.0306 Manning's Equation

A. The widely-used Manning's equation was introduced earlier as an example of a non-dimensionally homogeneous equation. The equation, named after the Irish engineer who proposed it in the late 1800's, is given by:

$$V = \frac{C_u}{n} R^{2/3} S_0^{1/2} \qquad \text{(eq. 3-42)}$$

where V is the flow velocity, C_u is a constant that depends on the system of units used ($C_u = 1.0$ in the SI, and $C_u = 1.486$ in the *ES*), is the hydraulic radius, S_O is the channel bed longitudinal slope, and n is the Manning's resistance coefficient. Manning's *n-values* are available from a variety of sources; some values are provided in the next section, Table 2. Manning's resistance coefficients for open channel flow.

B. Many times, it is preferable to write the Manning's equation in terms of the water discharge Q = VA:

$$Q = VA = \frac{C_u}{n} A R^{2/3} S_0^{1/2} \qquad \text{(eq. 3-43)}$$

C. Since the hydraulic radius is defined as R = A/P, the Manning's equation can also be written as:

$$Q = \frac{C_u}{n} \frac{A^{5/3}}{P^{2/3}} S_o^{1/2} \qquad \text{(eq. 3-44)}$$

D. The values of A and P depend on the cross-sectional geometry and the flow depth. If A and P are known, calculation of the discharge Q (see above), Manning's n, or the bed slope S_o, is straightforward:

$$n = \frac{C_u}{Q} \frac{A^{5/3}}{P^{2/3}} S_o^{1/2} \qquad \text{(eq. 3-45)}$$

$$S_o = \left(\frac{Qn}{C_u}\right)^2 \frac{P^{4/3}}{A^{10/3}} \qquad \text{(eq.3-46)}$$

E. The *USDA-NRCS Hydraulics Formula* program allows for the calculation of the discharge *Q* for a variety of cross-sections. Examples are provided below. (See section 634.0307).

F. Calculations involving the geometric parameters (e.g., flow depth, channel width) are more complicated because the geometric characteristics A and P are raised to fractional powers (5/3, 2/3), and because the geometric characteristics are a function of the parameters. Solving the nonlinear equations for any of the geometric parameters typically requires an iterative procedure. The calculations are best done with a spreadsheet, which uses a numerical analysis method to automate the iterative procedure.

G. Although the Darcy-Weisbach equation was originally developed for pipe flow (see section 4.3, Darcy-Weisbach Equation and Friction Factor), it has been adapted for open channel flow. Refer to NEH 654.0609(d) for information on applying the Darcy- Weisbach equation to open channel flow.

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H. Manning's Resistance Coefficient

(1) Manning's resistance coefficient, *n* (see equation 42), is assumed to be a dimensionless number in modern-day practice so that the same n-value can be used in both the English and International Systems of units. The n-value depends on many factors including:
   (i) Roughness of the channel lining
   (ii) Changes in channel alignment
   (iii) Changes in cross-section geometry
   (iv) Presence of obstructions
   (v) Presence of vegetation
   (vi) Water depth

(2) The selection of Manning's n depends heavily on the practitioner's experience. Photos and descriptions of channels and floodplains for which n-values have been calculated, based on measured discharges and high-water marks, are useful (See Barnes, 1967, or Fasken, 1963). Photo galleries of n-values have been established online.

(3) When an n-value must be selected for a channel outside a practitioner's experience, a more regimented approach is sometimes helpful (See Cowan, 1956; this method is also presented in Chow, 1959 and Fasken, 1963). In Cowan's approach, n-values are selected and summed for five channel conditions:
   (i) Bed material
   (ii) Degree of surface irregularity
   (iii) Variation of channel cross section
   (iv) Effect of obstructions
   (v) Vegetation

(4) The summed n-value may be further adjusted based on the degree of channel meandering.

(5) After selection, n-values should be calibrated with any available high-water marks and gaged flow data. In the calibration process, Manning's n-values should be reasonably adjusted to match observed water surface profiles.

(6) The retardance potential of a grass-lined open channel, including vegetated earthen spillways, can be better evaluated with a retardance curve index rather than Manning's n (See Temple, et.al., 1987). The retardance curve index is based on the grass stem length and stem density.

(7) The following table (figure 3-15) shows typical values of Manning's resistance coefficients for constructed channels lined with different materials, as well as for natural streams, excavated earth channels, and floodplains. The values were compiled from many of the following references: Barnes (1967), Cowan (1956), Chaudhry (1993), Chow (1959), Fasken (1963), French (1985), Mays (1999), Munson et al. (1998), Streeter et. al. (1998), and Temple et al. (1987).

(8) Also see NEH654.0609C for additional discussion and examples of determining Manning's resistance coefficient.

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Figure 3-15: Manning's resistance coefficients for open channel flow

Surface Lining	Manning's n-value	Surface Lining	Manning's n-value
Concrete, finished	0.012	Clean, straight, natural streams	0.030
Concrete, unfinished	0.014	*Excavated earth channels*	
Gravel	0.029	Clean	0.022
Earth	0.025	With gravel	0.025
Wood	0.012	With brush	0.030
Clay tile	0.014	With cobbles, stones	0.035
Brickwork	0.015	*Floodplains*	
Asphalt	0.016	Pasture, farmland	0.035
Masonry	0.025	Light brush	0.050
Smooth steel	0.012	Heavy brush	0.075
Corrugated metal	0.022	Trees	0.150

634.0307 Calculations in Uniform Flow

A. Calculation of discharge using Manning's formula is straightforward for a variety of cross-sections by using the *USDA-NRCS Hydraulics Formula* program. Three examples are illustrated below. An example of normal depth calculation is also shown.

B. Example – Trapezoidal channel solution using *USDA-NRCS Hydraulics Formula* program

(1) A trapezoidal channel of width $b = 6.5\ ft$ and side slope $z = 1.5$ flows at a depth $d = 3.2\ ft$ on a slope $S_o = 0.0005$. Use a Manning's $n = 0.018$, and calculate the discharge.

(2) Solution:

(i) The solution is presented below in figure 3-16 using the NRCS Hydraulics Formula program for a trapezoidal cross-section:

(ii) The result is Q = 106.12 cfs, with a velocity V = 2.93 fps. The program also shows the critical depth which is developed in Section 3.9.

(iii) Note that a trapezoidal cross-section with side slope $z = 0$ represents a rectangular cross section. Also, a trapezoidal cross-section with bottom width $b = 0$ represents a triangular cross section.

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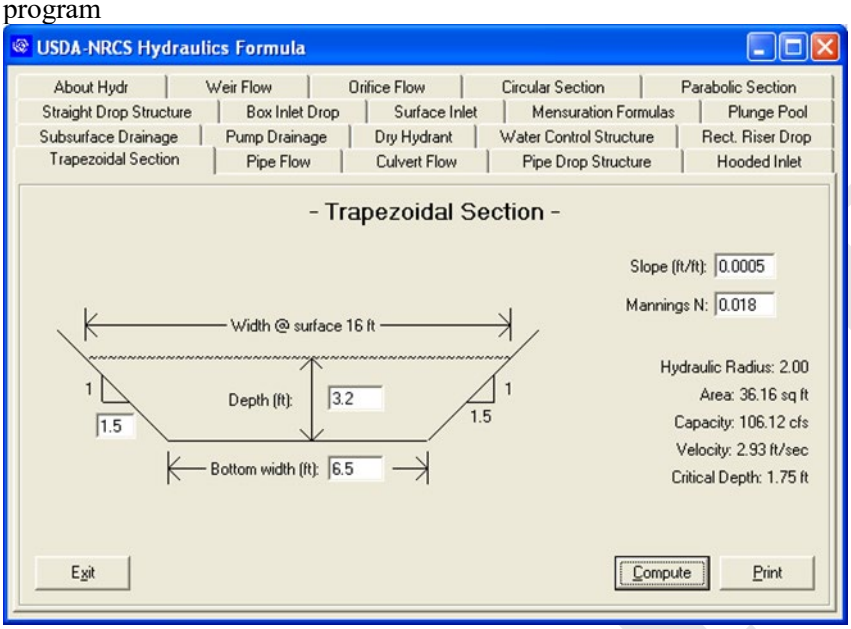
Figure 3-16: Trapezoidal Section solution using the NRCS Hydraulics Formula program



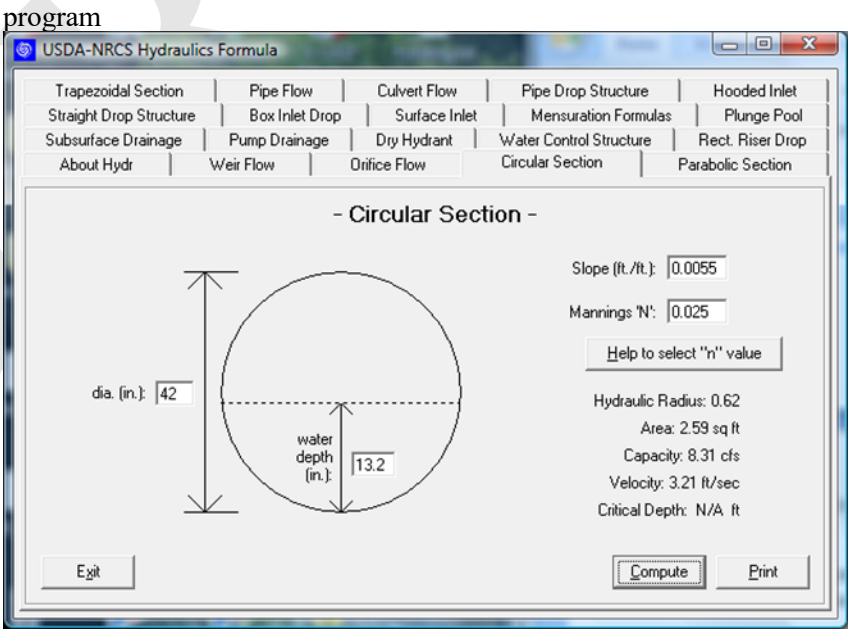
C. Example – Circular channel solution using the NRCS Hydraulics Formula program

(1) A circular channel of diameter $D = 3.5\ ft = 3.5\ ft \times 12\ in = 42\ in$ flows at a depth $d = 1.1\ ft = 13.2\ in$ on a slope $S_o = 0.0055$. Using a Manning's $n = 0.025$, calculate the discharge.

(2) Solution:

(i) The solution is presented below (figure 3-17) using the NRCS Hydraulics Formula program for a circular cross-section:

Figure 3-17: Circular channel solution using the NRCS Hydraulics Formula program



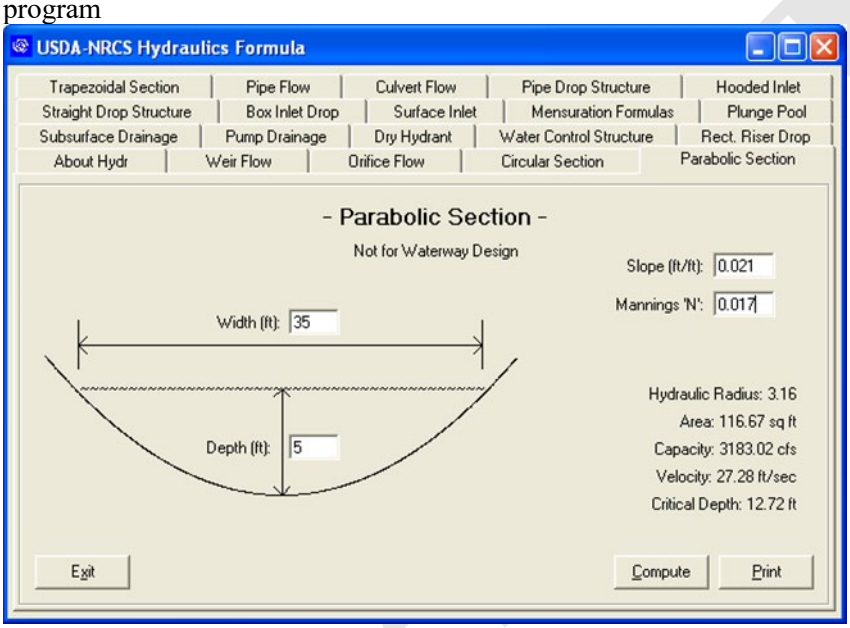
D. Example – Parabolic channel solution using the NRCS Hydraulics Formula program

(1) A parabolic channel flows at a depth d = 5 ft and top width T = 35 ft on a slope So = 0.021. Using a Manning's n = 0.017, calculate the discharge.

(2) Solution:

(i) The solution is presented below (figure 3-18) using the NRCS Hydraulics Formula program for a parabolic cross-section:

Figure 3-18: Parabolic channel solution using the NRCS Hydraulics Formula program



E. Example – Normal depth in a wide channel

(1) Consider a channel with a Manning's $n = 0.015$ laid on a slope $S_o = 0.00461$. Suppose that the channel has a width $b = 30\,ft$ and it carries a flow $Q = 100\,cfs$. Determine the normal depth, d_o.

(2) Solution:

(i) A wide channel is an approximately rectangular channel whose width b is at least 10 times larger than its depth, i.e., $b/d>10$. For wide channels, the hydraulic radius can be approximated by the flow depth (equation 3-33). The Manning's equation for this case can be written as:

$$Q = bd_o \frac{C_u}{n} d_o^{2/3} \sqrt{S_o}$$

(ii) Solving for the normal depth:

$$d_o = \left(\frac{Q_n}{C_u b\sqrt{S_o}}\right)^{3/5} = \left(\frac{100(0.015)}{1.486(30)\sqrt{0.00461}}\right)^{3/5} = 0.66ft$$

634.0308 Specific Energy in Open Channels

A. The specific energy in an open channel is the sum of energy heads referred to the channel bed, i.e., the flow depth added to the velocity head:

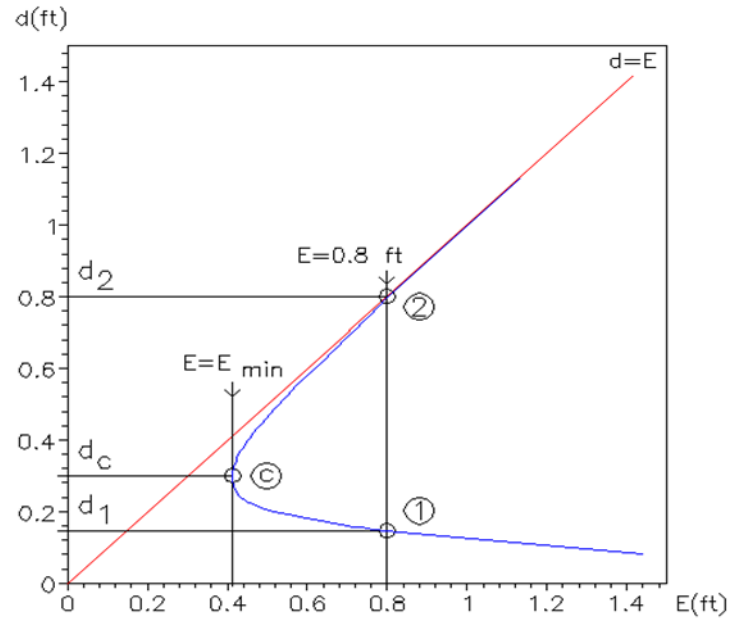
$$E = d + \frac{V^2}{2g} \qquad \text{(eq. 3-47)}$$

B. In terms of the discharge Q, with $V = Q/A$, and A = cross-sectional area, the specific energy is written as:

$$E = d + \frac{Q^2}{2gA^2} \qquad \text{(eq. 3-48)}$$

C. A *specific energy diagram* is a plot of the depth of flow, d, versus the specific energy, E. Figure 3-19 shows the specific energy diagram corresponding to a symmetric trapezoidal open channel of bottom $b = 2\ ft$ and side slopes $z = 1.5$ carrying a flow $Q = 20\ cfs$.

Figure 3- 19: Specific energy curve for a trapezoidal open channel.



D. In figure 3-19, the vertical axis represents the depth of flow and the horizontal axis the specific energy. Notice that the curve approaches the line $d = E$ asymptotically as the depth of flow increases. Also, the lower branch of the curve approaches the value $d = 0$ as the specific energy E increases. The shape of the curve shown is typical of specific energy diagrams in open channel flow. A vertical line corresponding to a specific energy $E = 0.8\ ft$ is shown. Notice that this line intersects the specific energy curve at two points (1) and (2), indicating that there are two possible flow depths that would produce the same specific energy. These are referred to as *alternate depths*, d_1 and d_2. The specific energy diagram of figure 3-19 also shows that there is a point, (c), where the specific energy is a minimum, $E = E_{min}$, for a given cross-section and discharge. This condition is known as *critical flow* and the corresponding flow depth is referred to as *critical depth*. The subject of critical flow is discussed in the following section 634.0309.

E. Example – Specific energy diagram for a rectangular channel cross-section

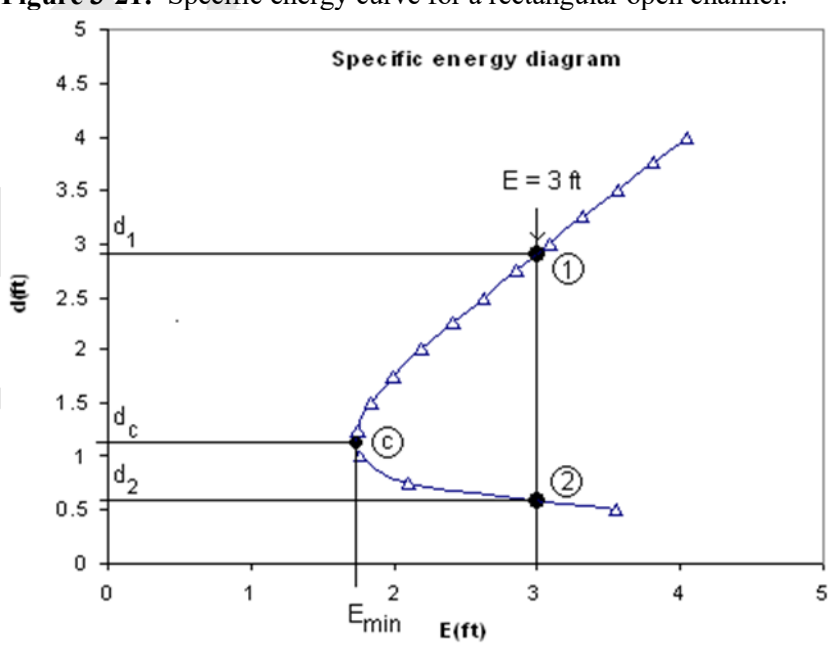
(1) A diagram such as that of Figure 3-19 or 3-21 can be used to determine, graphically, the alternate depths of flow in a given channel. For a rectangular channel of width *b* = *5 ft* and carrying a flow of 35 *cfs*, a table of values of the specific energy may be produced as shown in figure 3-20:

Figure 3-20: Estimating specific energy values

d(ft)	A(ft^2)	V(fps)	V^2/2g (ft)	E(ft)
0.50	2.50	14.00	3.04	3.54
0.75	3.75	9.33	1.35	2.10
1.00	5.00	7.00	0.76	1.76
1.10	5.50	6.36	0.63	1.73
1.25	6.25	5.60	0.49	1.74
1.50	7.50	4.67	0.34	1.84
1.75	8.75	4.00	0.25	2.00
2.00	10.00	3.50	0.19	2.19
2.25	11.25	3.11	0.15	2.40
2.50	12.50	2.80	0.12	2.62
2.75	13.75	2.55	0.10	2.85
3.00	15.00	2.33	0.08	3.08
3.25	16.25	2.15	0.07	3.32
3.50	17.50	2.00	0.06	3.56
3.75	18.75	1.87	0.05	3.80
4.00	20.00	1.75	0.05	4.05

(2) The resulting specific energy diagram is shown below.

Figure 3-21: Specific energy curve for a rectangular open channel.



(3) To find the alternate depths corresponding to a specific energy $E = 3\ ft$ for this case, draw a vertical line at that value of E and find the values of d where the line $E = 3\ ft$ intercepts the specific energy diagram. From the figure, the values for the alternate depths are estimated as $d_1 = 2.9\ ft$ and $d_2 = 0.6\ ft$. The specific energy diagram also reveals that the critical depth is approximately $d_c = 1.1\ ft$ corresponding to a specific energy $E_{min} = 1.7\ ft$.

634.0309 Critical Flow

A. The specific energy diagrams shown in figure 3-19 and figure 3-21 indicate that there is a point (c) where the specific energy is minimal ($E = E_{min}$) for a given cross-section and discharge. This point represents a condition known as *critical flow*, and the corresponding depth of flow is known as the *critical depth*. Critical flow is important in the analysis of open channel flow because it represents conditions of minimal energy. Critical flow can be used for the practical measurement of fluid flows, as in the case of broad-crested weirs or Parshall flumes (see section 634.0502 – *Measurements in Open Channels*).

C. To determine an equation that describes critical flow conditions, one can start from the definition of the specific energy in terms of the discharge, equation 3-48, written as:

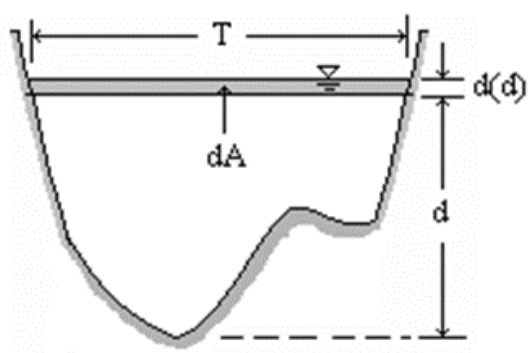
$$E(d) = d + \frac{Q^2}{2g\big(A(d)\big)^2} = d + \frac{Q^2}{2g}(A(d))^{-2}$$

D. Specific energy E, as well as, the area A of the channel's cross-section, is a function of the flow depth d. To find the conditions of minimum energy, take the derivative of $E(d)$ with respect to d and set it equal to zero:

$$\frac{dE}{d(d)} = 1 + (-2)\frac{Q^2}{2g}\big(A(d)\big)^{-3}\frac{dA}{d(d)} = 1 - \frac{Q^2}{gA^3}\frac{dA}{d(d)} = 0$$

E. The derivative $dA/d\ (d)$ can be shown to be equal to the top width of the cross-section, as illustrated in the figure below.

Figure 3-22: Relationship of cross-sectional area dA, flow depth $d(d)$, and top width T .



F. In the figure above, the increment in area, *dA*, due to a small increment in depth *d(d)*, is *dA = T d(d)*; thus, *dA/d(d) = T*, and a critical flow equation can be written as:

$$\frac{Q^2 T_c}{g A_c^3} = 1 \qquad \text{(eq. 3-49)}$$

where the subscript, *c* is added to emphasize critical flow conditions.

G. Re-writing to incorporate the *critical velocity,* $V_c = Q/A_c$:

$$\frac{Q^2 T_c}{g A_c^3} = \frac{(Q/A_c)^2 T_c}{g A_c^3} = \frac{V_c^2}{g(A_c/T_c)} = 1$$

H. The ratio *A/T* was defined earlier as the *hydraulic depth* ($D_h = A/T$), thus, an equation for critical velocity can be written as:

$$\frac{V_c^2}{g(D_h)_d} = 1 \qquad \text{(eq. 3-50)}$$

I. The left-hand side of the above equation is the square of the *Froude number*. This dimensionless number is relevant in open channel flow, and is defined, in general, as:

$$F_r = \frac{V}{\sqrt{g(D_h)}} \qquad \text{(eq. 3-51)}$$

J. Thus, the conditions of critical flow require that the Froude number be equal to 1. Another result that can be derived from the critical conditions equation (equation 3-50) is that the velocity head is half of the hydraulic depth:

$$\frac{V^2}{2g} = \frac{D_h}{2} \qquad \text{(eq. 3-52)}$$

K. Critical depth may be calculated with the *USDA-NRCS Hydraulics Formula(s)* program as shown in the examples Trapezoidal channel solution using USDA-NRCS Hydraulics Formula Program and Parabolic channel solution using USDA-NRCS Hydraulics Formula Program, provided in section 634.0307.

L. Calculations of critical depth for prismatic open channels often involve solving nonlinear equations, which requires an iterative procedure. The calculations are best done with a spreadsheet, which uses a numerical analysis method to automate the iterative procedure.

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634.0310 Flow Types

A. For any energy larger than the minimum, $E > E_{min}$, there are two alternate depths of flow, as indicated in the specific energy diagrams of Figure 3-19 and Figure 3-21. One of the depths, d_1 is larger than the critical depth ($d_1>d_c$), corresponding to a *subcritical flow*, while the second one, d_2, is smaller than the critical depth, and corresponds to a *supercritical flow*.

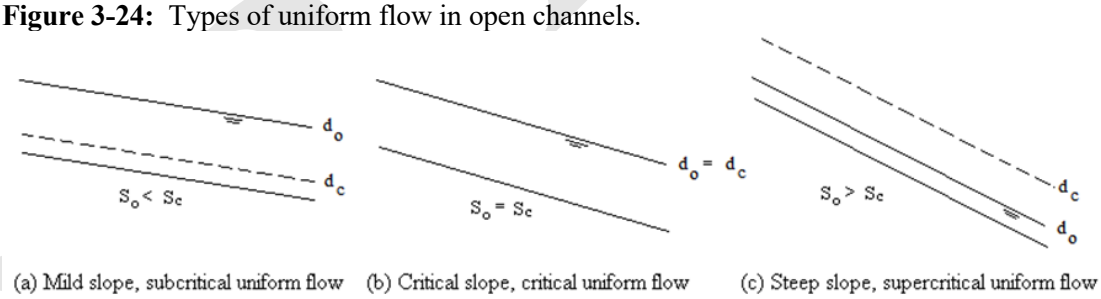
B. It can be shown by calculation with the Froude number equation that for *subcritical flow* the Froude number is less than 1 ($Fr<1$), while for *supercritical flow* the Froude number is greater than 1 ($Fr>1$).

C. If an open channel is laid on a slope S_o such that the normal depth of flow d_o is equal to the critical depth of flow d_c ($d_o=d_c$) for a given discharge Q, then the channel bed slope S_o is said to be the *critical slope* for that flow, i.e., $S_o = S_c$. The critical slope for a channel can be found by replacing $d_o = d_c$ in the Manning's equation. If the channel bed slope is smaller than the critical slope ($S_o<S_c$), the normal depth of flow is larger than the critical depth ($d_o>d_c$), and the channel is said to have a *mild slope*. On the other hand, if the channel bed slope is larger than the critical slope ($S_o>S_c$), the normal depth of flow is smaller than the critical depth ($d_o<d_c$), and the channel is said to have a *steep slope*. The different types of uniform flow possible in an open channel, and their corresponding slopes, are summarized in figure 3-23 and figure 3-24.

Figure 3-23: Types of uniform flow in open channels.

Type of flow	Flow depth	Slope	Type of slope	Froude Number
Subcritical	$d_o > d_c$	$S_o < S_c$	Mild	$Fr < 1$
Critical	$d_o = d_c$	$S_o = S_c$	Critical	$Fr = 1$
Supercritical	$d_o < d_c$	$S_o > S_c$	Steep	$Fr > 1$

Figure 3-24: Types of uniform flow in open channels.



D. It's worth noting that the celerity, or wave speed, c, of a surface wave in a shallow open channel of depth d is the same as the critical velocity V_c. If the flow is *subcritical*, $c>V$, and wave fronts from a surface disturbance will travel downstream at a speed $c+V>0$, while travelling upstream at a speed $c-V>0$. Here, V is the flow velocity. Thus, *surface disturbances in subcritical flow can travel both upstream and downstream from their point of origin*. On the other hand, if the flow in *supercritical*, $V>c$, the velocity of surface disturbances travelling downstream is still positive $c+V>0$, but that of disturbances traveling upstream is negative $c-V<0$. This last result indicates that these disturbances cannot travel upstream. Thus, surface disturbances in supercritical flow can only travel downstream from points of origin.

634.0311 Critical Flow in a Rectangular Channel

A. In a rectangular channel, the area and the top width are $A = bd$ and $T=b$. Thus, equation 3-49 produces the result:

$$\frac{Q^2 T_c}{g A_c^3} = \frac{Q^2 b}{g b^3 d_c^3} = \frac{Q/b^2}{g d_c^3} = 1$$

B. Introducing the unit discharge (or discharge per unit width of channel):

$$q = \frac{Q}{b} \qquad \text{(eq. 3-53)}$$

C. Substituting the *unit discharge* into the equation above, and solving for the critical depth:

$$d_c = \sqrt[3]{\frac{q^2}{g}} \qquad \text{(eq. 3-54)}$$

D. Example – Critical flow depth in a rectangular channel

(1) Consider a rectangular channel of width, $b = 30\ ft$ and carrying a flow $Q = 100\ cfs$. Determine the critical depth.

q = Q/b = 100 cfs/30 ft = 3.33 ft^2/s

$$d_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{3.33^2}{g}} = 0.70 ft$$

(2) Since for a rectangular channel the hydraulic depth is the same as the flow depth ($D_h = d$), the critical specific energy (minimum energy) is found to be equal to 3/2 of the critical depth (combining equations 3-47 and 3-52):

$$E_c = E_{min} = \frac{3}{2} d_c \qquad \text{(eq. 3-55)}$$

(3) To find the critical slope, we can use the Manning's equation (equation 3-44), with $d = d_c$, $A = bd_c$, $P = b + 2d_c$, and $S_o = S_c$:

$$Q = \frac{C_u}{n} \frac{(b d_c)^{5/3}}{(b + 2d_c)^{2/3}} S_o^{1/2}$$

(4) Solving for critical slope:

$$S_c = \left(\frac{Qn}{C_u}\right)^2 \frac{(b+2d_c)^{4/3}}{(b d_c)^{10/3}} \qquad \text{(eq. 3-56)}$$

E. Example – Critical slope in uniform open-channel flow with rectangular cross-section

(1) A rectangular channel of width $b = 4\ ft$, and Manning's $n = 0.012$, is laid on a slope $S_o = 0.00015$ and carries a discharge $Q = 20\ cfs$. (a) Determine the critical depth of flow. (b) Determine the critical slope. (c) What type of uniform flow is to be expected in this channel?

(2) First, determining the unit discharge:

$$\mathrm{q} = \frac{\mathrm{Q}}{\mathrm{b}} = \frac{20\ cfs}{4\ ft} = 5\ ft^2/s$$

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(3) The critical depth is calculated as:

$$d_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{(5\frac{ft^2}{s})^2}{32.2\frac{ft}{s^2}}} = 0.92ft$$

(4) The critical slope is:

$$S_c = \left(\frac{Qn}{C_u}\right)^2 \frac{(b+2d_c)^{4/3}}{(bd_c)^{10/3}} = \left(\frac{(20)(0.012)}{1.486}\right)^2 \frac{(4+2(0.91))^{4/3}}{(4(0.91)^{10/3}}$$

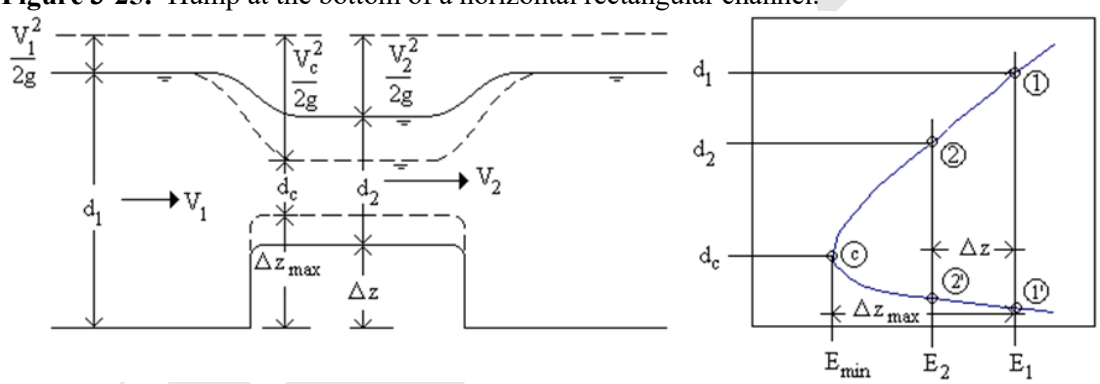
$$= 0.00368$$

(5) Since $S_o<S_c$, the channel slope is *mild*, and a *subcritical* uniform flow is to be expected in this channel.

634.0312 Obstacles in Open Channels

A. Consider a horizontal rectangular open channel that includes a hump of height Δz at the bottom as illustrated on the left-hand side of figure 3-25. The figure also shows the specific energy diagram for a given discharge Q in reference to the original channel bed.

Figure 3-25: Hump at the bottom of a horizontal rectangular channel.



B. Figure 3-25 shows the flow at a subcritical depth d_1 upstream of the hump, and at a subcritical depth d_2 over the hump. The figure also shows the energy heads in sections (1) and (2), assuming no energy losses over the hump. The energy equation written between sections (1) and (2) can be written as:

$$d_1 + \frac{V_1^2}{2g} = \Delta z + d_2 = \frac{V_2^2}{2g} \qquad \text{(eq. 3-57)}$$

C. Alternatively, this equation can be written as:

$$E_1 = \Delta z + E_2 \qquad \text{(eq. 3-58)}$$

D. The specific energy diagram shows the energy levels $E = E_1$ and $E = E_2$ separated by a distance Δz, as indicated by equation 3-58, with $E_1>E_2$. The depth of flow corresponding to energy level $E = E_2$ is $d_2 < d_1$, thus, the water surface over the hump drops as illustrated in figure 3-25.

E. If the flow depth is known at section (1), the specific energy E_1 can be calculated as:

$$E_1 = d_1 + \frac{V_1^2}{2g} = d_1 + \frac{Q^2}{2gb^2d_1^2}$$

F. Combining equations 3-57 and 3-58 and making use of the continuity equation gives an equation to determine the depth, d_2:

$$d_1 + \frac{Q^2}{2gb^2d_1^2} = E_1 - \Delta z \qquad \text{(eq. 3-59)}$$

G. Example – Change in channel bed elevation in a rectangular channel

(1) Refer to figure 3-25. A hump of height $\Delta z = 0.25\ ft$ is placed on a rectangular open channel of width $b = 5\ ft$ carrying a discharge $Q = 20\ cfs$. If the flow depth upstream of the hump is $d_1 = 2\ ft$, determine the flow depth atop the hump, d_2.

(2) Solution:

(i) Applying equation 3-59 and using several trials, two values were found for d_2, 1.73 ft and 0.42 ft. The iterative calculations may be facilitated by use of a spreadsheet. Since the flow for $d_1 = 2\ ft$ is subcritical, the correct value is $d_2 =$ 1.73 ft (subcritical flow).

(ii) To confirm $d_2 = 1.73$ ft, consider the specific energy diagram in figure 3-25. If the flow conditions upstream of the hump correspond to point (1) in the diagram, the flow conditions atop the hump would correspond to point (2). If the flow conditions upstream of the hump correspond to point (1') in the diagram, the flow over the hump would correspond to point (2'). In summary, if the flow upstream of the hump is subcritical, the flow above the hump should be subcritical (or at most, critical), while if the flow upstream of the hump is supercritical, the flow above the hump should be supercritical (or at most, critical).

(iii) To check whether the flow at sections (1) and (2) are subcritical or supercritical, one can calculate the Froude number for those sections:

$$F_{r1} = \frac{V_1}{\sqrt{gd_1}}; \quad F_{r2} = \frac{V_2}{\sqrt{gd_2}}$$

(3) The Froude numbers at sections (1) and (2) must both be either smaller than one ($Fr_1 < 1$ and $Fr_2 < 1$) for sub-critical flow, or larger than one ($Fr_1 > 1$ and $Fr_2 > 1$) for supercritical flow.

H. Example – Calculation of the Froude number in a rectangular channel

(1) For the previous example, Change in channel bed elevation in a rectangular channel, $d_1 = 2.00\ ft$, $Q = 20\ cfs$, $b = 5\ ft$, and $V_1 = Q/A = 2\ ft/s$. The Froude number at section (1) is:

$$F_{r1} = \frac{V_1}{\sqrt{gd_1}} = \frac{2.00\ {}^{ft}/_{s}}{\sqrt{32.2\ {}^{ft}/_{s^2}\,(2.00ft)}} = 0.249 < 1$$

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(2) Thus, the flow upstream of the hump for this example is subcritical. Two possible depths of flow atop the hump were found, $d_2 = 1.73\ ft$ and $d_2 = 0.42\ ft$. The Froude numbers corresponding to these depths are:

$$For\ d_2 = 1.73ft, \qquad F_{r2} = \frac{V_2}{\sqrt{gd_2}} = \frac{20cfs/(5ft)(1.73ft)}{\sqrt{(32.2\,{}^{ft}/_{s^2})(01.73ft)}} = 0.309 < 1$$

$$For\ d_2 = 0.42ft, \qquad F_{r2} = \frac{V_2}{\sqrt{gd_2}} = \frac{20cfs/(5ft)(0.42ft)}{\sqrt{(32.2\,{}^{ft}/_{s^2})(0.42ft)}} = 2.589 > 1$$

(3) Thus, $d_2 = 1.73\ ft$ corresponds to a subcritical flow atop the hump, while $d_2 = 0.42\ ft$, corresponds to supercritical flow atop the hump. Since the flow upstream was subcritical, the correct answer is $d_2 = 1.73\ ft$.

(4) There is a possibility that the hump height Δz takes the value Δz_{max} so that the flow conditions on top of the hump are critical. In the specific energy diagram of figure 3-25, critical flow conditions correspond to point (c), by making $\Delta z = \Delta z_{max}$, $E_2 = E_{min}$. If the flow atop the hump is critical, the depth of flow d_2 (above the hump) is the critical depth d_c, which, for a rectangular channel, can be calculated using equations 3-54 or 3-55.

(5) If critical flow conditions are achieved atop the hump, it may be possible to measure the depth of flow in that location ($d_2 = d_c$). The discharge can then be calculated from equations 3-53 and 3-54, combined and rearranged:

$$Q = b\sqrt{gd^3} \qquad \text{(eq. 3-60)}$$

(6) This unique relationship between the flow discharge Q and the depth atop the hump under critical conditions d_c allows the use of relatively high humps as a discharge-measuring device known as a *broad-crested weir*. See section 634.0511, Broad-crested Weirs, and section 634.0513, Long-throated Flumes for additional examples and applications.

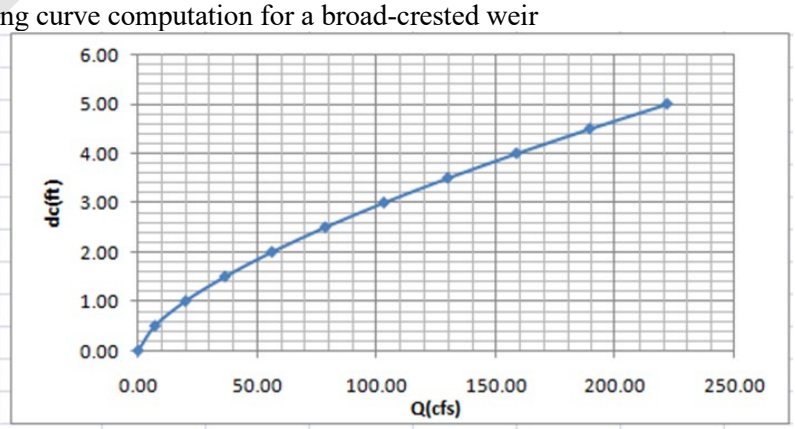
I. Example – Rating curve for a broad-crested weir

(1) Draw the rating curve (i.e., Q –vs.-d_c) for a broad-crested weir in a channel that is *3.5-ft* wide, for the range $0 < d_c < 5.0\ ft$.

(2) The table below was developed by applying equation 3-60 to produce the rating curve as shown below.

Figure 3-26: Rating curve computation for a broad-crested weir

dc(ft)	Q(cfs)
0.00	0.00
0.50	7.02
1.00	19.86
1.50	36.49
2.00	56.17
2.50	78.51
3.00	103.20
3.50	130.05
4.00	158.89
4.50	189.59
5.00	222.05



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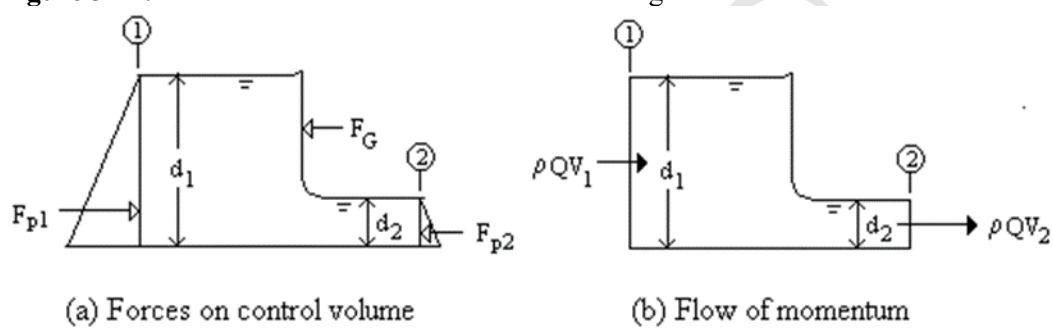
634.0313 Momentum Analysis in Open Channels

A. The energy loss represented by h_f in the energy equation is not easily determined in the analysis of a hydraulic jump (see the following section 3.12 for hydraulic jump analyses). The principle of impulse-momentum or conservation of momentum is more easily applied (than the energy equation) to analyses involving high internal energy changes, such as hydraulic jump or sluice gate flow analyses. Momentum may be thought of as "mass-in-motion" and an impulse as change in momentum. The principle of conservation of momentum is used to determine forces on moving fluids. Consider, for example, the case of the flow under a sluice gate illustrated in figure 3-9. Figure 3-27 below shows a sluice-gate control volume with the forces acting on it, and the flow of momentum through the control surfaces.

Figure 3-27: Forces and flow of momentum for sluice gate flow.



B. Figure 3-27(a) shows hydrostatic forces F_{p1} and F_{p2} acting in the upstream and downstream sections of the control volume for the flow under a sluice gate. Included also in figure 3-27(a) is the force F_G that the gate exerts on the control volume. By Newton's third law (principle of action-reaction), the flow exerts a force $-F_G$ in the opposite direction on the gate. Figure 3-27(b) shows the flow of momentum in (ρQV_1) and out (ρQV_2) of the control volume through the control surfaces at sections (1) and (2), respectively.

C. The principle of impulse-momentum states that the sum of forces on the control volume is equal to the net flow of momentum out of the control volume, i.e., momentum flow out minus momentum flow in. This principle can be expressed as the following vector equation:

$$\sum F = \Delta(pQV) = (pQV)_{out} - (pQV)_{in} \qquad \text{(eq. 3-61)}$$

D. Specifically, for the case illustrated in Figure 3-27:

$$F_{p1} - F_G - F_{p2} = pQV_2 - pQV_1$$

from which it follows that:

$$F_G = \left(F_{p1} + pQV_1\right) - \left(F_{p2} + pQV_2\right)$$

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E. The hydrostatic forces F_{p1} and F_{p2} can be calculated by using equation 2-5 (210-NEH-634-2, section 634.0203C):

$$F_{p1} = (\omega h_c A)_1 \quad and \quad F_{p2} = (\omega h_c A)_2$$

where $h_c A$ stands for the first moment of area with respect to the free surface of a given cross-section (h_c is the depth of the centroid of the cross-section, and A is the area). Also, replacing $V_1 = Q/A_1$ and $V_2 = Q/A_2$, and using $\rho = \omega/g$, the expression for F_G becomes:

$$F_G = \omega\left[\left(h_c A + \frac{Q^2}{gA}\right)_1 - \left(h_c A + \frac{Q^2}{gA}\right)_2\right]$$

F. The quantities between parentheses in the equation above is defined as the *momentum function* for open channel flow:

$$M_F = h_c A + \frac{Q^2}{gA} \qquad \text{(eq. 3-62)}$$

G. The force on the gate can now be written as:

$$F_G = \omega[(M_F)_1 - (M_F)_2] \qquad \text{(eq. 3-63)}$$

H. For a rectangular channel, $A = bd$, and $h_c = d/2$, thus, $h_c A = ½\, bd^2$. Also, using the concept of the unit discharge (discharge per unit width), $q = Q/b$, the term $Q^2/gA = q^2 b/gd$. The momentum function (equation 3-62) becomes:

$$M_F = h_c A + \frac{Q^2}{gA} = \frac{1}{2} bd^2 + \frac{q^2 b}{gd} = b\left(\frac{d^2}{2} + \frac{q^2}{gd}\right)$$

I. Continuing, a *unit momentum function* (or *momentum function per unit width*) for a rectangular open channel can be defined as:

$$m_F = \frac{M_F}{b} = \frac{d^2}{2} + \frac{q^2}{gd} \qquad \text{(eq. 3-64)}$$

J. A *unit momentum function diagram* for a rectangular cross-section is a plot of the channel depth (d) against the unit momentum function (mF), as illustrated in the following example.

K. Example – Momentum function diagram for a rectangular channel

(1) A rectangular channel of width $b = 10\ ft$ carries a discharge $Q = 132\ ft^3/s$; produce a unit momentum function diagram for this channel.

(2) Solution:

(i) The unit discharge is:

$$q = \frac{Q}{B} = \frac{132\ ft^3/s}{10\ ft} = 13.2\ ft^2/s$$

(ii) The unit momentum function diagram, below, was developed by applying equation 3-64:

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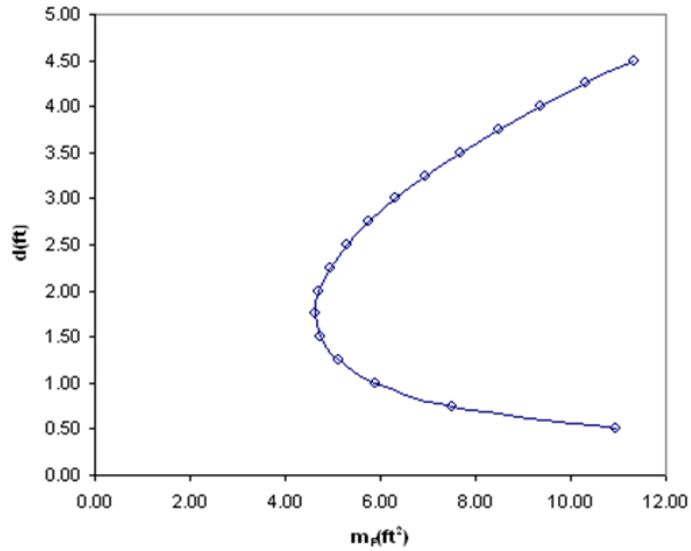
Figure 3-28: Unit momentum function diagram

d(ft)	$m_F(ft^2)$
0.50	10.95
0.75	7.50
1.00	5.91
1.25	5.11
1.50	4.73
1.75	4.62
2.00	4.71
2.25	4.94
2.50	5.29
2.75	5.75
3.00	6.30
3.25	6.95
3.50	7.67
3.75	8.47
4.00	9.35
4.25	10.30
4.50	11.33



L. Example – Calculation of force on sluice gate

(1) In the example Bernoulli's principle applied to sluice gate flow, which used the sluice gate of figure 3-9, the values $b = 10\,ft$, $d_1 = 3.5\,ft$, and $d_2 = 1.0\,ft$ were used to calculate $Q = 132\,ft/s^3$, i.e, $q = Q/b = 13.2\,ft^2/s$. Calculate the force that the flowing water exerts on the gate.

(2) Solution:

(i) The force can be calculated by using the unit momentum function as follows:

$$(m_F)_1 = \frac{d^2{}_1}{2} + \frac{q^2}{gd_1} = \frac{(3.5ft)^2}{2} + \frac{\left(13.3\frac{ft^2}{s}\right)^2}{\left(32.2\frac{ft}{s^2}\right)(3.5ft)} = 7.67ft^2$$

$$(m_F)_2 = \frac{d^2{}_2}{2} + \frac{q^2}{gd_2} = \frac{(1.0ft)^2}{2} + \frac{\left(13.3\frac{ft^2}{s}\right)^2}{\left(32.2\frac{ft}{s^2}\right)(1.0ft)} = 5.91ft^2$$

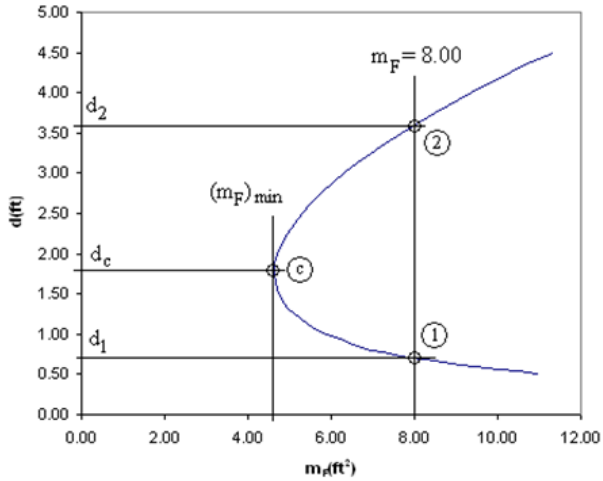
(ii) With $\omega = 62.4\,lb/ft^3$, the force on the gate, as given by equation 3-63, is:

$$F_G = \omega[(M_F)_1 - (M_F)_2] = \omega b[(m_F)_1 - (m_F)_2]$$

$$F_G = 62.4\frac{lb}{ft^3}(10ft)\left[7.67ft^2 - 5.91ft^2\right] = 1098.24lb$$

(iii) An analysis of the unit momentum function diagram used in the example Momentum function diagram for a rectangular channel, above, indicates that there are two possible depths associated with a given value of m_F. This is illustrated in the figure below. These two depths are referred to as *conjugate depths*.

Figure 3-29: Conjugate depths in the unit momentum function diagram for a rectangular open channel.

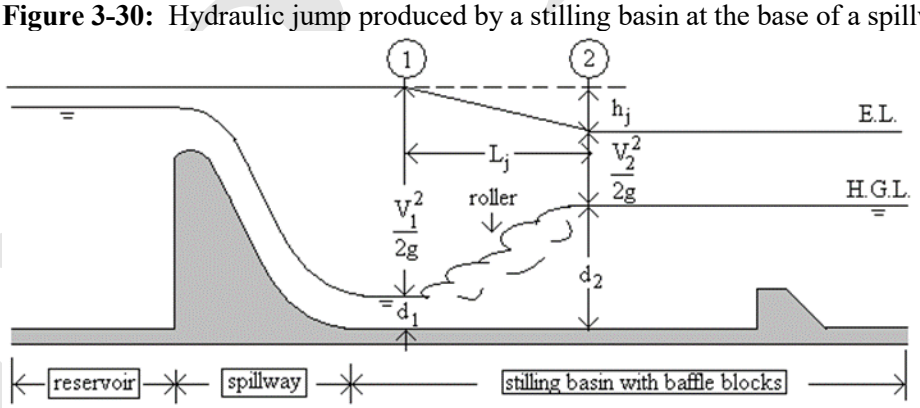


(iv) The unit momentum function diagram also shows that there is a point where the unit momentum function becomes a minimum, $m_F = (m_F)_{min}$. It can be shown that this point of minimum momentum function corresponds to critical flow. Thus, for a given value of the momentum function there is a subcritical and a supercritical depth of flow, the conjugate depths.

634.0314 Hydraulic Jumps

A. A hydraulic jump consists of a sudden increase of water depth in an open channel from a supercritical depth ($d_1 < d_c$) to a subcritical depth ($d_2 > d_c$). A hydraulic jump could occur at the foot of a spillway as illustrated in the following figure.

Figure 3-30: Hydraulic jump produced by a stilling basin at the base of a spillway.



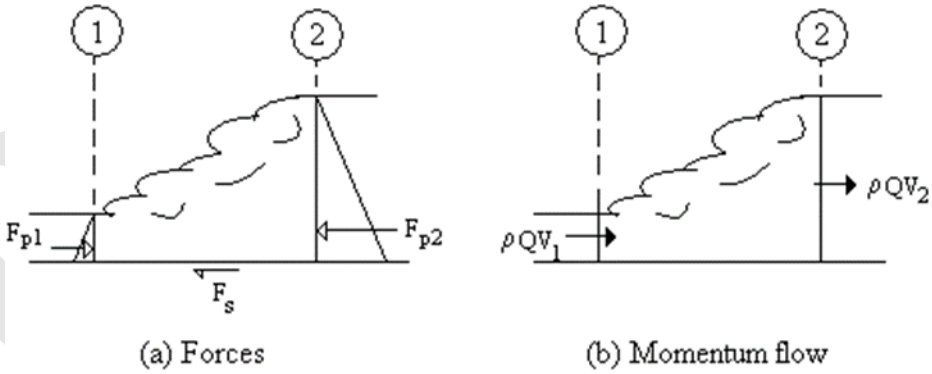
B. The figure below, shows a photograph of a hydraulic jump at the base of a model of a dam spillway.

Figure 3-31: Hydraulic jump observed at the foot of a model spillway (Courtesy of the Utah Water Research Laboratory).

C. The hydraulic jump is typically an abrupt raise in the water surface showing a rough surface with strong turbulence and producing a large amount of air entrainment. This area of strong turbulence and marked air entrainment is referred to as the *roller* of the hydraulic jump. The sketch of the hydraulic jump shown in figure 3-30 indicates an energy loss h_j through the jump.

D. Since energy is not conserved through the jump, analysis of the hydraulic jump is better performed using the impulse-momentum principle. Forces on a control volume enclosing the jump, as well as the momentum flow through the corresponding control surfaces, are shown below.

Figure 3-32: Forces and flow of momentum for a hydraulic jump.



E, Besides the hydrostatic forces F_{p1} and F_{p2}, the control volume is affected by a shear force at the wetted perimeter F_s. Using the principle of impulse-momentum for the jump results in an equation like that of the force on a sluice gate (equation 3-63):

$$F_G = \omega[(M_F)_1 - (M_F)_2]$$

F. Since the hydraulic jump occurs along a relatively small length of the channel, the shear force F_s is negligible, and the impulse-momentum principle for the hydraulic jump results in:

$$(M_F)_1 = (M_F)_2$$

G. If the hydraulic jump occurs in a rectangular channel, the principle of impulse-momentum produces the result:

$$\frac{d_1^2}{2} + \frac{q^2}{gd_1} = \frac{d_2^2}{2} + \frac{q^2}{gd_2}$$

H. The depths of flow upstream (d_1) and downstream (d_2) of the jump are known as conjugate depths.

I. Algebraic manipulation of the above equation gives the following results for a horizontal rectangular-channel hydraulic jump:

(1) Unit discharge (discharge per unit width):

$$q = \sqrt{\frac{gd_1d_2(d_1+d_2)}{2}} \qquad \text{(eq. 3-65)}$$

(2) Ratio of depths:

$$\frac{d_2}{d_1} = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8q^2}{gd_2^3}}\right] \qquad \text{(eq. 3-66)}$$

or,

$$\frac{d_1}{d_2} = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8q^2}{gd_2^3}}\right] \qquad \text{(eq. 3-67)}$$

(3) The *energy head loss* is equal to the difference in specific energy before and after the jump:

$$h_f = \frac{(d_2-d_1)^3}{4d_1d_2} \qquad \text{(eq. 3-68)}$$

(4) The length of the jump, L_j, cannot be determined from energy or momentum considerations. However, experimental results reveal that:

$$4 < \frac{L_j}{d_2} < 6 \qquad \text{(eq. 3-69)}$$

(5) Thus, the average value can be used as a first approximation to the jump length:

$$L_j = 5d_2 \qquad \text{(eq. 3-70)}$$

J. Example – Discharge, head loss, and length of a hydraulic jump in a rectangular channel

(1) A hydraulic jump is observed in a rectangular channel and the upstream and downstream depths are measured to be $d_1 = 0.50\ ft$ and $d_2 = 3.5\ ft$, respectively. Please refer to figure 3-30 as a schematic. Determine (a) the discharge per unit width, (b) the energy head loss through the jump, and (d) an approximation to the jump length.

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(2) Solution:

(i) The unit discharge is (equation 3-65):

$$q = \sqrt{\frac{g d_1 d_2 (d_1 + d_2)}{2}} = \sqrt{\frac{32.2\frac{ft}{s^2}(0.50ft)(3.5ft)(0.50ft + 3.5ft)}{2}}$$

$$= 10.62\frac{ft^2}{s}$$

(ii) The energy head loss is (equation 3-68):

$$h_f = \frac{(d_2 - d_1)^3}{4d_1d_2} = \frac{(3.5ft - 0.50ft)^3}{(4)(0.5ft)(3.5ft)} = 3.86ft$$

(iii) An approximation to the jump length is (equation 3-70):

$$L_j = 5d_2 = 5 \times 3.5\ ft = 17.5\ ft$$

K. Example – Flow depth, head loss, and length of hydraulic jump in a rectangular channel

(1) A hydraulic jump takes place in a rectangular channel of width *b = 5.5 ft* that carries a discharge *Q = 25 cfs*. Please refer to figure 3-30 as a schematic. If the depth upstream of the jump is determined to be $d_1 = 0.75\ ft$, by means of water surface profile calculations, (see section 3.15) determine: (a) the depth downstream of the jump, (b) the energy head loss through the jump, and (c) an approximation to the jump length.

(2) Solution:

(i) The unit discharge is:

$$q = \frac{Q}{b} = \frac{25\ cfs}{5.5\ ft} = 4.55\ ft^s/s$$

(ii) From equation 3-66, d_2 is found:

$$d_2 = \frac{d_1}{2}\left[-1 + \sqrt{1 + \frac{8q^2}{gd_2^3}}\right] = \frac{0.75}{2}\left(-1 + \sqrt{1 + \frac{8(4.55\frac{ft^2}{s})^2}{(32.2\frac{ft}{s^2})(0.75ft)}}\right)$$

$$= 0.987ft$$

(iii) The energy head loss is (equation 3-68):

$$h_f = \frac{(d_2 - d_1)^3}{4d_1d_2} = \frac{(0.987ft - 0.75ft)^3}{4(0.987ft)0.75ft)} = 0.0045ft$$

(iv) An approximation to the jump length is (equation 3-70):

$$L_j = 5d_2 = 5 \times 0.987\ ft = 4.94\ ft$$

L. Figure 3-33 illustrates a hydraulic jump occurring over an obstacle or a baffle block.

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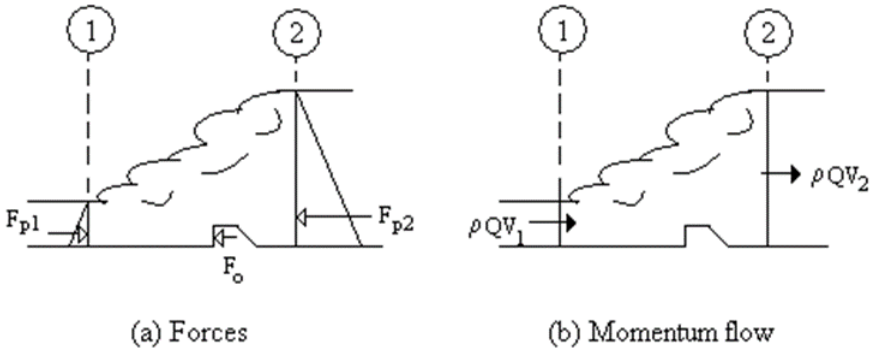
Figure 3-33: Hydraulic jump over a baffle block.



M. For the case illustrated in figure 3-33, equations 3-65 through 3-68 no longer apply because the force that the obstacle or block exerts on the flow is not negligible. However, the principle of impulse-momentum may be applied in the same manner as it was for sluice- gate flow:

$$F_o = \omega[(M_F)_1 - (M_F)_2] \qquad \text{(eq. 3-71)}$$

N. Where $(M_F)_1$ and $(M_F)_2$ are the momentum functions (equation 3-63) for the sections upstream and downstream of the hydraulic jump, respectively. For a rectangular cross-section, the above equation can be written as:

$$F_o = \omega b[(m_F)_1 - (m_F)_2] \qquad \text{(eq. 3-72)}$$

O. Where ω is the specific weight of water, b is the channel width, and the unit momentum function, m_F, is calculated by equation 3-64.

P. Example – Calculation of force on an obstacle producing a hydraulic jump in a rectangular channel

(1) A hydraulic jump in a rectangular channel of width $b = 10\,ft$ is produced by an obstacle at the channel bed. Please refer to figure 3-33 as a schematic. If the depths upstream and downstream of the jump are determined to be $d_1 = 0.95\,ft$ and $d_2 = 1.25\,ft$, respectively, and the discharge in the channel is $Q = 80\,ft/s^3$. Calculate the force that the flowing water exerts on the obstacle.

(2) Solution:

(i) The unit discharge is:

$$q = \frac{Q}{b} = \frac{80\,cfs}{10\,ft} = 8\,ft^s/s$$

(ii) The force can be calculated by using the unit momentum function (equation 3-64):

$$(m_F)_1 = \frac{d^2}{2} + \frac{q^2}{gd} = \frac{(0.95ft)^2}{2} + \frac{\left(8\frac{ft^2}{s}\right)^2}{32.3\frac{ft}{s^2}(0.95ft)} = 2.543ft^2$$

(iii) With $\omega = 62.4\,lb/ft^3$, the force on the obstacle, as given by equation 3-71, is:

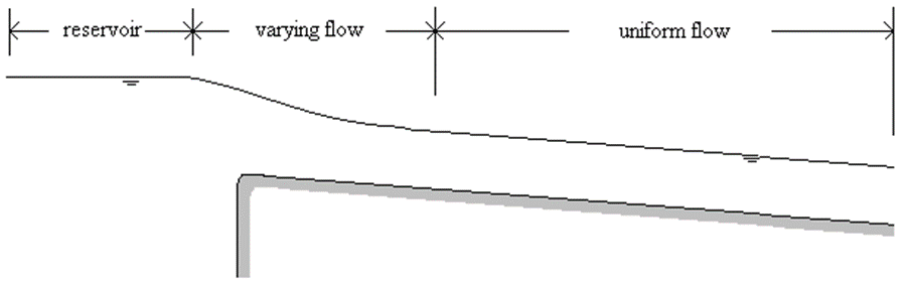
$$F_o = \omega[(M_F)_1 - (M_F)_2] = \omega b[(m_F)_1 - (m_F)_2]$$

$$F_o = 62.4\frac{lb}{ft^3}(10ft)[2.543ft^2 - 2.371ft^2] = 107.6lb$$

634.0315 Varying Open Channel Flow

A. Uniform flow has a constant depth and is achieved when the driving force (gravity) is in balance with the resisting force (shear forces) on the channel boundary. The entrance from a reservoir into a long open channel, as illustrated in figure 3-34, below, may include a zone of varying flow depth, before uniform flow is achieved.

Figure 3-34: Varying flow from a reservoir leading to uniform flow in an open channel.



B. A zone of varying flow depth, as the one illustrated in figure 3-34, is referred to as *gradually varied flow* (GVF). Gradually varied flow zones may be very long. Hydraulic jumps (figure 3-30) or flows over an obstacle (figure 3-33), on the other hand, are examples of *rapidly varied flow*, with the flow depth changing quickly. Flow depths generally vary over the entire length of natural channels.

634.0316 Gradually-varied Flow

A. Figure 3-8 shows the energy heads in a gradually varied flow between two sections separated by a horizontal distance Δx. The slopes shown in that figure include:

B. Slope of the energy line, $S_f = h_f/L$, where h_f= energy head loss, and L = length between sections (1) and (2) measured along channel.

C. Slope of the water surface, $S_w = (WS_1\text{-}WS_2)/\Delta x$, where WS_1 and WS_2 are the water surface elevations at sections (1) and (2)

D. Slope of the channel bed, $S_o = (z_1\text{-}z_2)/\Delta x$, where z_1 and z_2 are the bed elevations at sections (1) and (2)

E. Because typically the channel bed slope S_o is small, the length of channel L is approximately equal to the horizontal length Δx between sections (1) and (2), i.e., $L \approx \Delta x$, and $S_f \approx h_f/\Delta x$.

F. Two equations useful in calculating GVF parameters include the energy equation (see section 634.0302), written as:

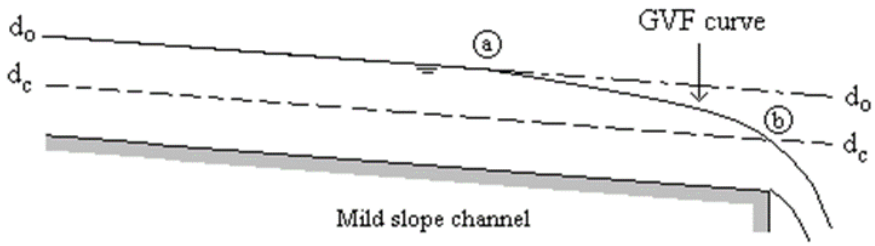
$$z_1 + d_1 + \frac{V^2}{2g} = z_1 + d_1 + \frac{V^2}{2g} + h_f \qquad \text{(eq. 3-73)}$$

G. And Manning's equation, used to estimate the energy slope, S_f:

$$S_f = \left(\frac{Qn}{C_u}\right)^2 \left(\frac{1}{\left((A)(R_h^{2/3})\right)^2}\right) \qquad \text{(eq. 3-74)}$$

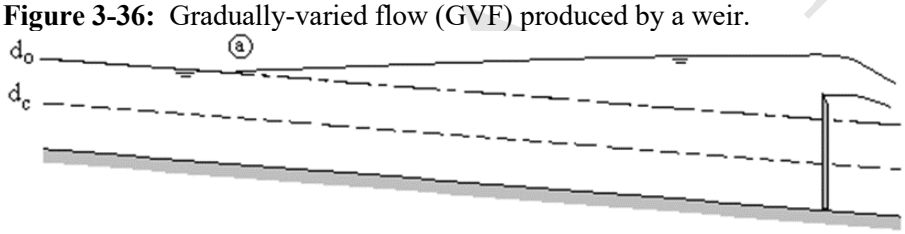
H. Figures 3-35 and 3-36 illustrate two common forms of GVF. In figure 3-35 a uniform flow in a mild-slope channel approaches an overfall. The GVF curve approaches the line of uniform flow (d_o) asymptotically at (a), while approaching the line of critical flow (d_c) almost perpendicularly at (b). The GVF curve in the figure below is referred to as a *drawdown curve*.

Figure 3-35: Gradually-varied flow (GVF) near an overfall.



I. Another example of GVF is shown in figure 3-36. In this case, a weir across a channel forces the water depth above the normal depth of flow. The figure shows a GVF curve that approaches the normal depth line asymptotically at point (a). The curve in the figure below is referred to as a *backwater curve*.

Figure 3-36: Gradually-varied flow (GVF) produced by a weir.



634.0317 Classification of Gradually-Varied Flow

A. GVF curves cannot cross the lines of normal depth (d_o) or of critical depth (d_c). Thus, *backwater curves* or *drawdown curves* must be contained within the region limited by the channel bed and the closest of the d_o or d_c lines, the region between those lines, or the region above the highest of the d_o and d_c lines. These two or three regions are indicated in figure 3-37. GVF curves are classified according to the type of slope of the channel and the region where they occur. For example, a curve above the d_o line in a mild-slope channel would be classified as *M1*, and so on. The different types of gradually varied flow curves are illustrated in figure 3-37.

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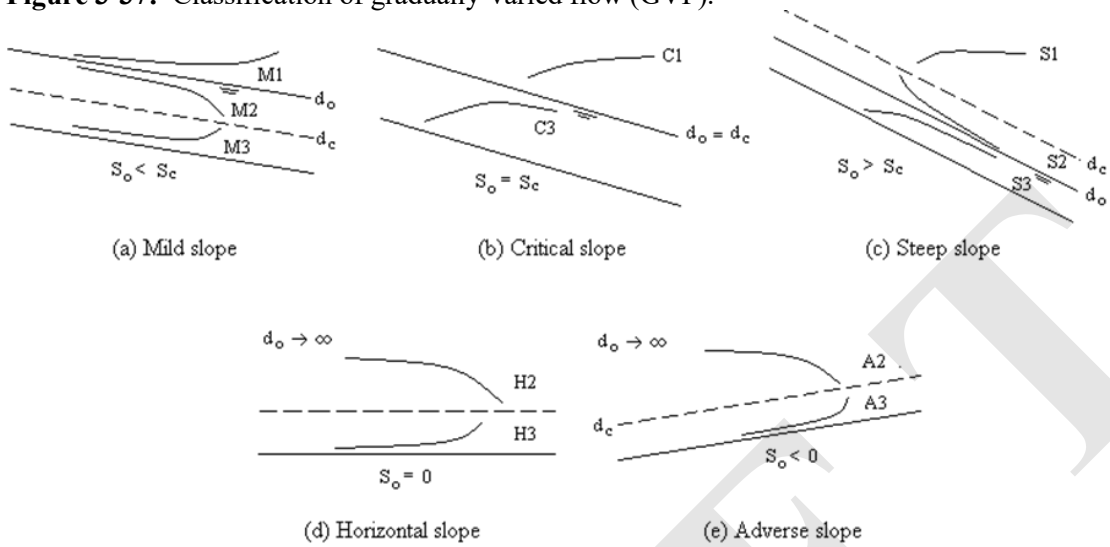
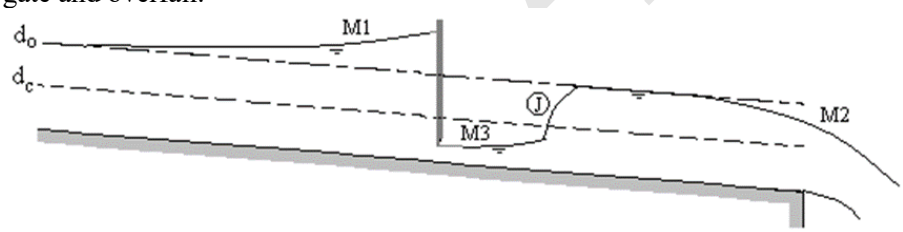
Figure 3-37: Classification of gradually-varied flow (GVF).



B. The curves of figure 3-37 can be used to sketch the type of gradually-varied flow expected on a given channel.

C. The following figure illustrates the type of curves that can be generated by a sluice gate placed across the channel, so that it produces a supercritical flow under the gate. The channel ends with an overfall.

Figure 3-38: Gradually-varied flow (GVF) curves in a mild-slope open channel with a sluice gate and overfall.



D. The sluice gate in figure 3-38 produces an *M1* curve upstream, and an *M3* curve downstream of the gate. The *M3* curve ends in a hydraulic jump (J) that raises the water level to the normal depth of flow before reaching the overfall. The overfall produces an *M2* curve at the downstream end of the channel.

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634.0318 Standard step method

A. Manning's equation provides a good estimate of flow depth, when uniform flow conditions exist. See section 634.0307, Calculations in Uniform Flow. Because uniform flow conditions of constant depth and discharge do not normally exist in natural streams and in many reaches of constructed channels, other methods, such as the standard step, are needed to more accurately calculate flow profiles.

B. The *standard step method* calculates a gradually varied flow (GVF) profile by solving the energy equation with an iterative procedure. One way of applying the method is by varying the flow depths. The calculations start at a point where the depth is known, and the depth is changed by small increments or decrements until reaching a specified depth value. For each depth increment or decrement, the distance between sections, Δx, for the given depth change, is calculated.

C. To develop an equation for the standard step method one may start from equation 3-73, re- written as follows:

$$z_1 + E_1 = z_2 + E_2 + h_f \qquad \text{(eq. 3-75)}$$

D. Using z_1-$z_2 = S_o\Delta x$ and $h_f = S_f\Delta x$, we can solve for Δx, the horizontal distance between sections (1) and (2):

$$\Delta x = \frac{E_1 - E_1}{S_f - S_o} \qquad \text{(eq. 3-76)}$$

E. The value of the energy slope to use is based on the Manning's equation by using the average velocity and hydraulic radius for sections (1) and (2):

$$\bar{V} = \frac{1}{2}(V_1 + V_2), \overline{R_h} = \frac{1}{2}(R_{h1} + R_{h2}) \qquad \text{(eq. 3-77)}$$

F. The energy slope is calculated using Manning's equation as:

$$S_f = \left(\frac{nV}{C_u R_h^{2/3}}\right)^2 \qquad \text{(eq. 3-78)}$$

G. The calculation procedure starts by selecting a depth d_1 and then postulating a depth $d_2 = d_1 \pm \Delta d$, e.g., you could have $d_1 = 2.5\ ft$ and $d_2 = 2.6\ ft$ (with $\Delta d = 0.1\ ft$). Then, proceeding to calculate the areas (A_1, A_2), wetted perimeters (P_1, P_2), hydraulic radii (R_{h1}, R_{h2}), velocities (V_1, V_2), specific energies (E_1, E_2), average hydraulic radius (R_h), average velocity (V), energy slope (S_f), and finally the increment Δx from equation 3-76 is calculated. The procedure is repeated then by taking $d_1 = d_2$ and postulating a new value of d_2. Tables of the calculation results are shown in the following example.

H. Example – Gradually-varied flow calculation in a rectangular channel

(1) A rectangular flume is *5-ft* wide ($b = 5ft$) and carries a flow $Q = 60\ cfs$. The bed slope is $S_o = 0.0006$, and the Manning's resistance coefficient is $n = 0.012$. At a certain section the depth is $d_1 = 3\ ft$. Find the distance Δx to the section where the depth is *2.5 ft*.

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(2) Solution:

(i) Using depth decrements of *Δd = -0.1 ft*, the depths *d = 3.0 ft, 2.9 ft, 2.8 ft, 2.7 ft, 2.6 ft, and 2.5 ft* are used in the calculation. Calculations are shown for the depths, *3.0 ft and 2.9 ft*. For a rectangular cross section, the area and wetted perimeter are given by *A = bd = 5d*, *P = b + 2d = 5+2d*, respectively. Thus:

- For d_1 = 3 ft:

$$A_1 = 5\text{ ft} \times 3\text{ ft} = 15ft^2$$
$$P_1 = 5 + (2 \times 3) = 11\ ft$$
$$R_{h1} = A_1/P_1 = 15/11 = 1.36\ ft$$
$$V_1 = Q/A_1 = 60\ cfs/15\ ft^2 = 4\ ft/s$$
$$E_1 = d_1 + {V_1}^2/2g = 3\ ft + (4\ ft/s)^2/(2 \times (32.2\ ft/s^2))$$
$$= 3.2484\ ft$$

- For d_2 = 2.9 ft

$$A_1 = 5\text{ ft} \times 2.9\text{ ft} = 14.5ft^2$$
$$P_1 = 5 + (2 \times 2.9) = 10.8\ ft$$
$$R_{h1} = A_1/P_1 = 14.5/10.8 = 1.34\ ft$$
$$V_1 = Q/A_1 = 60\ cfs/14.5\ ft^2 = 4.14\ ft/s$$
$$E_1 = d_1 + {V_1}^2/2g = 2.9\ ft + (4.14\ ft/s)^2/(2 \times (32.2\ ft/s^2))$$
$$= 3.166\ ft$$

(ii) The average velocity and hydraulic radius are:

$$V = (V_1 + V_2)/2 = 4.07\ ft/s$$
$$R_h = (R_{h1} + R_{h2})/2 = 1.35\ ft$$

(iii) The energy slope is calculated as follows:

$$S_f = \left(\frac{nV}{C_u R_h^{2/3}}\right)^2 = \left(\frac{(0.012)(4.07)}{1.486(1.35^{2/3})}\right)^2 = 0.000721$$

(iv) Finally, the distance between cross sections is calculated as:

$$\Delta x = \frac{E_1 - E_1}{S_f - S_o} = \frac{3.2484ft - 3.166ft}{0.000721 - 0.0006} = 680ft$$

(v) The standard step solution is presented in the following tables.

Figure 3-39: Standard step solution example -table 1

Depth d	Area A	Wetted Perimeter P	Hydraulic Radius R=A/P	Flow Velocity V	Specific Energy E	Average Hydraulic Radius R_{avg}	Average Velocity V_{avg}
3.000	15.000	11.000	1.364	4.000	3.248	-	-
2.900	14.500	10.800	1.343	4.138	3.166	1.353	4.069
2.800	14.000	10.600	1.321	4.286	3.085	1.332	4.212
2.700	13.500	10.400	1.298	4.444	3.007	1.309	4.365
2.600	13.000	10.200	1.275	4.615	2.931	1.286	4.530
2.500	12.500	10.000	1.250	4.800	2.858	1.262	4.708

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Figure 3-40: Standard step solution example – table 2

Depth d	Energy Slope S	Spec. En. Increment ΔE	Distance Increment Δx	Cumulative Distance x
3.0	-	-	-	0
2.9	0.000721	0.0826	680.11	680.11
2.8	0.00079	0.0807	425.5	1105.61
2.7	0.000867	0.0785	293.53	1399.14
2.6	0.000957	0.076	213.03	1612.15
2.5	0.001059	0.073	158.91	1771.05

(vi) The cumulative distance to the section where the depth is *2.5 ft* is shown to be *1771 ft*.

I. The best results in the standard step method are achieved by using an increment or decrement of depth as small as possible. Thus, implementation of the standard step solution in spreadsheet software facilitates the calculation, especially for the trapezoidal, parabolic, and circular shapes that occur in prismatic open channels.

J. To achieve best results in the standard step method, the step computations should be carried upstream for subcritical flows and downstream for supercritical flows.

K. For analyses of gradually-varied flow in prismatic or natural channels, one can use the U.S. Army Corps of Engineers' Hydrologic Engineering Center's HEC-RAS software, (RAS stands for *River Analysis System*.). The latest version of HEC-RAS can be downloaded at: http://www.hec.usace.army.mil/software/hec-ras/

L. HEC-RAS utilizes an iterative standard step method to solve for gradually varied flow. Details on the operation of HEC-RAS are available at the website shown above.

634.0319 Sediment Transport

Earth-lined channels may carry sediments if the water velocities are large enough to produce erosion of the channel linings. Rivers may carry significant sediment load when the water discharge increases. The ability of a river to erode and carry sediments depends on the hydraulic characteristics of the stream as well as on the sediment properties. Refer to NEH 654, Chapter 13, for additional sediment transport information.

634.0320 Sediment Properties

A. The size of sediments can be determined by performing a sieve test on a sample of sediments. For very fine sediments a settling test performed in a settling tube may be necessary to determine the size distribution of the particles. Typically, sediment size follows a log-normal distribution of probabilities, with a *median size* D_{50}, meaning the size for which 50% of the sample mass is retained. The diameters D_{84} and D_{16} represent the diameters for which *15.9%* and *84.1%* of the sample mass is retained. The *standard deviation* of the logarithms of the diameters is given by:

$$\sigma = \sqrt{\frac{D_{84}}{D_{16}}} \qquad \text{(eq. 3-79)}$$

B. If the diameters of the sediment sample do not follow a log-normal distribution, the *geometric mean* can be used as a representative value of the sediment size:

$$D_g = \sqrt{D_{85} D_{16}} \qquad \text{(eq. 3-80)}$$

C. Example - Sediment size data analysis

(1) Sediment size data from a sieve analysis can be used to determine the characteristic diameters for a sediment sample. This information is useful in the prediction of sediment transport discharges in rivers and streams. The table below shows the results from a sieve analysis of a sample of sandy sediment:

Figure 3-41: Sediment data from sieve analysis

Sieve opening (mm)	Amount retained (g)
0.495	0.85
0.417	1.56
0.351	3.08
0.295	3.82
0.246	5.35
0.208	5.69
0.175	4.31
0.147	5.06
0.124	2.37
0.104	1.16
0.088	0.21
0.074	0.12
Pan	0.04

(2) Using a *probability-logarithmic* plot, determine the values of D_{16}, D_{50}, and D_{84}, calculate the standard deviation of the logarithms, and the geometric mean.

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(3) From the table given above, we can produce the following table (figure 3-42) to determine the percentage finer than the given sieve size.

Figure 3-42: Determination of percent finer than given sieve size

Sieve opening (mm)	Weight retained (g)	% weight retained	Cumulative % weight retained	Percent finer
0.495	0.85	2.53	2.53	97.470
0.417	1.56	4.64	7.17	92.830
0.351	3.08	9.16	16.33	83.670
0.295	3.82	11.36	27.69	72.310
0.246	5.35	15.91	43.6	56.400
0.208	5.69	16.92	60.52	39.480
0.175	4.31	12.82	73.34	26.660
0.147	5.06	15.05	88.39	11.610
0.124	2.37	7.05	95.44	4.560
0.104	1.16	3.45	98.89	1.110
0.088	0.21	0.62	99.51	0.490
0.074	0.12	0.36	99.87	0.130
pan	0.04	0.12	100.00	0.000
sum	33.62	100.00	-	-

(4) The *probability-logarithmic* plot is shown below. The *x* axis denotes the sieve opening, in *mm*, in the logarithmic scale, while the *y* axis displays the *percentage finer* in the probability scale. The plot also shows the lines corresponding to *84.1%-, 50%-*, and *15.9%-finer* which are used to determine the values of D_{16}, D_{50}, and D_{84}. These values are:

$$D_{16} = 0.15 \text{ mm}, D_{50} = 0.23 \text{ mm}, D_{84} = 0.34 \text{ mm}$$

(5) The standard deviation of the logarithms of the diameters is:

$$\sigma = \sqrt{\frac{D_{84}}{D_{16}}} = \sqrt{\frac{0.34mm}{0.15mm}} = 1.505$$

(6) The geometric mean of the diameters is:

$$D_g = \sqrt{D_{84}D_{16}} = \sqrt{(0.34mm)(0.15mm)} = 0.225mm$$

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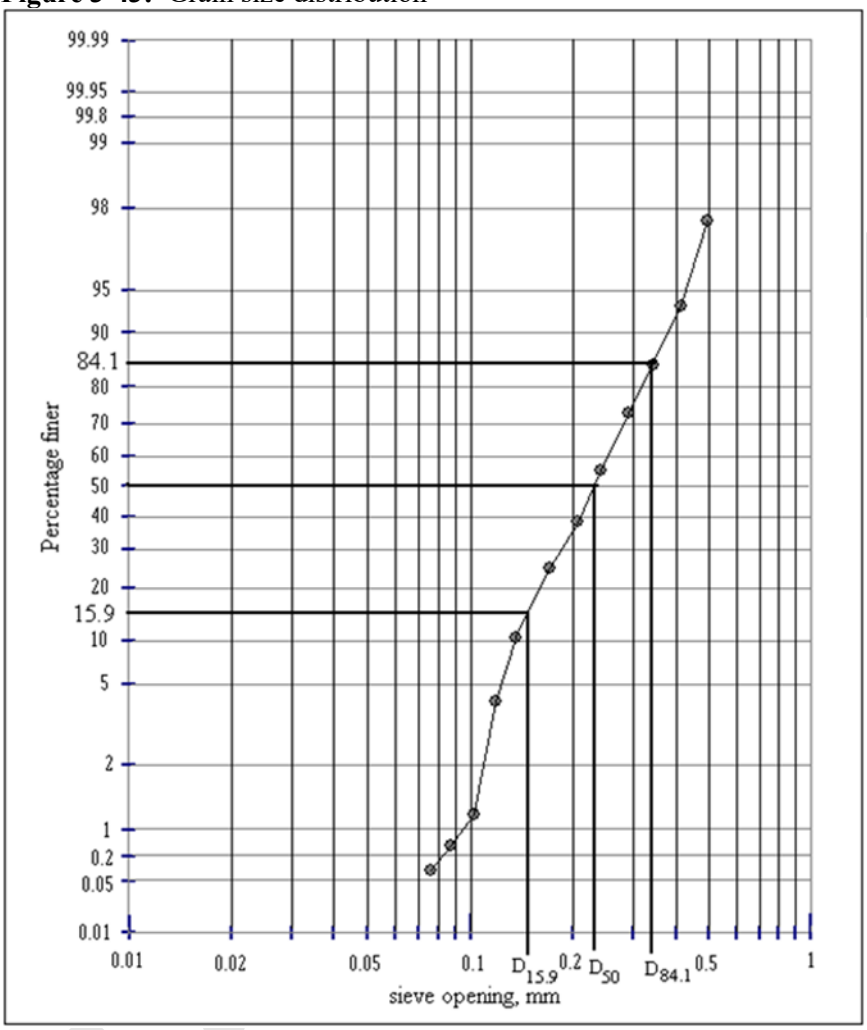
Figure 3-43: Grain size distribution



D. In the analysis of suspended sediment, i.e., those sediment particles that are entrained in the moving fluid, it's important to consider the settling velocity of the sediments. Theoretical analysis of settling velocities indicate that the terminal velocity w of a spherical particle of diameter D is given by:

$$w = \sqrt{\frac{4gD}{3C_D}\left(\frac{\rho_s - \rho}{\rho}\right)} = \sqrt{\frac{4gD}{3C_D}(S_s - 1)} \qquad \text{(eq. 3-81)}$$

E, Where C_D is the drag coefficient, g is the acceleration of gravity, and ρ and ρ_s are the densities of the fluid and of the solid sediment particles, respectively. A typical value used for the specific density (gravity) of the solid sediment particles (sand) is $S_s = \rho_s / \rho = 2.65$. The drag coefficient is related to the *Reynolds number* of the particle, namely:

$$R_e = \frac{wD}{\nu} \qquad \text{(eq. 3-82)}$$

where ν is the viscosity of water.

F. For laminar flow, i.e., $Re < 2$:

$$C_D = \frac{24}{R_e}\left(1 + \frac{3}{16}R_e - \frac{19}{1280}R_e^2 + \frac{71}{20480}R_e^3 + \cdots\right) \quad \text{(eq. 3-83)}$$

G. The approximation $C_D = 24/Re$, applies for $Re < 0.5$. For turbulent flow, with $Re < 800$, the drag coefficient on a spherical particle is approximated by:

$$C_D = \frac{24}{R_e}(1 + 0.105R_e^{0.687}) \quad \text{(eq. 3-84)}$$

H. Example - Determining settling velocity of spherical sediments

(1) Determine the settling velocity of a spherical sediment particle with a diameter $D = 0.23\ mm = 7.54\times10^{-4}\ ft$, in water at $60\ ^oF$ ($\nu = 1.217\times10^{-5}\ ft^2/s$). Use $S_s = 2.65$ for the specific gravity of the solid particles. (This type of information is useful for designing sediment settling basins).

(2) Solution:

(i) Determining the settling velocity, w, is a trial and error solution, since the settling velocity is related to a drag coefficient, which is related to the *Reynolds number*, which is, in turn, related to the settling velocity (the parameter to be calculated).

(ii) For efficient calculations, an iterative procedure involving the above equations 3-81 thru 3-84 may be programmed in a spreadsheet to obtain the settling velocity, $w = 0.0929\ ft/s$, with $Re = 5.76$, (in the turbulent regime).

I. The results from the previous example apply to spherical particles only. Natural sediments may have other shapes; therefore, the effect of that shape must be taken into account when calculating the settling velocity. A simple approach consists in multiplying the velocity calculated with equation 3-81 by a shape factor ψ:

$$w = \psi w_{sphere} = \psi\sqrt{\frac{4gD}{3C_D}\left(\frac{\rho_s-\rho}{\rho}\right)} = \psi\sqrt{\frac{4gD}{3C_D}(S_s - 1)} \quad \text{(eq. 3-85)}$$

J. Some typical values of the shape factor are provided in the table below.

Figure 3-44: Shape factors for sediment settling velocity

Particle shape	Shape factor, ψ
sphere	1.000
cube-octahedron	0.906
octahedron	0.846
cube	0.806
tetrahedron	0.670

K. Example – Determination of settling velocity for non-spherical sediments

(1) In the previous example, a settling velocity $w_{sphere} = 0.0929\ ft/s$ for a spherical particle was found. If the particle shape is actually an octahedron, what is the settling velocity?

(2) Solution - From the table, a shape factor $\psi = 0.846$ for an octahedron. The settling velocity for the particle is:

$$w = \Psi \cdot \omega_{sphere} = 0.846 \times 0.0929\ ft/s = 0.0786\ ft/s$$

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L. For additional information on sediment properties and settling velocity calculations, refer to: Raudkivi (1976), ASCE (2006), and NEH 654, Chapter 7, Basic Principles of Channel Design; Chapter 8, Threshold Channel Design; Chapter 9, Alluvial Channel Design; and Chapter 13, Sediment Impact Assessments.

634.0321 Threshold of Sediment Motion

A. Channels may be classified as threshold or alluvial. Sediment passes through a threshold channel with little impact on the channel boundary. An alluvial or movable-bed channel is more active, with an exchange of sediment between the channel boundary and the flow.

B. In a threshold channel, the applied forces of the flow are less than the threshold for movement of the boundary material. Equation 3-28 (section 634.0304) provides an expression for the average shear stress at the bed of an open channel in uniform conditions, and is repeated here:

$$\tau_0 = \omega \cdot R_h \cdot S_0$$

C. Define the shear velocity, v_*, as:

$$v_* = \sqrt{\frac{\tau_o}{\rho}} \qquad \text{(eq. 3-86)}$$

D. Although this is not a velocity that can be measured in the flow, the quantity defined above has the units of a velocity, and, being related to the bed shear stress, is appropriately named the shear velocity.

E. The bed shear stress, τ_o, and the shear velocity, v_*, (u_* is used interchangeably with v_*) can be used to define a set of parameters to analyze the threshold of sediment motion in open channels. The two parameters to consider are a *Reynolds number* based on the shear velocity:

$$\frac{v_* D}{\nu} \qquad \text{(eq. 3-87)}$$

and a dimensionless shear stress:

$$\frac{\tau_o}{(\omega_s - \omega)D} = \frac{\tau_o}{(S_s - 1)\omega D} \qquad \text{(eq. 3-88)}$$

where ν and ω ("γ" is used interchangeably with "ω") are the kinematic viscosity and specific weight of water, respectively, and D and ω_s are the diameter and the specific weight of the sediments. The specific gravity of the sediments is $S_s = \omega_s / \omega$ ($S_s = 2.65$ for sand).

F. NEH 654, Chapter 8 (Threshold Channel Design), describes the origin and meaning of the Shield's diagram. This diagram, shown in figure 3-45, and based on the *Reynolds number* and the dimensionless shear stress (equations 3-87 and 3-88), has been extensively used for determining the threshold of sediment motion in open channel flow. For a given flow, points above the Shields's curve indicate sediment motion, whereas points below the Shield's curve would show no sediment motion.

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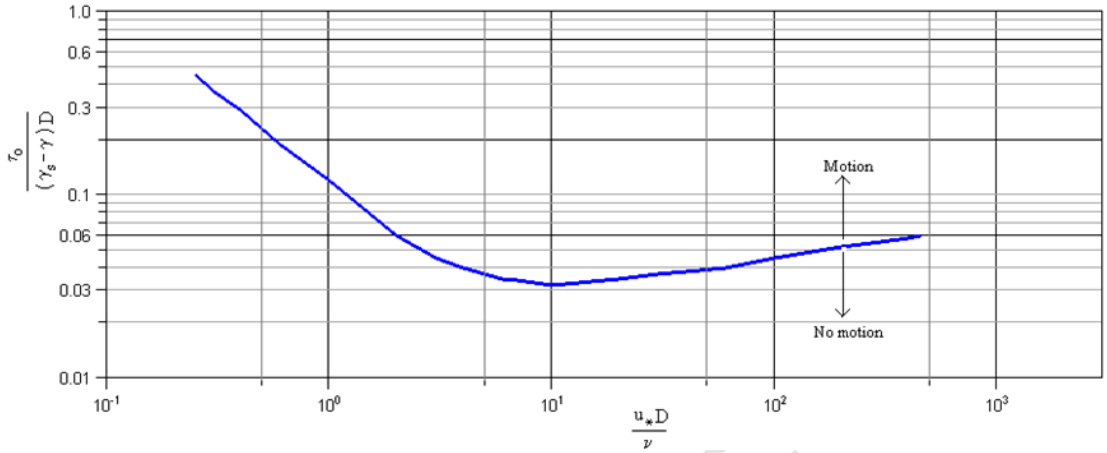
Figure 3-45: Shield's diagram for determining the threshold of sediment motion in open channel flow.



G. Example - Sediment motion threshold analysis using Shield's diagram

(1) A rectangular channel of width $b = 10\,ft$, with Manning's $n = 0.015$, and laid on a slope $S_o = 0.005$, flows at a normal depth $d_o = 0.5\,ft$. Using Shield's diagram, determine if the flow is able to move sediment of diameter $D = 0.20\,mm = 6.56\times10^{-4}\,ft$. Use $S_s = 2.65$ as the specific gravity of the sediments, and $\nu = 1.217\times10^{-5}\,ft^2/s$ for the viscosity of water.

(2) Solution:

(i) The bed shear stress and the shear velocity are calculated as follows:

$$A = bd_0 = 10\,ft \times 0.5\,ft = 5\,ft^2$$

$$P = b + 2d_0 = 10\,ft + (2 \times 0.5\,ft) = 11\,ft0$$

$$R_h = A/P = 5\,ft^2/11\,ft = 0.4545\,ft$$

$$\tau_0 = \omega R_h S_0 = 62.4\ lb/ft^2 \times 0.4545\,ft \times 0.005 = 0.1418\ lb/ft^2$$

$$v_* = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{\frac{g\omega R_h S_o}{\omega}} = \sqrt{gR_hS_o} = \sqrt{32.2\frac{ft}{s^2}(0.4545ft)(0.005)}$$

$$= 0.2705ft/s$$

(ii) The parameters of the Shield's diagram are calculated as follows:

$$\frac{v_* D}{v} = \frac{(0.2705ft/s)(6.56x10^{-4}ft)}{1.217x10^{-5}} = 14.6$$

$$\frac{\tau_o}{(S_s-1)\omega D} = \frac{0.1418\frac{lb}{ft^2}}{(2.65-1)\left(\frac{62.4lb}{ft^3}\right)(6.56x10^{-4}ft)} = 2.1$$

(iii) The point (14.6, 2.1) in the Shield's diagram is well above Shield's curve; therefore, sediment motion will occur for the flow specified above. Normally, sediment motion is assured if the shear stress parameter is a factor of "2" above the curve. And likewise, if the shear stress parameter is less than half of the curve value, then sediment motion will not occur.

H. More recent work indicates that Shield's diagram does not account for the absence of stream bed forms, the sporadic entrainment of sediment particles at low shear stress, or the effects of non-uniform bed material. See NEH 654.0804(b) to calculate a refined allowable shear stress parameter.

I. Grass linings have been widely used to protect the erodible soil boundaries of waterways, floodways, and reservoir auxiliary spillways. An effective stress design approach is provided in the basic reference USDA AH 667. Waterway Design Tool software has been developed to design grassed waterways, following the procedures of AH 667. Refer to NEH Part, 650, Chapter 7, Grassed Waterways, for example designs of trapezoidal and parabolic-shaped grassed waterways, using an extensive set of design tables. Also, refer to NEH 654.0806 for more information and an example problem on the threshold design of a grass-lined channel.

634.0322 Suspended Sediment Load

A. Once sediment particles are moving, the finest particles may be entrained in the flow and kept in suspension by turbulent motions while being carried downstream by the bulk flow. The sediment thus transported is known as the *suspended sediment load*.

B. The concentration $C(y)$ of suspended sediment, in units of sediment mass per unit liquid mass, can be calculated using the following equation:

$$\frac{C(y)}{C_a} = \left(\left(\frac{d-y}{d-a}\right)\frac{a}{y}\right)^2 \qquad \text{(eq. 3-89)}$$

where y is the distance from the channel bed, C_a is the concentration measured at a reference level $y = a$ close to the channel bed, d is the flow depth, and z is a parameter calculated as:

$$z = \frac{w}{\beta \kappa v_*} \qquad \text{(eq. 3-90)}$$

C. In this parameter, w is the settling velocity of the sediment particles, β is the ratio of the sediment transport coefficient to the turbulent kinematic viscosity in the flow, κ is von Karman's constant (related to the viscous stress model commonly used in open channel flow), and v_* is the shear velocity. The value of β is close to 1.0 for fine sediments and decreases as the particle size increases. Von Karman's constant has been measured to be $k = 0.40$ in clear water, and lower for sediment-laden water.

D. If we measure the concentration at the midpoint of the depth, i.e., $C_{md} = C(d/2)$, equation 3-89 can be written as follows ($a = d/2$):

$$\frac{C(y)}{C_{md}} = \left(\frac{d-y}{y}\right)^2 \qquad \text{(eq. 3-91)}$$

E. The suspended sediment discharge G_{ss} (mass per unit volume) in a rectangular channel of width b, can be calculated by integrating the product $C(y) \cdot v(y)$ over the depth of flow, with $v(y)$ being the flow velocity distribution in the vertical:

$$G_{ss} = \int_A C(y) \cdot v(y) \cdot dA = \int_A C(y) \cdot v(y) \cdot b \cdot dy \qquad \text{(eq. 3-92)}$$

F. A typical velocity distribution in turbulent open channel flow is the logarithmic distribution given by:

$$v(y) = V + \frac{v_*}{\kappa}\left[1 + ln\left(\frac{y}{d}\right)\right] \qquad \text{(eq. 3-93)}$$

where V is the mean flow velocity. The point near the channel bed where the velocity becomes zero is located at a distance y_o from the channel bed. The value of y_o can be found from the above equation by making $v(y_o) = 0$, resulting in:

$$y_o = (d)exp\left[-\left(\frac{\kappa V}{v_*}\right) + 1\right] \qquad \text{(eq. 3-94)}$$

G. This value becomes the lower limit for the integral of equation 3-92, the upper limit being the depth of flow d. The suspended sediment discharge per unit width can be calculated as:

$$g_{ss} = \frac{G_{ss}}{b} = \int_{y_o}^{d} C(y)v(y)dy = \int_{y_o}^{d} C_{md}\left(\frac{d-y}{y}\right)^2\left(\frac{v_*}{\kappa}\left[1 + ln\left(\frac{y}{d}\right)\right]\right)dy \qquad \text{(eq. 3-95)}$$

H. A dimensionless unit sediment discharge can be obtained by dividing g_{ss} by the product qC_{md}, where $q = Vd$ is the unit water discharge (or discharge per unit width). This dimensionless unit sediment discharge can be written as:

$$g_{ss}^* = \frac{g_{ss}}{qC_m} = \left(1 + \frac{v_*}{\kappa V}\right)I_1(z, \eta_o) + \frac{v_*}{\kappa V}I_2(z, \eta_o) \qquad \text{(eq. 3-96)}$$

where $\eta_o = y_o/d$, and the integrals $I_1(z, \eta_o)$ and $I_2(z, \eta_o)$, are calculated as:

$$I_1(z, \eta_o) = \int_{\eta_o}^{1}\left(\frac{1-\eta}{\eta}\right)^z d\eta \qquad \text{(eq. 3-97)}$$

$$I_1(z, \eta_o) = \int_{\eta_o}^{1}\left(\frac{1-\eta}{\eta}\right)^z \ln(\eta)\, d\eta \qquad \text{(eq. 3-98)}$$

I. Example – Suspended sediment discharge calculation

(1) Consider a rectangular stream of width $b = 20\,ft$, laid on a slope $S_o = 0.00025$, with Manning's $n = 0.025$, flows at a normal depth $d_o = d = 5.0\,ft$. The mid-depth concentration is measured to be $C_{md} = 0.05\ gm/lt = 0.00312\ lb/ft^3$. The settling velocity of the particles has been calculated to be $w = 0.12\,ft/s$. Calculate the suspended sediment discharge G_{ss}, assuming that the parameter $\beta = 0.88$.

(2) Solution:

(i) Proceeding to calculate the mean velocity and shear velocity:

$$A = bd_0 = 20\,ft \times 5\,ft = 100\,ft^2$$

$$P = b + 2d_0 = 20\,ft + (2 \times 5\,ft) = 30\,ft0$$

$$R_h = A/P = 100\,ft^2/30\,ft = 3.33\,ft$$

$$V = \frac{C_u}{n}R_h^{2/3}\sqrt{S_o} = \frac{1.486}{0.025}(3.33)^{2/3}\sqrt{0.00025} = 2.097ft/s$$

$$v_* = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{\frac{g\omega R_h S_o}{\omega}} = \sqrt{gR_hS_o} = \sqrt{32.2\frac{ft}{s^2}(3.33ft)(0.00025)}$$

$$= 0.1638ft/s$$

(ii) The lower limit in the integral of equation 3-94 is calculated wth κ= *0.40* as:

$$y_o = (d)exp\left[-\left(\frac{\kappa V}{v_*}\right)+1\right] = (5ft)exp\left[-\left(\frac{\frac{(0.4)2.097ft}{s}}{\frac{0.1638ft}{s}}\right)+1\right]$$

$$= 0.0109ft$$

while the lower limit of the integrals in equations 3-97 and 3-98 is:

$$\eta_o = y_o/d = 0.0109ft/5ft = 0.00219$$

(iii) The parameter *z* from equation 3-90 is calculated with $\kappa = 0.40$ as follows:

$$z = \frac{w}{\beta\kappa v_*} = \frac{0.12ft/s}{(0.88)(0.40)0.1638ft/s} = 2.081$$

(iv) The integrals in equations 3-97 and 3-98 can be calculated by using numerical integration in a spreadsheet, which calculates the integrals, I_1 = 677.1623 and I_2 = -3553.72.

(v) The dimensionless unit suspended sediment discharge is calculated with equation 3-96:

$$g_{ss}^* = \left(1+\frac{0.1638}{0.40(2.097)}\right)677.1623 + \frac{0.1638}{0.42.097} - 3553.72 = 115.52$$

(vi) The unit suspended sediment discharge is:

$$g_{ss} = g_{ss}^* q C_m = (115.52)\frac{(2.097)100}{2}0.00312 = \frac{3.78lb}{s}/ft$$

(vii) The suspended sediment discharge is:

$$G_{ss} = g_{ss}(20) = 75.58lb/s$$

634.0323 Bed Sediment Load

A. Coarser sediment particles carried by a stream may move by rolling and saltation along the bed of the channel constituting what is known as *bed sediment load*. Refer to NEH 654, Chapter 9, for bed sediment load formulae and where to apply. Using the *Meyer-Peter-Muller* formula due to its simplicity*:*

$$g_B = 8\omega\sqrt{gD^3}\left(\frac{S_s}{S_s-1}\right)\left[\frac{R_h S_o}{D}\left(\frac{k}{k'}\right)^{3/2} - 0.047(S_s-1\right]^{3/2} \qquad \text{(eq. 3-99)}$$

B. In this formula, g_B is the bed sediment discharge (mass per unit time) per unit width of a rectangular channel, ω is the specific weight of water, g is the acceleration of gravity, $D = D_{50}$ is the sediment particle diameter, S_s is the specific gravity of sediments (e.g., $S_s = 2.65$ for sand), R_h is the hydraulic radius (which can be taken as $R_h = d$, the depth of flow, for a wide channel), and S_o is the bed slope. The coefficients *k* and *k'* are *Stickler coefficients* defined by the following equations:

$$V = kR_h^{2/3}\sqrt{S_o} = \frac{C_u}{n}R_h^{2/3}\sqrt{S_o} \qquad \text{(eq. 3-100)}$$

$$V = k'R_h^{2/3}\sqrt{S'} = \frac{C_u}{n}R_h^{2/3}\sqrt{S'} \qquad \text{(eq. 3-101)}$$

C. The first expression for V is Manning's equation with the coefficient k accounting for the overall channel resistance, namely, bed form resistance (e.g., dunes, bars) and grain friction resistance. This velocity is defined in terms of the bed slope S_o. The second expression for V is Manning's equation with the coefficient k' accounting only for the grain friction resistance. This equation uses a slope S' that corresponds to the grain friction resistance. Thus, S' can be defined as a grain-friction energy slope.

D. The ratio k/k', that appears in the Meyer-Peter-Muller equation, can take values between 0.5 and 1.0, with $k/k' = 1.0$ when no bed forms are present, and $k/k' = 0.5$ when strong bed forms are present. The coefficient k' can be calculated, using metric units, as:

$$k' = \frac{26m^{1/3}/s}{D_{90}^{1/6}} \qquad \text{(eq. 3-102)}$$

where D_{90} is the diameter of bed sediments for which 90% of the material is finer. In the English system of units the equation to use is:

$$k' = \frac{38.63ft^{1/3}/s}{D_{90}^{1/6}} \qquad \text{(eq. 3-13)}$$

E. Example – Bed sediment load rate calculation

(1) A rectangular stream of width $b = 10\ ft$, laid on a slope $S_o = 0.00025$, with Manning's $n = 0.025$, flows at a normal depth $d_o = d = 1.5\ ft$. The sediment particles specific gravity is $S_s = 2.65$, and the diameters $D_{50} = D = 0.20\ mm = 0.20\times10^{-3}\ m = 6.56\times10^{-4}\ ft$, and $D_{90} = 0.28\ mm = 0.28\times10^{-3}\ m$. Determine the bed sediment load rate (lb/s) for this stream.

(2) Solution:

(i) The calculations proceed as follows:

$$A = bd_0 = 10\ ft \times 1.5\ ft = 1.5\ ft^2$$

$$P = b + 2d_0 = 10\ ft + (2 \times 1.5\ ft) = 13\ ft0$$

$$R_h = A/P = 10\ ft^2/13\ ft = 1.15\ ft$$

$$k' = \frac{26m^{1/3}/s}{D_{90}^{1/6}} = \frac{26}{(0.28x10^{-3})^{1/6}} = 101.65$$

$$k = \frac{C_u}{n} = \frac{1.486}{0.025} = 59.44$$

$$\frac{k}{k'} = 0.5847$$

$$g_B = 8\omega\sqrt{gD^3}\left(\frac{S_s}{S_s - 1}\right)\left[\frac{R_h S_o}{D}\left(\frac{k}{k'}\right)^{3/2} - 0.047(S_s - 1\right]^{3/2} = K_1K_2$$

with (note that K_1 and K_2 are used only to aid computation)

$$K_1 = (8)\left(62.4\frac{lb}{ft^3}\right)\sqrt{32.2\frac{ft}{s^2}(6.56x10^{-4}ft)^3}\left(\frac{2.65}{1.65}\right) = \frac{\frac{0.07644lb}{ft}}{s}$$

$$K_2 = \left[\frac{1.15ft}{6.56x10^{-4}}(0.5847)^{3/2} - 0.047(2.65-1\right]^{3/2} = 0.0407$$

$$g_B = K_1K_2 = \left(\frac{\frac{0.07644lb}{ft}}{s}\right)(0.0407) = \frac{\frac{0.0031lb}{ft}}{s}$$

(ii) For the width *b = 10 ft*, the bed sediment load rate is:

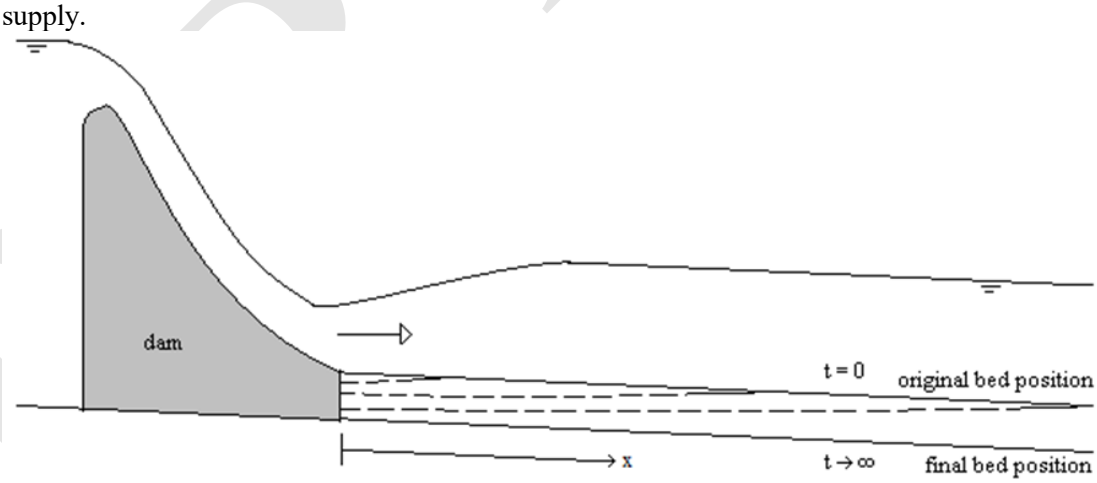
$$G_B = g_B b = 0.0031\ lb/ft/s \times 10\ ft = 0.031\ lb/s$$

634.0324 Scour and Deposition in Channels

A. A channel in equilibrium, from the point of view of sediment transport, is one which is not degrading (losing bed material) nor aggrading (gaining bed material). A decrease in the sediment supply to the channel may cause degradation. For example, if a dam has been built in a stream that typically carries sediment, the water downstream from the dam may contain much less sediment than before and, most likely, would pick up local materials to make up for the loss. On the other hand, an increase in the sediment supply to a channel may cause aggradation as the channel may not be able to carry the additional supply. Refer to NEH 654, Chapter 13, for additional information on local scour calculations.

B. Consider a degrading channel, of infinite length, as consequence of the building of a dam. This situation is depicted in the following figure.

Figure 3-46: Channel bed degradation downstream of a dam due to reduction in sediment supply.



C. Let S_o *be* the slope of the original bed position and S_∞ be the slope of the final bed position. The solution that describes the change in the bed elevation with respect to time *t* and space *x*, measured from the dam foot, is given by (see Raudkivi, 1976):

$$z(x,t) = (S_\infty - S_o)\left[2\sqrt{\frac{Kt}{\pi}}exp\left(\frac{x^2}{4Kt}\right) - x(erfc)\left(\frac{x}{2\sqrt{Kt}}\right)\right] \qquad \text{(eq. 3-104)}$$

where *K* is a parameter defined in terms of an equation of flow (the Chezy equation) and the sediment discharge expression (equation 3-107), and *erfc* is the complementary error function defined as:

$$erfc = 1 - erf(\theta) = 1 - \int_0 \exp(-\xi)\, d\xi \qquad \text{(eq. 3-105)}$$

and *erf* is the error function.

D. The Chezy equation for the flow:

$$V = C\sqrt{dS_f} \qquad \text{(eq. 3-106)}$$

where *C* is the Chezy coefficient, *d* is the flow depth (the equation is given for a wide channel), and S_f is the energy slope (slope of the energy line).

E. The sediment discharge expression is given in terms of the flow velocity *V* as an empirical equation:

$$g_s = aV^b \qquad \text{(eq. 3-107)}$$

where g_s is the unit sediment discharge (sediment discharge per unit width), and *a* and *b* are constant values. The parameter *K* in equation 3-104 is calculated as:

$$K = \frac{C^2 bqa^{3/2} g_s^{1-3/b}}{3(1-\lambda)} \qquad \text{(eq. 3-108)}$$

where $q = Vd$ is the water discharge per unit width, λ is the porosity of the bed sediments (typical value, $\lambda = 0.40$), g_s, *a*, and *b* are defined in equation 3-107, and *C* is the Chezy coefficient (see equation 3-106). *Porosity* of soils or sediments is defined as the ratio of the volume of voids in the material to the total volume (solids + voids).

F. Defining the amount of degradation about the dam as:

$$z_o = 2(S_\infty - S_o)\sqrt{\frac{Kt}{\pi}} \qquad \text{(eq. 3-109)}$$

G. With this definition, equation 3-104 can be written to give a dimensionless bed elevation:

$$\frac{z}{z_o} = \exp\left(-\frac{x^2}{4Kt}\right) - \frac{x}{2}\sqrt{\frac{\pi}{Kt}erfc\left(\frac{x}{2\sqrt{Kt}}\right)} \qquad \text{(eq. 3-110)}$$

H. Defining the initial and final sediment supply as:

$$g_{so} = K(1-\lambda)S_o \qquad \text{(eq. 3-111)}$$

and,

$$g_{s\infty} = K(1-\lambda)S_\infty \qquad \text{(eq. 3-112)}$$

I. The parameter z_o can be written as:

$$z_o = \frac{2(g_{s\infty} - g_{so})}{1-\lambda}\sqrt{\frac{t}{\pi K}} \qquad \text{(eq. 3-113)}$$

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J. Notice that the solution provided by equation 3-104, applies equally to degradation ($g_{so}>g_{s\infty}$) or aggradation ($g_{so}<g_{s\infty}$).

K. Example – Bed aggradation

(1) A laboratory flume is set at a slope $S_o = 0.00356$ and water flows at a uniform depth $d_o = 2$ *inches* $= 0.167$ *ft*, and a velocity $V = 1.31$ *ft/s*. Sediment is supplied into the flow at a rate $g_{so} = 1.598\times10^{-4}\ ft^2/s$ until the channel reaches equilibrium. The estimated value of the parameter K is $K = 0.0748\ ft^2/s$, and the porosity of sediments is $\lambda = 0.40$. Without changing any of the flow conditions, the sediment supply rate is increased to $g_{s\infty} = 7.99\times10^{-4}\ ft^2/s$, so that the channel bed starts aggrading. (a) Compute the maximum amount of aggradation after $t = 1$ *hr* $= 3600$ *s*, and (b) compute the amount of aggradation at a point $x = 2.0$ *ft* downstream of the point of sediment injection, after $t = 1$ *hr* $= 3600$ *s*. Assume that the channel is of infinite length.

(2) Solution;

(i) The maximum amount of aggradation at any given time occurs at position $x = 0$, and is given by equation 3-113:

$$z_o = \frac{2(g_{s\infty} - g_{so})}{1-\lambda}\sqrt{\frac{t}{\pi K}}$$

$$z_o = \frac{2\left(7.99x10^{-4}\,{ft^2}/{s} - 1.598x10^{-4}\,{ft^2}/{s}\right)}{1-\lambda}\sqrt{\frac{3600s}{(3.1416)0.0748\,{ft^2}/{s}}}$$

$$= 0.2637ft = 3.16\ in$$

(ii) To find the aggradation elevation for $x = 2$ *ft* and $t = 3600$ *hr*, use equation 3-110:

$$z = z_o\left(exp\left(-\frac{x^2}{4Kt}\right) - \frac{x}{2}\sqrt{\frac{\pi}{Kt}}erfc\left(\frac{x}{2\sqrt{Kt}}\right)\right)$$

(iii) The value of the function *erfc* may be found by use of an appropriate spreadsheet. The argument of the function for this example is:

$$\frac{x}{2\sqrt{Kt}} = \frac{2.0ft}{2\sqrt{0.0748\,{ft^2}/{s}\,3600s}} = 0.0609$$

(iv) The spreadsheet gives *erfc(0.0609) = 0.931367*. The required aggradated elevation is:

$$z = 0.2637ft\left(exp\left(-\frac{(2.0ft)^2}{(4)0748\,{ft^2}/{s})3600s}\right) - \frac{2ft}{2}\sqrt{\frac{3.1416}{0.0748\,{ft^2}/{s}\,3600s}}(0.931367)\right)$$

$$z = 0.2362ft = 2.83in$$

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L. Example – Bed degradation downstream of dam.

(1) The initial slope of a river channel is $S_o = 0.0094$. It is estimated that, after construction of a dam, the resulting slope will be $S_\infty = 0.0020$. The parameter K is estimated to be $10750\ ft^2/day$. The initial bed elevation at the dam site is $z_s = 1500\ ft$ above mean sea level. Compute the expected bed elevation at a number of points along the stream channel after, (a) two months (60 days), and (b) 2 years (730 days) of the construction of the dam.

(2) Solution:

(i) The elevation at the dam site for $t = 60\ days$, is calculated as:

$$z_o = 2(0.0020 - 0.0094)\sqrt{\frac{10750\frac{ft^2}{day}60day}{3.1416}} = 6.71ft$$

(ii) And for $t = 730\ days$, the elevation z_o is calculated as:

$$z_o = 2(0.0020 - 0.0094)\sqrt{\frac{10750\frac{ft^2}{day}730day}{3.1416}} = -23.39ft$$

(iii) In the following table and plot, the degradation elevations z for $t = 0$, $t = 60\ day$, and $t = 730\ day$, for a number of values of *"x"* are presented. The bed elevation is calculated as follows:

$$Bed\ elevation(x) = Bed\ elevation(x = 0) + z(x) - s_0(x)$$

(iv) The calculations and plot of the data were performed with an appropriate spreadsheet.

Figure 3-47: Expected bed elevation computations for bed degradation downstream of dam

			t = 0	Bed	t = 60 d	Bed	t = 730 d	Bed
$x/2(Kt)^{1/2}$	z/z_o	x(ft)	z(ft)	elev (ft)	z(ft)	elev (ft)	z(ft)	elev (ft)
0.00	1.0000	0.00	0.00	1500.00	-6.71	1493.29	-23.39	1476.61
0.15	0.7565	240.94	0.00	1497.74	-5.07	1492.66	-17.70	1480.04
0.30	0.5569	481.87	0.00	1495.47	-3.73	1491.74	-13.03	1482.44
0.45	0.3983	722.81	0.00	1493.21	-2.67	1490.53	-9.32	1483.89
0.60	0.2764	963.74	0.00	1490.94	-1.85	1489.09	-6.47	1484.48
0.75	0.1858	1204.68	0.00	1488.68	-1.25	1487.43	-4.35	1484.33
0.90	0.1209	1445.61	0.00	1486.41	-0.81	1485.60	-2.83	1483.58
1.05	0.0760	1686.55	0.00	1484.15	-0.51	1483.64	-1.78	1482.37
1.20	0.0462	1927.49	0.00	1481.88	-0.31	1481.57	-1.08	1480.80
1.35	0.0271	2168.42	0.00	1479.62	-0.18	1479.44	-0.63	1478.98
1.50	0.0153	2409.36	0.00	1477.35	-0.10	1477.25	-0.36	1476.99
1.65	0.0083	2650.29	0.00	1475.09	-0.06	1475.03	-0.19	1474.89
1.80	0.0044	2891.23	0.00	1472.82	-0.03	1472.79	-0.10	1472.72

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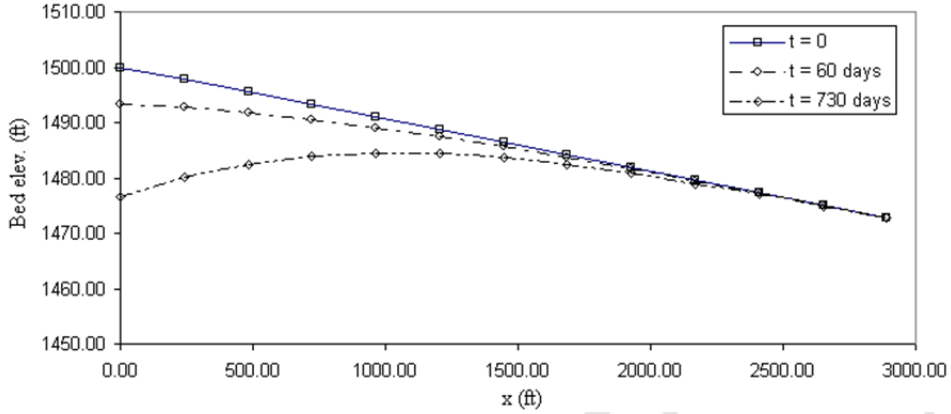
Figure 3-48: Expected bed elevation along stream channel downstream for bed degradation downstream of dam



M. In some instances, a control on the downstream end of a channel of length L may exist such that the bed elevation at the downstream end remains constant even as the channel aggrades or degrades. If the starting bed slope is S_o and the ending bed slope is S_∞, the elevation of the channel bed $z(x,t)$ is given by the following equation (see Raudkivi, 1976):

$$z(x,t) = S_\infty(L-x) + \frac{8L(S_o - S_\infty)}{\pi^2}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}\exp\left(\frac{(2k-1)^2\pi^2 K t}{4L^2}\right)\cos\left(\frac{(2k-1)\pi x}{2L}\right) \quad \text{(eq. 3-114)}$$

The coefficient K is calculated as in equation 3-108.

N. Example – Bed degradation with constant downstream elevation

(1) This example uses the same data as in the previous example, i.e., $S_o = 0.0094$, $S_\infty = 0.0020$, and $K = 10{,}750\ ft^2/day$; however, a control of constant elevation occurs at the downstream end of a channel length $L = 1.5\ mi = 7920\ ft$. The initial bed elevation at the dam site is $z_S = 1{,}500\ ft$. Compute the expected bed elevation at a number of points along the river channel after, (a) two months (60 days), (b) 2 years (730 days), and (c) 10 years (3650 days) of the construction of the dam. (Note: the value of S_∞ can be estimated from the terrain geometry and selected to produce a channel with a mild slope.)

(2) Solution:

(i) The calculations and plot of the data, shown in figure 3-49, were performed with an appropriate spreadsheet.

Figure 3-49: Expected bed elevation with a constant downstream elevation

time (d):	0	Bed	60	Bed	730	Bed	3650	Bed
x(ft)	z(ft)	elev (ft)	z(ft)	elev (ft)	z(ft)	elev (ft)	z(ft)	elev (ft)
0.00	74.44	1500.00	67.74	1493.30	51.06	1476.62	25.99	1451.55
500.0	69.75	1495.31	66.10	1491.66	49.87	1475.43	24.94	1450.50
1000.0	65.05	1490.61	63.30	1488.86	48.32	1473.88	23.79	1449.35
1500.0	60.35	1485.91	59.62	1485.18	46.40	1471.96	22.54	1448.10
2000.0	55.65	1481.21	55.38	1480.94	44.14	1469.70	21.20	1446.76
2500.0	50.95	1476.51	50.87	1476.43	41.55	1467.11	19.77	1445.33
3000.0	46.25	1471.81	46.23	1471.79	38.66	1464.22	18.25	1443.81
3500.0	41.55	1467.11	41.54	1467.10	35.49	1461.05	16.64	1442.20
4000.0	36.85	1462.41	36.85	1462.41	32.08	1457.64	14.96	1440.52
4500.0	32.15	1457.71	32.15	1457.71	28.44	1454.00	13.21	1438.77
5000.0	27.45	1453.01	27.45	1453.01	24.61	1450.17	11.39	1436.95
5500.0	22.75	1448.31	22.75	1448.31	20.63	1446.19	9.53	1435.09
6000.0	18.05	1443.61	18.05	1443.61	16.51	1442.07	7.61	1433.17
6500.0	13.35	1438.91	13.35	1438.91	12.29	1437.85	5.66	1431.22
7000.0	8.65	1434.21	8.65	1434.21	8.00	1433.56	3.68	1429.24
7500.0	3.95	1429.51	3.95	1429.51	3.66	1429.22	1.68	1427.24
7920.0	0.00	1425.56	0.00	1425.56	0.00	1425.56	0.00	1425.56

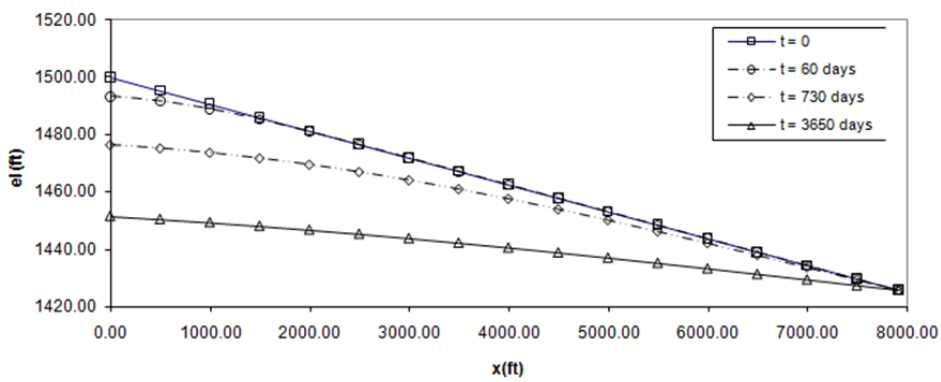



Figure 3-50: Expected bed elevation along stream channel with a constant downstream elevation