United States Department of Agriculture

Natural Resources Conservation Service

Part 634 Hydraulics
National Engineering Handbook

Chapter 2 Hydrostatics

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Part 634 – Hydraulics

Chapter 2 - Hydrostatics

634.0200 General

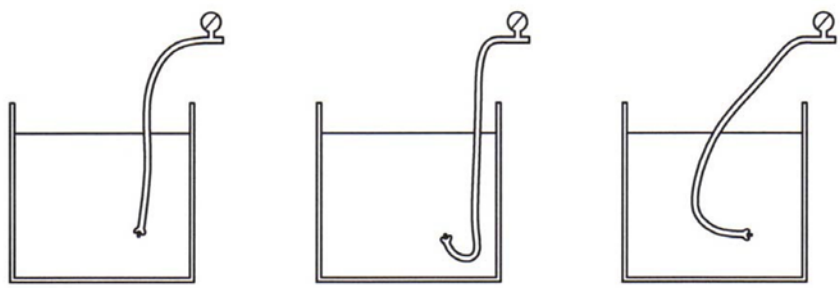
Hydrostatics refers to the study of water (and other liquids) at rest. Such is the case, for example, of water contained in a storage tank with no flow in or out. Subjects of interest in hydrostatics include determination of pressures in a fluid, instruments for measuring pressure, calculation of forces on submerged surfaces (e.g., on gates or tank walls), and study of buoyancy.

634.0201 Hydrostatic Pressure Relationships

A. Pressure is defined as the force per unit area that a fluid (liquid or gas) exerts on a surface submerged in the fluid. The pressures discussed here are those within a body of water at rest.

(1) Consider, for example, a water tank as shown in the figure 2-1 below.

Figure 2-1: Schematic of pressure measurement in a liquid



A small probe of area A is introduced at a point in the tank as shown. If the force that the water exerts on the probe tip is F, then the pressure at that point is:

$$p = \frac{F}{A} \qquad \text{(eq. 2-1)}$$

where: p = pressure
F = force
A = area

(2) Changing the orientation of the probe's tip at the same point, does not change the force exerted by the water on the tip. Thus, the pressure at any given point does not change with the orientation of the surface acted upon. Pressure is said to be isotropic, i.e., it is independent of the orientation in which it is measured.

(3) The table in figure 2-2 lists some of the most commonly used units of pressure in the English System (ES) and International System (SI). Notice that pressure can also be expressed in terms of the height of a liquid column as shown below. In this table, H_20 and Hg are the chemical symbols of water and mercury, respectively.

Figure 2-2: Commonly used units of pressure

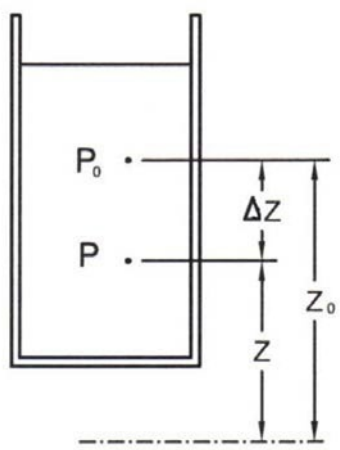
Unit of Pressure	Definition or Equivalent
Basic Units	
psf	1 lb/ft^2 = 0.006944 psi = 47.88 Pa
psi	1 lb/in^2 = 144 psf = 6,894.76 Pa
Pa (Pascal)	1 N/m^2 = 0.02088 psf = 0.000145 psi
Other Units	
kPa (kilo Pascal)	1,000 Pa
MPa (mega Pascal)	1,000 kPa = 1,000,000 Pa
bar	14.50 psi = 100,000 Pa
mb (millibar)	0.001 bar = 0.014504 psi = 100 Pa
atm (atmosphere)	14.7 psi = 101,325 Pa
Height of Liquid Column	
1 m H_20	1.4209 psi = 9796.85 Pa
1 ft H_20	0.4331 psi = 2986.08 Pa
1 in H_20	0.03609 psi = 248.84 Pa
1 mm Hg	0.01934 psi = 133.32 Pa
1 in Hg	0.4912 psi = 3,386.39 Pa

(4) Pressure, within a liquid at rest, increases linearly with depth. Referring to figure 2-3, if the pressure at elevation z_0 is known to be p_0, then the pressure p at elevation z, is given by the hydrostatic law:

$$p = p_0 + \omega \Delta z$$

where ω is the specific weight of the liquid, and $\Delta z = z - z_0$ is the difference in elevation of the two points of interest. The elevations z and z_0 are measured from any common horizontal level or datum.

Figure 2-3: Pressures within a liquid at rest



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B. Atmospheric pressure

Atmospheric pressure refers to the pressure exerted by the weight of the air in the atmosphere. As there is greater weight of air above lower versus higher levels on the surface of the earth, atmospheric pressure decreases as elevation increases. Exhibit 5 shows the typical values of atmospheric pressure at different elevations above mean sea level. Atmospheric pressure is measured with an instrument called a barometer. Typical values at mean sea level and at a temperature of 59º F (15º C) are:

$$p_{atm} = 1\ atm = 2{,}116.22\ psf = 14.70\ psi = 101.33\ kPa$$

$$p_{atm} = 760\ mm\ Hg = 407.19\ in\ H_2O = 29.92\ in\ Hg$$

C. Absolute pressure and gage pressure.

(1) Absolute pressure refers to the pressure measured with a zero-value corresponding to a perfect vacuum, i.e., the total absence of matter in a volume. Absolute pressure (p_{abs}), therefore, is always a positive quantity. Barometric pressure is an example of absolute pressure. To emphasize that a certain quantity is reported in units of absolute pressure, sometimes 'a' or 'abs' is added to the units of pressure. For example, atmospheric pressure is written as:

$$p_{atm} = 2{,}116.22\ psfa = 14.70\ psia = 101.33\ kPa - abs$$

$$p_{atm} = 760\ mmHg \cdot abs = 427.19\ in\ H_2O \cdot abs = 29.29\ in\ Hg \cdot abs$$

(2) On the other hand, when measuring pressure with manometers (see next section), it is possible to shift the zero value of the scale to the level of atmospheric pressure. Thus, in this gage pressure scale, pressures above atmospheric will be positive, while those below atmospheric will be negative.

(3) Absolute and gage pressures are related by the following equation:

$$p_{abs} = p_{gage} + p_{atm} \qquad \text{(eq. 2-2)}$$

For example, if the atmospheric pressure is p_{atm} = 13.5 psi at a given location, and if a pressure gage on a pipe reads p_{gage} = -12 psi, then the corresponding absolute pressure is:

$$p_{abs} = p_{gage} + p_{atm} = \text{-12 psi +13.5 psi = 1.5 psi}$$

D. Gage pressure distribution in liquids.

(1) Consider a tank open to the atmosphere. The gage pressure at the free surface of the tank would be zero, by definition. The gage pressure at any point located at a depth h below the free surface will be given by:

$$p_{gage} = 0 + \omega h = \omega h \qquad \text{(eq. 2-3)}$$

(2) If the tank is closed and the free surface is pressurized at pressure p_s, then the pressure at a point at depth h below the free surface will be subject to a pressure given by:

$$p = p_s + \omega H$$

(3) The pressure p, calculated above, would be an absolute or a gage pressure depending on whether p_s is given as an absolute or gage pressure, respectively.

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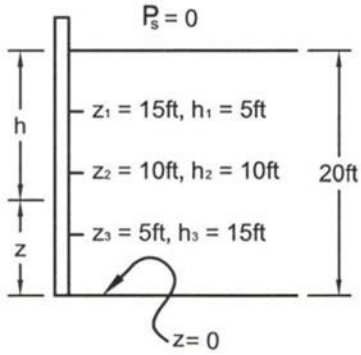
E. Example – Pressure at depth in water

As an example, consider the water contained in a reservoir to a depth of 20-ft, as illustrated in Figure 2-4.

Figure 2-4: Calculation of pressures at different depths



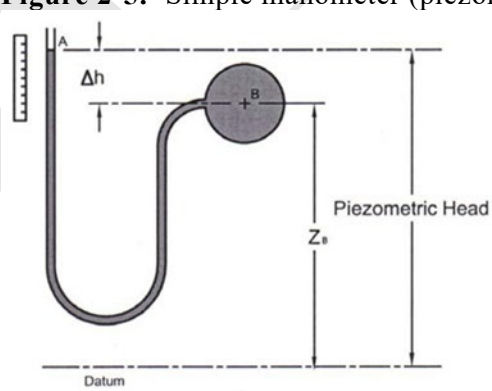
(1) Calculating the pressure at points 1, 2, and 3, located at elevations z_1 = 15 ft, z_2 = 10 ft, and z_3 = 5 ft, measured from the reservoir's bottom, requires that the water depths be calculated first:
   (i) h_1 = 20 ft – z_1 = 20 ft – 15 ft = 5 ft
   (ii) h_2 = 20 ft – z_2 = 20 ft – 10 ft = 10 ft
   (iii) h_3 = 20 ft – z_3 = 20 ft – 5 ft = 15 ft

(2) Then using ω = 62.4 lb/ft3 for the specific weight of water, the corresponding (gage) pressures are:
   (i) $p_1 = \omega\, h_1$ = 62.4 lb/ft^3 x 5 ft = 312 lb/ft^2 = 312/144 lb/in^2 = 2.2 psi
   (ii) $p_2 = \omega\, h_2$ = 62.4 lb/ft^3 x 10 ft = 624 lb/ft^2 = 624/144 lb/in^2 = 4.3 psi
   (iii) $p_3 = \omega\, h_3$ = 62.4 lb/ft^3 x 15 ft = 936 lb/ft^2 = 936/144 lb/in^2 = 6.5 psi

634.0202 Piezometers and Manometers

A. Manometers are instruments used in the measurement of pressure. The simplest manometer consists of a u-tube with a leg attached to the point where the pressure will be measured, and the other leg open to the atmosphere. Such a manometer is illustrated in figure 2-5.

Figure 2-5: Simple manometer (piezometer)



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B. The circle centered at B may represent, for example, the cross-section of a flowing pipe. The elevation of point A (open to the atmosphere) with respect of the pipe centerline B is equal to Δh. Now, suppose that the liquid (e.g., water) in the pipe and manometer has a specific weight ω, then, according to the hydrostatic law, the pressure at B is given by:

$$p_B = p_A + \omega \, \Delta h$$

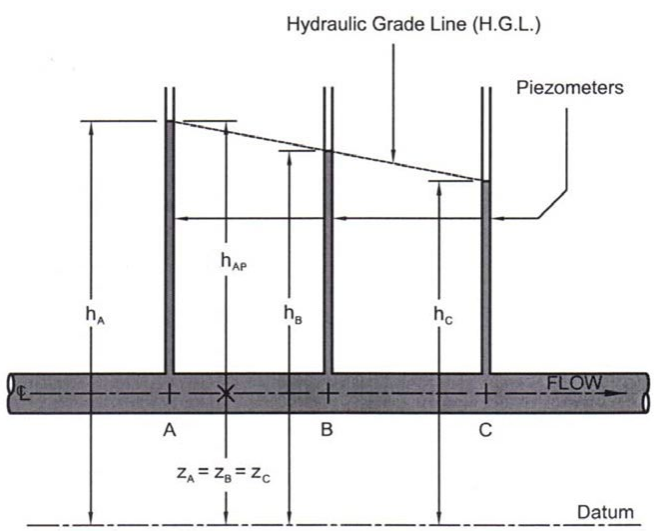
C. If we use gage pressure to report our result, then we can take $p_A = 0$, and the pressure at the pipe centerline will be simply:

$$pB = \omega \, \Delta h$$

D. In pipeline flow we are often interested in determining the so-called piezometric head of the flow at a given location. The piezometric head, as illustrated in figure 2-5, above, is the sum of the pressure head ($\Delta h = p_B/\omega$) plus the elevation of the pipe centerline z_B. Thus, a manometer, as the one illustrated above, is also known as a piezometer, for it shows the piezometric head in a pipe flow.

E. In many cases, piezometers are simply vertical tubes attached to the top of the pipeline, as illustrated in figure 2-6.

Figure 2-6: Piezometers on a horizontal pipe flow



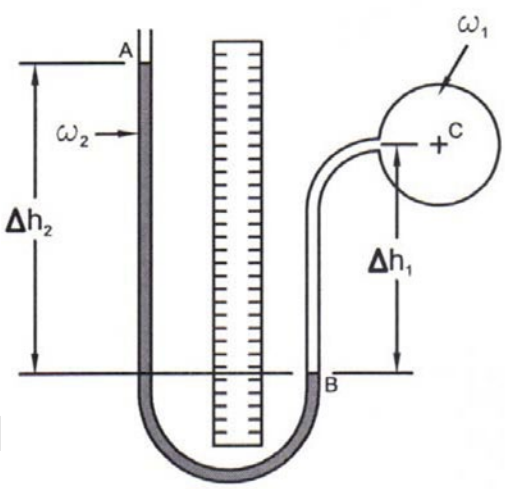
F. The piezometers in figure 2-6 above show the location of the hydraulic grade line (HGL), which, in a horizontal pipeline, illustrates the decrease in pressure along the pipeline in the direction of the flow. The piezometers in the figure show that the piezometric head decreases from A to B to C, thus, $h_A > h_B > h_C$. The centerline elevation of points A, B, and C is the same, i.e., $z_A = z_B = z_C$, therefore, the pressure heads (for point A, the pressure head is $h_A - z_A$) will be such that $h_{AP} > h_{BP} > h_{CP}$.

G. The photograph in figure 2-7, below, shows piezometers located before and after an orifice meter in a transparent pipeline, typically used in a laboratory setting. An orifice meter is used to measure flow discharge in a pipe. The piezometers are used to measure the pressure variation about the orifice meter.0

Figure 2-7: Piezometers near an orifice meter in a pipeline

H. Figure 2-8 below shows a U-tube manometer being used to determine the pressure at the centerline of a flowing pipe (point C). The flowing fluid has a specific weight ω_1, while the manometric fluid has a specific weight ω_2.

Figure 2-8: U-tube manometer



I. For the case shown in figure 2-8, above, point A is open to the atmosphere, thus, using gage pressures, we can write $p_A = 0$. The interface between the two liquids, point B, is called a meniscus. Point B is located at a depth Δh_2 with respect to the free surface meniscus A. Thus, the gage pressure at point B can be calculated as:

$$p_B = p_A + \omega_2 \Delta h_2 = 0 + \omega_2 \Delta h_2 = \omega_2 \Delta h_2$$

J. On the other hand, within the flowing fluid, the pressure at point B can be written as (hydrostatic law):

$$p_B = p_C + \omega_1 \Delta h_1$$

K. Equating the two expressions found above for p_B one can write:

$$p_C + \omega_1 \Delta_1 = \omega_2 \Delta h_2$$

from which it follows that:

$$p_C = \omega_2 \Delta h_2 - \omega_1 \Delta h_1$$

L. If the flowing fluid is water, $\omega_1 = \omega_w$ (the specific weight of water), and $\omega_2 = S_m \omega_w$, where S_m is the specific gravity of the manometric fluid (e.g., for mercury, $S_m = 13.56$). Thus, one can write:

$$p_C = S_m \omega_w \Delta h_2 - \omega_w \Delta h_1 = \omega_w (S_m \Delta h_2 - \Delta h_1)$$

M. For example, to determine the pressure at point C in the figure above, given that the manometric fluid is mercury ($S_m = 13.56$), and that $\Delta h_2 = 6$ in = 6/12 ft = 0.5 ft, $\Delta h_1 = 2$ in = 2/12 ft = 0.167 ft, and $\omega_w = 62.4$ lb/ft^3, using the equation above:

$$p_C = \omega_w (S_m \Delta h_2 - \Delta h_1) = (62.4\ lb/ft^3)\ (13.56 \times 0.5\ ft - 0.167\ ft) = 412.7\ lb/ft^2$$

or

$$p_c = \frac{412.7\ lb}{144\ in^2} = 2.87\ psi \approx 3\ psi$$

N. The photograph in figure 2-9 shows a u-tube manometer used to measure flow discharge in the pipe located behind the manometer. The manometer legs in the photograph are attached to tubes connected to a Venturi meter (not shown). The manometric fluid shown is a red manometric fluid with a specific gravity $S_m = 0.75$.

Figure 2-9: U-tube manometer for flow measurement in a pipeline

O. Rules for manometer calculations.

(1) The following rules apply to the calculation of pressures in tube manometers involving many fluids:

(i) Start at a point of known pressure or at a point where the pressure is required.

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(ii) Following the tube manometer to the next meniscus, add the product (specific weight depth) if the meniscus is located below the starting point, or subtract the same product if the meniscus is located above the starting point.

(iii) Continuing the path to the next meniscus, add the product (specific weight x depth) if the meniscus is located below the previous meniscus, or subtract the same product if the meniscus is located above the previous meniscus.

(iv) When reaching the ending point in the manometer path, make the resulting sum equal to the pressure at the ending point.

- For example, for the piezometer in figure 2-5, one can write:

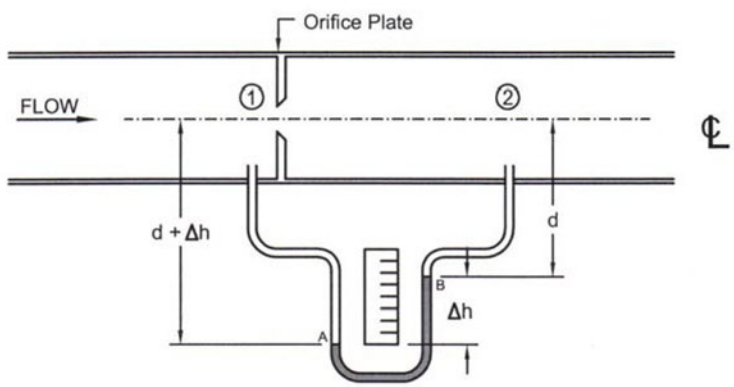
$$pA + \omega\, \Delta h = pB$$

- For the manometer of figure 2-8, one can write:

$$p_A + \omega_2\, \Delta h_2 - \omega_1\, \Delta h_1 = p_C$$

- As an additional example, consider the case in which a manometer at an orifice plate, illustrated in figure 2-10, below.

Figure 2-10: Schematic of manometric measurements at an orifice plate in a pipe



-To determine the difference in pressures between points 1 and 2, use the rules for manometer calculations presented above; start at point 1 and first write:

$$p_1 + \omega_w\, (d+\Delta h)$$

-The above expression reaches from point 1 to meniscus A using water for the specific weight. From point A to point B, use the specific weight of the manometric fluid, and subtract the amount $\omega_m \Delta h$ from the expression above, thus, producing:

$$p_1 + \omega_w\, (d + \Delta h) - \omega_m\, \Delta h$$

-From point B to point 2, subtract the amount $\omega_w d$ from the expression above, and make the result equal to the pressure at point 2:

$$p_1 + \omega_w\, (d + \Delta h) - \omega_m \Delta h - \omega_w d = p_2$$

-Expanding the expression above:

$$p_1 + \omega_w d + \omega_w\ \Delta h - \omega_m \Delta h - \omega_w d = p_2$$

-Eliminating the terms $+\ \omega_w d$ and $-\ \omega_w d$, one gets:

$$p_1 + \omega_w\ \Delta h - \omega_m \Delta h = p_2$$

-Algebraic manipulation of this equation allows writing:

$$\Delta p = p_1 - p_2 = (\omega_m - \omega_w)\,\Delta h$$

-The resulting equation can be simplified by introducing the specific gravity of the manometric fluid, i.e., $\omega_m = S_m \omega_w$, thus:

$$\Delta p = \omega_w\,\Delta h\,(S_m - 1)$$

-If $\omega_w = 62.4$ lb/ft^3, $\Delta h = 8$ in = 8/12 ft = 0.666 ft, and $S_m = 13.56$ (for mercury), the difference in pressure, Δp is:

$$\Delta p = \omega_w\,\Delta h\,(S_m - 1) = (62.4\ lb/ft^3)\,(0.666\ ft)\,(13.56 - 1)$$
$$= 521.97\ lb/ft^2$$

or

$$\Delta p = \frac{521.97\ lb}{144\ in^2} = 3.62\ psi$$

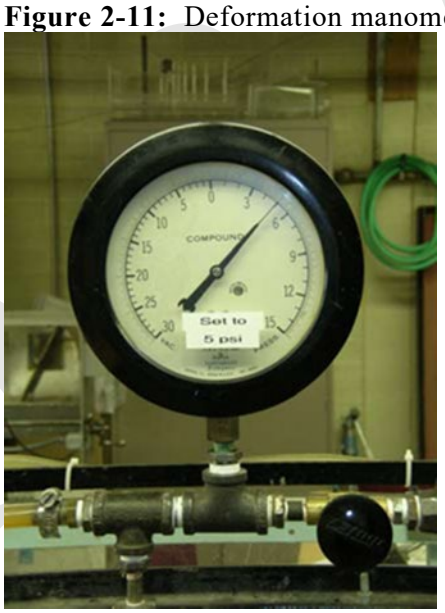
(2) Orifice plates may be used to measure flow in pipelines. The pressure difference, Δp, can be related to the pipeline discharge by calibration or by theoretical analysis. Examples of such analyses are presented in the next section.

P. Deformation Manometers.

(1) Deformation manometers, such as the Bourdon manometer shown in figure 2-11, utilize the deformation of spiral tubes or of diaphragms to measure pressure. These manometers are calibrated by manufacturers or in the laboratory and provide a direct reading of the pressure at the manometric tap.

(2) Modern deformation manometers have digital readouts, making the reading of the pressure straightforward. The Bourdon manometer shown in figure 2-11 has an analog scale, with the pressure marked by the pointer attached to the spiral tube located inside the manometer.

Figure 2-11: Deformation manometer (Bourdon manometer)



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634.0203 Forces on Submerged Plane Surfaces

A. The calculation of the size, direction, and location of the forces on submerged surfaces is essential in the design of dams, bulkheads, water control gates, and other related appurtenances.

B. Horizontal surface.

(1) The hydrostatic law indicates that pressure varies with depth. Thus, a horizontal surface within a liquid at rest is subject to the same pressure p over the entire surface, and the resultant force F on the surface is given by:

$$F = pA = \omega hA \qquad \text{(eq. 2-4)}$$

where A is the area of the surface.

(2) Figure 2-12 shows that the pressure on top of the horizontal surface is represented by vertical arrows of the same height pointing towards the surface. The pressure arrows and the horizontal surface form a three-dimensional figure known as the pressure prism. The resultant force vector on any surface coincides with the centroid (also referred to as the center of mass or center of gravity) of the pressure prism. The force on a horizontal surface will be vertical and applied to the centroid C of the surface as illustrated in figure 2-13. The point of application of the force is referred to as the center of pressure P.

Figure 2-12: Force on a submerged horizontal surface

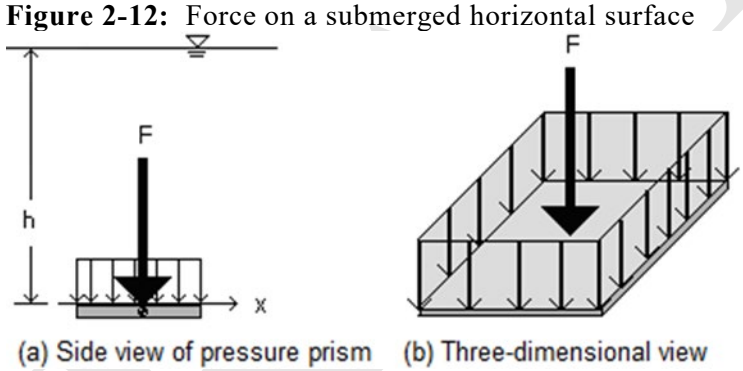
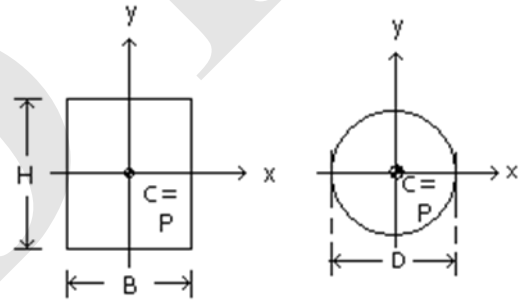


Figure 2-13: Point of application of a force on a submerged horizontal surface



(3) Example 2 – Hydrostatic force on horizontal area

(i) A circular tank 15 ft in diameter is filled with water to a depth of 2 ft. Determine the magnitude and location of the vertical force that the water applies on the tank bottom.

(ii) The tank has a diameter D = 15 ft, therefore, the area of the tank bottom is that of a circle:

$$A = \frac{\pi D^2}{4} = \frac{(3.1416)(15\ ft)^2}{4} = 176.2\ ft^2$$

(iii) The magnitude of the pressure applied to the bottom of the tank can be calculated by using the specific weight of water, ω_W = 62.4 lb/ft^3 and depth of the water in the tank, h = 2 ft:

$$p = \Delta h = (62.4\ lb/ft^3)\ (2\ ft) = 124.8\ lb/ft^2$$

(iv) The magnitude of the force applied on the tank bottom is:

$$F = pA = (124.8\ lb/ft^2)\ (176.71\ ft^2) = 22{,}053.4\ lb$$

(v) The force is applied at the center of the circular bottom and is equivalent to the weight of the water.

C. Inclined surface.

(1) For a surface located on an inclined plane, the pressure increases linearly from the top of the surface to the bottom of the surface. The magnitude of the force on the surface is calculated as:

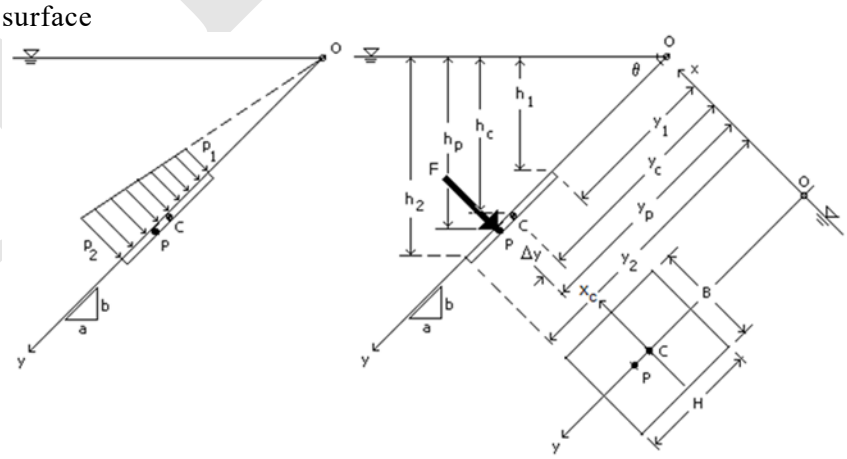
$$F = p_c A = \omega h_c A \qquad \text{(eq. 2-5)}$$

where $p_c = \omega h_c$ is the pressure at the centroid of the figure. The force, F, is represented by the volume of the pressure prism.

(2) Figure 2-14, below, shows the pressure distribution along an inclined surface and the resulting force on a rectangular region laid on the inclined surface. The rectangular region could represent a gate on the slope of a dam or dike. The inclined surface is located at an angle θ with respect to the horizontal. Alternatively, the slope can be indicated by the proportion zH (horizontal):1V (vertical) as shown in the figure. For the case illustrated in figure 2-14, the angle θ is related to the slope by:

$$tan(\theta) = \frac{b}{a} = \frac{1}{z} \qquad \text{(eq. 2-6)}$$

Figure 2-14: Pressure distribution, force, and center of pressure on an inclined surface



(a) Pressure distribution (b) Force and center of pressure

(3) Figure 2-14 shows a system of coordinate axes, *x* and *y*, located on the inclined surface. The system is selected so that the x-axis is located along the free surface. Points on the inclined surface can be located by either their depth h or their y coordinate along the surface. These two distances are related by:

$$h = y \cdot \sin(\theta) \qquad \text{(eq. 2-7)}$$

(4) Point 1 is located at the top of the gate while point 2 is located at the bottom of the gate. The gate dimensions are *B* (width) and *H* (height). Point C represents the centroid of the gate, while point P represents the point of application of the hydrostatic force *F* on the gate, the center of pressure.

(5) Unlike the case of a horizontal surface, the center of pressure on an inclined surface is located below the centroid of the surface along the plane of the surface, by a distance given by:

$$\Delta y = \frac{I_c}{(A) y_c} \qquad \text{(eq. 2-8)}$$

(6) In this formula, y_c is the location of the centroid of the surface measured from the free surface along the plane of the surface, and I_c is the moment of inertia of the surface with respect to a centroidal axis parallel to the x axis (i.e., an axis through point C). Thus, the center of pressure will be located at a distance:

$$y_p = y_c + \Delta y = y_c + \frac{I_c}{A\, y_c} \qquad \text{(eq. 2-9)}$$

(1) For the rectangular and circular figures of figure 2-13, with the x axis representing the centroidal axis xc, the centroidal moments of inertia are the following:

(i) Rectangular area: $I_c = \frac{1}{12} BH^3$ (eq. 2-10)

(ii) Circular area: $I_c = \frac{\pi}{64} D^4$ (eq. 2-11)

(2) Since most gates are either rectangular or circular, equations 2-10 and 2-11 will be useful for calculating the center of pressure of those types of gates by using equations 2-8 and 2-9.

(10) Example 3 – Hydrostatic force on inclined area

(i) For the rectangular gate illustrated in figure 2-14, $B = 3$ ft, $H = 4.5$ ft, $a = 2$, $b = 1$, $y_1 = 5$ ft, determine the force on the gate and the location of the center of pressure.

(ii) The slope is specified by the numbers $a = 2$ and $b = 1$, i.e., 2H:1V. The corresponding angle θ is calculated as:

$$\theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{1}{2}\right) = 26.57°$$

(iii) While the top of the gate is located at a distance $y_1 = 5$ ft measured along the slope, the bottom of the gate will be located at:

$$y_2 = y_1 + H = 5\,ft + 4.5\,ft = 9.5\,ft$$

(iv) The centroid is located midway between y_1 and y_2:

$$y_c = \frac{y_1 + y_2}{2} = \frac{5\,ft + 9.5\,ft}{2} = 7.25\,ft$$

(v) The depth of the centroid is:

$$h_c = y_c \sin(\theta) = 7.25\,ft \times \sin(26.57°) = 3.24\,ft$$

(vi) Thus, the pressure at the centroid is (equation 2-7):

$$p_c = \omega h_c = 62.4\,lb/ft^3 \times 3.24\,ft = 202.18\,lb/ft^2$$

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(vii) The area of the gate is:

$$A = BH = 3\,ft \times 4.5\,ft = 13.5\,ft^2$$

(viii) The force on the gate (equation 2-5):

$$F = p_c A = 202.18\,lb/ft^2 \times 13.5\,ft^2 = 2{,}729.43\,lb$$

(viii) To calculate the location of the center of pressure, start by calculating the centroidal moment of inertia using equation 2-10:

$$I_c = \frac{1}{12}\,BH^3 = \frac{1}{12}\,(3\,ft)(4.5ft) = 22.78\,ft^4$$

(ix) The distance between the centroid C and the center of pressure is calculated with equation 2-8:

$$\Delta y = \frac{I_c}{A\,y_c} = \frac{22.78\,ft^2}{(13.5\,ft^2)(7.25\,ft)} = 0.23\,ft$$

(x) The center of pressure is located at the distance (equation 2-9):

$$y_p = y_c + \Delta y = 7.25\,ft + 0.23\,ft = 7.48\,ft.$$

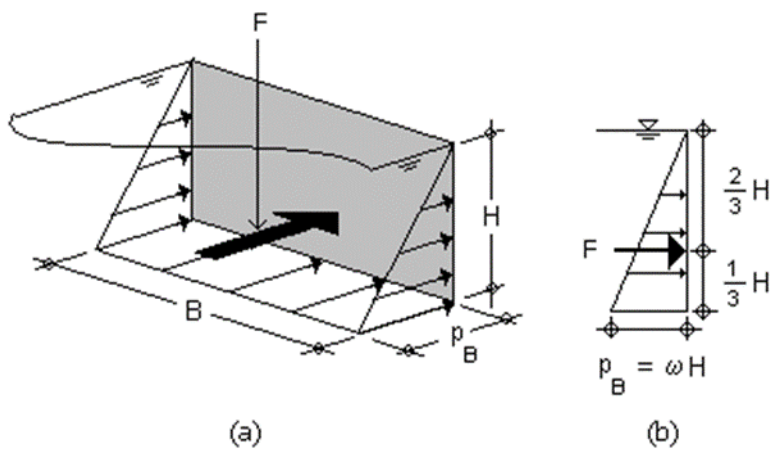
(xi) And the depth of the center of pressure is (equation 2-7):

$$h_p = y_p \sin(\theta) = 7.48\,ft \times \sin(26.56^o) = 3.35\,ft.$$

D. Vertical surface.

(1) In figure 2-15, a rectangular surface of width *B* is located on a vertical plane and the free surface of the water reaches to a depth *H*.

Figure 2-15: Pressure prism (a) and force location (b) for a vertical rectangular surface



(2) The triangular distribution in figure 2-15(b) represents the pressure distribution on the vertical surface. The pressure at the bottom is given by $p_B = \omega H$. The force F on the surface is equal to the volume of the pressure prism:

$$F = \left(\frac{1}{2}P_B H\right) B = \frac{1}{2}\omega\, B\, H^2 \qquad \text{(eq. 2-12)}$$

(3) Using equations 2-9 and 2-10 and the area of this rectangular surface, A = BH, one can prove that the location of the center of pressure (point of application of the force) is given by:

$$y_p = h_p = \frac{2}{3}H \qquad \text{(eq. 2-13)}$$

(4) Thus, the force is applied at 2/3 of the depth measured from the surface, or 1/3 of the depth measured from the bottom as indicated in figure 2-15 (b). Examples of this type surface include vertical gates and flashboards.

(5) Example 4 – Hydrostatic force on vertical gate

(i) A wooden vertical gate with a width of 10 ft is used to close a canal. If the water depth on the gate is 2.5 ft, determine the hydrostatic force on the gate and its location.

(ii) Solution:

- For this case B = 10 ft, H = 2.5 ft, and ω= 62.4 lb/ft^3, thus, the force is (equation 2-12):

$$F = \frac{1}{2}\,\omega\, B\, H^2 = \frac{1}{2}\,(62.4)(10\,ft)(2.5)^2 = 1950\,lb$$

- The force is located at a distance from the surface (equation 2-13):

$$y_p = h_p = \frac{2}{3}H = \frac{2}{3}\,2.5\,ft = 1.67\,ft$$

- The use of a spreadsheet application greatly facilitates calculation of forces on submerged plane surfaces.

634.0204 Buoyancy

A. Bouyancy Forces

Buoyancy is the upwards force experienced by solid bodies submerged in liquids or gases. Archimedes' principle states that a solid body submerged in a fluid (i.e., liquid or gas) experiences a vertical upward force (the buoyancy force) equal to the weight of the volume of fluid it displaces. Thus, the buoyancy force, F_B, experienced by a body of volume V submerged in a fluid of specific weight ω, is given by:

$$F_B = \omega V \qquad \text{(eq. 2-14)}$$

B. Buoyancy Applications

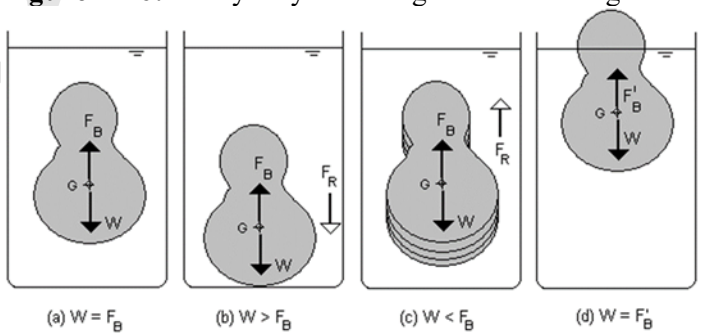
(1) A solid body submerged is also acted upon by its own weight, which can be calculated as

$$W = \omega_s\, V \qquad \text{(eq. 2-15)}$$

where ω_s is the specific weight (weight per unit volume) of the solid material.

(2) A solid body fully submerged in water is subject to its own weight W (equation 2-15) and the buoyancy force (equation 2-14) exerted by the water on the solid body. Figure 2-16 illustrates several possibilities in terms of force equilibrium when a solid body, fully submerged in water, is released.

Figure 2-16: Buoyancy and weight forces acting on a submerged body



(3) If the net resulting force, F_R, on the body, i.e., the difference between its weight W and the buoyancy force F_B, is zero, the body is said to be neutrally buoyant and it will remain in place within the water as illustrated in figure 2-16 (a).

(4) The net resulting force, F_R, is a downward force if the weight is larger than the buoyancy force ($W>F_B$), i.e., if the specific weight of the solid is larger than that of water $\omega_s > \omega$, e.g., metal). In this case the solid body will sink into the liquid until reaching the bottom of a container, as illustrated in figure 2-16 (b).

(5) On the other hand, if the weight is smaller than the buoyancy force ($W<F_B$), i.e., if the specific weight of the solid is smaller than that of water ($\omega_s < \omega$, e.g., wood), when released, the body will float upwards towards the free surface as illustrated in figure 2-16 (c). A floating body will reach equilibrium when its weight is balanced by an equal buoyancy force F'_B, calculated based on the volume of water displaced below the flotation line (V_b). Therefore, for the case illustrated in figure 2-16 (d):

$$W = F'_B = \omega V_b \qquad \text{(eq. 2-16)}$$

C. Example 5 – Buoyancy force calculation – depth of a loaded barge

(1) A small barge with a rectangular bottom 4 ft by 3 ft is to be used to carry 400 lb of construction materials. What is the minimum depth needed to carry such weight?

(2) Solution - The weight W = 400 lb must be balanced by the buoyancy force on the barge, namely, $F'_B = 62.4\ lb/ft^3 \times 4\ ft \times 3\ ft \times h$, thus:

$$h = \frac{400}{\left(62.4\ \frac{lb}{ft^3}\right)(4\ ft)(3\ ft)} = 0.53\ ft = (0.53\ ft)\left(\frac{12in}{ft}\right) = 6.4in$$

D. Example 6 – Buoyancy force calculation – flotation safety of CMP inlet

(1) Figure 2-17 below shows a cylindrical CMP (corrugated metal pipe) vertical inlet of diameter d = 36 in = 3 ft, supported on a square footing of side length L, and height h_b =13.5 in = 1.125 ft. The inlet footing is covered with saturated soil up to a depth h = 7 ft which corresponds to the level of the inlet crest. The CMP inlet weighs ω_{CMP} = 38 lb/ft, the buoyant weight of saturated soil on the footing is ω_{SS} = 60 lb/ft^3, and the concrete in the footing weighs ω_C = 150 lb/ft3. Determine the size L of footing required to make the CMP inlet safe from flotation with a factor of safety $F.S.$= 1.5. Assume that there is no frictional resistance between the inlet walls and the surrounding soil, and that no shear forces act on the outlet conduit. The minimum length L must be $L = d + 1$ ft to allow for placement of reinforcing steel bars outside of the inlet with 3 in concrete cover and 2 in clearance between bars and inlet.

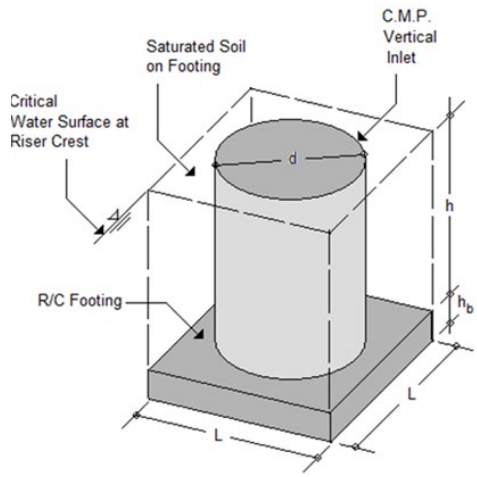
(2) Solution:

(i) The factor of safety, $F.S.$ for this case, is defined as the ratio of the net downward forces (ΣF_d) to the net upward forces (ΣF_u):

$$F.S. = \frac{\sum F_d}{\sum F_u}$$

(ii) Here, $\underline{\Sigma}$, indicates summation of forces. By requiring that the factor of safety be <u>F.S.</u> = 1.5, the downward forces are 50% larger than the upward forces ensuring that the inlet will be safe from flotation.

Figure 2-17: Cylindrical CMP vertical inlet on a square footing



(iii) Using the minimum length of footing:

$$L = d + 1ft = 3\ ft + 1\ ft = 4\ ft$$

(iv) The downward forces in this problem include the weight of the inlet, W_{CMP}, the buoyant weight of saturated soil, WSS, and the weight of footing, W_C. With the minimum length, these forces are calculated as follows:

$$W_{CMP} = \omega_{CMP}\ (h+hb) = 38\ lb/ft \times (7\ ft + 1.125\ ft) = 308.75\ lb$$

$$W_{SS} = \omega_{SS}\,(L2h - \pi d^2/4\ h) = 60\ lb/ft^3 \times ((4\ ft)^2 - 3.1416 \times (3ft)^2/4) \times (7\ ft)$$
$$= 3{,}751.2\ lb$$

$$W_C = \omega_C L^2 h_b = 150\ lb/ft^3 \times (4\ ft)^2 \times 1.125\ ft = 2{,}700\ lb$$

Thus,

$$\Sigma F_d = W_{CMP} + W_{SS} + W_C = 308.8\ lb + 3{,}751.2\ lb + 2{,}700\ lb = 6{,}760\ lb$$

(v) The upward forces include the buoyancy forces on the riser, F_{BCMP}, and on the footing, F_{BC}, which are calculated as:

$$F_{BCMP} = \omega_w\ (\pi d^2/4)\ h = 62.4\ lb/ft^3 \times (3.1416 \times (3\ ft)^2/4) \times 7\ ft = 3{,}088\ lb$$

$$F_{BC} = \omega_w\ L^2 h_b = 62.4\ lb/ft^3 \times (4\ ft)^2 \times 1.125\ ft = 1{,}123\ lb$$

Thus,

$$\Sigma F_u = F_{BCMP} + F_{BC} = 3{,}087\ lb + 1{,}123\ lb = 4{,}210\ lb$$

(vi) The factor of safety for a side of footing L = 4 ft is:

$$F.S. = \frac{\sum F_d}{\sum F_u} = \frac{6760}{4210} = 1.61 > 1.5\ ok$$

(vii) Since the factor of safety calculated is larger than 1.5 the CMP inlet should be safe for buoyancy. If $F.S.$ < 1.5, try a larger footing side length or height, and recalculate the sum of forces and the factor of safety until it is larger than the required value of 1.5.

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