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Chapter 51 Earth Spillway Erosion Model

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628.5100   Introduction and general concepts

The earth spillway erosion model incorporated into the 1995 NRCS Water Resource Site Analysis Program combines basic physical principles with data gathered from performance of actual field spillways. This program predicts the extent of erosion expected from a given hydrograph through the spillway. The intended use of the model is to evaluate the potential for the flow to breach the spillway. To determine the potential for breach, the model iteratively analyzes various headcut formation and advance scenarios to determine the one representing the greatest risk of spillway breach. Each potential headcut is analyzed as if it were the only erosion occurring in the spillway. The eroded profile appearing in the model output is the combined profile resulting from all headcuts evaluated. Therefore, details of the predicted eroded surface may not exactly match those actually developed in the field.

The erosion model is two dimensional, considering only downward and upstream erosion components. Discharge per unit width of the spillway is used to compute hydraulic attack. Flow concentrations associated with three-dimensional flow are accounted for as described in this chapter. All spillway exit channel slopes are assumed to be of sufficient length for the flow to approach normal depth. To allow the model to identify the conditions representing the greatest risk of breach, the spillway profile and associated materials must be described along the flow path from the upstream end of the crest to the elevation of tailwater at the time of flow. Channel width and shape are treated as constant for all reaches. Although erosion computations are terminated at the time a headcut reaches the upstream end of the crest (identified by adverse slope), it is usually desirable to also describe inlet reaches for purposes of spillway rating computations.

The potential for headcut formation and advance is evaluated for each reach downstream of the spillway crest. A reach is defined as a length of spillway channel with constant slope, vegetal cover conditions, and exposed surface material. Reaches may be further subdivided by the model during computation based on the location at which erosion would most rapidly expose the downstream end of a subsurface material in the spillway.

For each reach or subreach evaluated, the erosion is assumed to be a three-phase process. These phases are sequential and may be described as:
- The failure of the vegetal cover protection (if any) and the development of concentrated flow
- The downward and downstream erosion associated with the concentrated flow that leads to formation of a vertical or near-vertical headcut in the vicinity of initial failure
- The upstream advance and deepening of the headcut resulting from flow over the vertical or near vertical face

Because the dominant erosion processes are different for each of these phases, a different set of relations is used to model each phase. To the extent possible, however, like processes are treated similarly within each phase.

628.5101   Erosion phases

(a) Phase 1

Phase 1 of the spillway erosion process is the failure of the vegetal cover, if any, and the development of concentrated flow. Observation of flows in the field and laboratory suggest that this phase is dominated by particle detachment associated with hydraulic shear at the surface. The key equations governing this action may be written as:

$$\tau_e = \tau_o\left(1 - C_F\right)\left(\frac{n_s}{n}\right)^2 \qquad [51-1]$$

and

$$\frac{d\varepsilon}{dt} = k_d\left(\tau_e - \tau_c\right) \qquad [51-2]$$

where, under the assumption of normal depth of flow:

$$\tau_o = \gamma dS \qquad [51-3]$$

and

τ_e = effective stress on the erodible boundary
τ_o = gross hydraulic stress on the spillway surface
C_F = vegetal cover factor
n_s = soil grain roughness of the erodible boundary
n = gross roughness (Manning's *n*) for the spillway
$d\varepsilon/dt$ = erosion or detachment rate
k_d = detachment rate coefficient
τ_c = threshold or critical stress for the erodible material
γ = unit weight of the water
d = normal depth of flow in the reach
S = surface slope of the reach

A more complete description of the variables of equation 51–1 and aids for their estimation are provided by Temple et al. (1987).

Combining these equations under the assumption of applied stress much greater than critical stress, and calibrating the resulting relation using data from field spillways results in the relation for estimating time of vegetal failure given by Temple and Hanson, 1994:

$$\int_0^{t_f} \gamma dS\left(1 - C_F\right)\left(\frac{n_s}{n}\right)^2 dt = 0.2I_w + 1 \qquad [51-4]$$

where:
t_f = time of phase 1 failure, hours
I_w = plasticity index of the erodible material

and the other variables are as previously defined with gamma (γ) expressed in pounds per cubic foot and d expressed in feet.

Equation 51–4 forms the basis for evaluating phase 1 failure for grass covers having adequate rooting depths when subjected to applied stress levels consistent with the data base from which the equation was developed. Observation of field spillways indicates that when root impenetrable materials exist near the surface, the dominant mode of vegetal cover failure changes from surface detachment to a mass stripping or rafting of the sod as a result of hydrodynamic forces at the material interface. To account for this, and for possible mass destruction of the vegetal cover at stresses higher than those represented in the original data base, the model assumes phase 1 failure to occur instantaneously any time the gross stress (γdS) exceeds the value computed from the relation (Temple and Hanson 1994):

$$\frac{D_r}{1.5} = 2^8\left(\frac{\tau_g}{13.5} - \frac{1}{2}\right)^9 + \frac{1}{2} \qquad [51-5]$$

where:
τ_g = limiting gross stress, pounds per square foot
D_r = depth to a root impenetrable layer (potential rooting depth), feet

Discontinuities in the vegetal cover are accounted for in phase 1 computations through adjustment of the stress modification terms of equation 51–1. Minor discontinuities in the cover are defined as those that have a maximum length in the direction of flow on the same order as stem length, flow depth, or both. For these discontinuities, C_F is set to 0 to represent the local area devoid of vegetal cover. Major discontinuities are those having a length greater than the flow depth or vegetal stem length. For these discontinuities,

the values of both n and C_F in equation 51–1 are those associated with the discontinuity. Except as described in later discussion, the model sets C_F to 0, and n to the greater of 0.02 or n_s for major discontinuities. Roads or trails perpendicular to the flow are examples of minor cover discontinuities, and roads or trails parallel to the flow are examples of major discontinuities.

The earth spillway erosion model uses equations 4 and 5 to compute the time of flow concentration development whether vegetation is present or not. However, when vegetal cover is not present either generally or locally, the hydraulic attack associated with phase 1 failure is relatively small. In all cases, the local erosion depth at the end of phase 1 is assumed to be 0.5 foot unless phase 1 failure is the result of sod stripping (equation 51–5) to a lesser depth.

(b) Phase 2

Phase 2 of the spillway erosion process is the surface detachment resulting from hydraulic shear stress in the region of concentrated flow. The same basic equations govern phase 2 as were used for phase 1 computations. However, once a flow concentrating discontinuity is formed, the vegetal cover is no longer effective in protecting the erodible boundary. If it is assumed that all of the roughness elements in the flow field may be detached by the flow, then equation 51–1 becomes simply:

$$\tau_e = \tau_o \qquad [51–6]$$

stating that the erosionally effective stress is equal to the gross stress within the discontinuity.

By further assuming supercritical flow in the discontinuity and that the discontinuity is wide enough that shear on vertical planes parallel to the flow may be ignored, but narrow enough that the water surface is governed by flow outside of the discontinuity, the gross stress within the discontinuity is reduced to:

$$\tau_o = \gamma(d + \Delta d)S \qquad [51–7]$$

where:
d   = flow depth computed for the spillway reach without considering the discontinuity
Δd = computed erosion depth within the discontinuity

The eroded depth, Δd, is computed by numerically integrating equation 51–2 using the stress computed from each preceding time step. The detachment rate coefficient of equation 51–2 is computed from the relation (Temple and Hanson 1994):

$$k_d = \frac{5.66\gamma}{\gamma_d} \exp\left[-0.121 c_\%^{0.406}\left(\frac{\gamma_d}{\gamma}\right)^{3.1}\right] \qquad [51–8]$$

where:
γ_d = dry bulk density of the material being eroded
$c_\%$ = percent clay of the material being eroded

and the other variables are as previously defined. The units of k_d when computed by equation 51–8 are (ft/h)/(lb/ft^2). k_d may alternately be determined using the jet test described in ASTM D5852. When determined using the jet test, the detachment rate coefficient becomes a direct input to the model.

The critical shear stress, τ_c, of equation 51–1 is computed using an iterative solution of Shields' diagram. As implemented, the computational routine assumes a constant water viscosity and sediment specific gravity which results in the particle diameter versus critical shear curve of figure 51–1. For fine grained materials typical of those supporting vegetation, this is little different from the zero critical shear assumption used in developing equation 51–4.

(c) Phase 3

When the erosion depth becomes greater than approximately the flow critical depth, the flow tends to break up on entry into the discontinuity and cause it to advance upstream as a vertical or near-vertical headcut. This upstream advance, combined with continued deepening, is phase 3 of the erosion process.

The model computes erosion depth for each time increment in phase 3 using equations 2 and 8 along with figure 51–1 in the same way as was done for phase 2. However, instead of computing applied stress using the assumption of maximum flow concentration (equation 51–7), the applied stress is taken to be the

greater of either the stress associated with normal flow depth on the eroding slope without flow concentration, or the stress computed by the relation:

$$\tau_o = \gamma d 0.011 \left(\frac{H}{d_c} \right)^{0.582} \qquad [51\text{--}9]$$

where:
d_c = the flow critical depth
H = the height of the overfall

and the other variables are as previously defined. Equation 51–9 is based on a regression analysis of laboratory data presented by Robinson (1992). In model computations, the height of the overfall is taken as the difference between the elevation of the eroded surface and the elevation of the original spillway surface at the computed location of the headcut.

Headcut advance rate is based on a threshold rate relation of the same general form as equation 51–2. This relation may be written as:

$$\frac{dX}{dt} = \begin{cases} C(A - A_o) & (A - A_o) > 0 \\ 0 & (A - A_o) \le 0 \end{cases} \qquad [51\text{--}10]$$

Figure 51–1 Critical stress for incipient motion computed from Shields' criteria with a sediment specific gravity of 2.65 and a kinematic viscosity of water of 10^{-5} ft^2/s

where:
dX/dt = headcut advance rate
A = hydraulic attack
A_o = attack threshold below which no movement occurs
C = a proportionality coefficient.

An energy based analysis of data from headcuts in field spillways yielded values for the parameters of equation 51–10 given by the relations (Moore et al. 1994, Temple and Moore 1994):
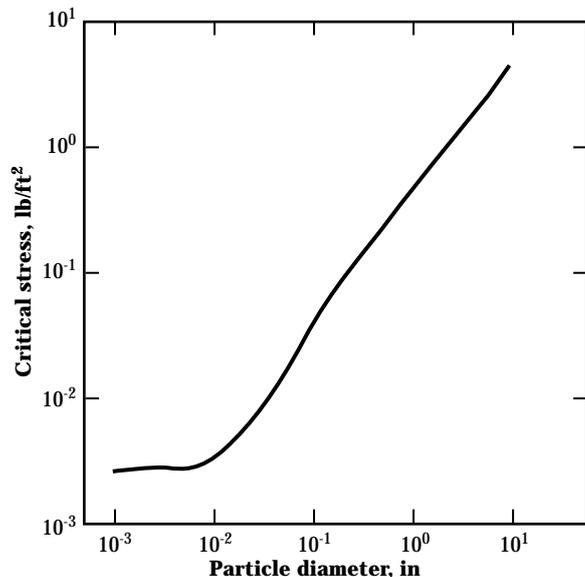
$$A = \left(qH \right)^{\frac{1}{3}} \qquad [51\text{--}11]$$

$$A_o = \begin{cases} \left[189 K_h^{\frac{1}{2}} \exp\left(\frac{-3.23}{\ln\left(101 K_h\right)} \right) \right]^{\frac{1}{3}} & K_h \ge 0.01 \\ 0 & K_h \le 0.01 \end{cases} \qquad [51\text{--}12]$$

and

$$C = \begin{cases} -0.79 \ln(K_h) + 3.04 & K_h < 18.2 \\ 0.75 & K_h \ge 18.2 \end{cases} \qquad [51\text{--}13]$$

where:
q = the unit discharge over the headcut, ft^3/s/ft
K_h = the dimensionless headcut erodibility index described in NEH 628, chapter 52, appendix 52B

with overfall height, H, given in feet and advance rate, dX/dt, given in feet per hour (positive upstream).

When multiple materials are exposed in the headcut, a composite K_h is used in equations 51–12 and 51–13. This composite is computed using a depth weighted averaging relation of the form (Temple and Moore 1994):

$$K_h = \exp\left[\frac{\sum h_i \ln(K_h)_i}{\sum h_i} \right] \qquad [51\text{--}14]$$

where:
h_i = exposed thickness of the ith material

and the summation is carried out over all of the exposed material. To avoid excessive impact of weak topsoil material, weaker surface material up to the lesser of 1 foot or a third of the headcut height is excluded from the summations of equation 51–14.

In carrying out computations, the downward erosion associated with a time step is computed first. The height of the headcut and the composite headcut erodibility index are then determined and used to compute the headcut advance distance for that step. Because vertical material thicknesses may be comparatively small, the time increment may be subdivided to account for a change in material when computing downward erosion. For purposes of computing the headward movement, the material profile is treated as constant during a single time step.

628.5102 Computation details

The three-phase computational model described is a simplification of complex physical processes. Therefore, numerous computational details must be considered to apply the model to generalized field problems. Some of these details, such as the selection of the previously described transition points between phases, are partly subjective based on judgment or observation and experience. The most significant of these details were included in the previous sections. Others that impact computations to a lesser extent or account for more unusual conditions are described here.

(a) Unit discharge

The primary hydraulic input for the model is the discharge per unit of spillway width as a function of time. Because the spillway is actually of a finite width with a known cross-sectional shape and total discharge as a function of time, it is necessary to convert from total discharge to unit discharge. This is done in the model by computing the critical depth for the entire cross-section and using this depth to compute the unit discharge that would exist in an infinitely wide channel at critical conditions. The energy coefficient is assumed to be unity for all calculations. The result is only slightly different than dividing total discharge by bed width for most spillways. However, the critical depth approach is applicable to channels with bed widths approaching zero.

(b) Nonvegetated conditions

The model identifies a reach surface as nonvegetated when Manning's *n* is input directly, C_F is entered as 0, and the potential rooting depth is not defined. In this case, a potential rooting depth of 0.5 foot is substituted into equation 51–5 to cause phase 1 failure to the 0.5-foot depth to be predicted with minimal flow. Computations then proceed in the phase 2 format, which is that appropriate for unlined channel conditions. This is essentially equivalent to assuming a nonvegetated surface with maximum vertical variation in the cross-section of 0.5 foot at the beginning of flow.

(c) Boundary roughness

The spillway may be divided into a maximum of 20 reaches for purposes of specifying flow resistance or other surface conditions. For any reach, flow resistance may be expressed in terms of Manning's *n* or a vegetal retardance curve index, C_I (Temple et al. 1987). When C_I is used to specify flow resistance, Manning's *n* is computed for each discharge by the equation:

[51–15]

$$n = \exp\left\{C_I\left[0.0133\ln^2(q) - 0.0954\ln(q) + 0.297\right] - 4.16\right\}$$

$$0.0025C_I^{2.5} \le q \le 36$$

with C_I related to vegetal properties by:

$$C_I = 2.5\left(h\sqrt{M}\right)^{\frac{1}{3}} \qquad [51\text{–}16]$$

where:
  h = vegetal stem length, feet
  M = stem density, stems per square foot

and the other variables are as previously defined with discharge in cubic feet per second per foot. Outside the bounds indicated for equation 51–15, n is set equal to the value of n at the nearest boundary. Equation 51–15 is essentially an extended numerical expression of the n-VR curves for flow resistance of grassed channels (USDA 1947).

Equation 51–15 was developed using data from laboratory channels that were smoothly graded before the establishment of the vegetal covers. Therefore, as C_I tends to 0 (no vegetal cover), n computed from equation 51–15 tends to a value of 0.0156. This value of n is lower than that expected for bare channels under field conditions. To account for this, equation 51–15 is further limited in the model to be greater than or equal to some base value, n_b. The value of n_b is arbitrarily set to 0.02 corresponding to a typical bare earth channel unless changed by user input. User change of this variable is discouraged for normal application because of the implications to the erosion computations described here. For consistency with equation 51–15, user entered values less than the base of 0.0156 are ignored.

For phase 1 erosion computations, the minimum value of Manning's *n* is computed as the greater of the specified n_b or n_s computed from the relation (Lane 1955):

$$n_s = \frac{d_{75}^{\frac{1}{6}}}{39} \qquad [51\text{–}17]$$

where:
  d_{75} = the representative diameter of the surface material, inches

In the case of major discontinuities, this minimum value becomes the value of n for use in computing effective stress by equation 51–1.

Roughness within the eroding area during phase 2 is determined similarly. In this case, erodible particle roughness is determined based on the deepest material exposed at the point of headcut formation. Because the assumption of random orientation of the earth material particles implicit in equation 51–17 is not always appropriate for larger materials, Manning's *n* within the phase 2 discontinuity is further limited to be less than or equal to the surface roughness of the spillway in the same vicinity. Note, however, that roughness within the discontinuity does not enter the calculation of phase 2 stress using equation 51–7. It is significant only for velocity calculations related to the determination of Froude number for purposes described later herein.

Roughness, n, in the eroding area is determined in the same fashion for phase 3 as for phase 2 without the upper bound. The use of n in phase 3 is to determine the normal flow depth, and, therefore, the stress associated with flow on the eroding slope. The greater of this stress and that computed by equation 51–9 is used in computing the downward erosion at the base of the headcut. The slope used in this normal depth stress computation is the least slope along the original flow path between the point of initial headcut formation and the elevation of the current base of the headcut. The discharge used is the unit discharge for the spillway without consideration of flow concentration. To avoid unrealistic stress calculations when downcutting is initially limited by material changes, this stress is considered only when the advance distance is less than or equal to twice the erosion depth.

Stress computed from this relation is generally less than that computed by equation 51–9 except when the headcut is formed on a very steep slope. Initial slopes greater than 50 vertical to 1 horizontal are treated as verticals, and the slope of the next reach downstream is used in phase 3 downcutting stress computations.

(d) Flow concentration

For the typical condition with supercritical flow in the spillway exit channel, effective stress within a major discontinuity in phase 1 and within the eroding area in phase 2, is computed as described using the constant frictional energy slope assumption implied by equations 1 and 7. However, as may be shown by spatially varied flow computations, the assumptions implied in the application of these equations to areas of flow concentration are not consistent with actual behavior when the flow is everywhere subcritical. Therefore, phase 1 and 2 stress computations for subcritical flow conditions are made using a constant discharge assumption rather than the constant frictional energy dissipation assumption. Using this approach, the energy slope in an area of flow concentration is computed from Manning's equation in the form:

$$S_e = \left[\frac{qn}{1.49(d + \Delta d)^{\frac{5}{3}}} \right]^2 \qquad [51–18]$$

where:
S_e = energy slope used in stress calculations

The other variables are as previously defined, and the foot-per-second unit system is implied. In applying equation 51–18, $d+\Delta d$ is the local flow depth and n is the local flow resistance in the area of flow concentration. The potential for error associated with the simplification of the subcritical flow stress computations is generally greater than that associated with the more common supercritical flow condition.

The lack of a tendency for the flow to concentrate in subcritical flow is also recognized in the computation of stress in the vicinity of minor discontinuities in phase 1 by modifying the cover factor adjustment. For subcritical flow depths in excess of 2 feet, the cover factor within a minor discontinuity is set to $(1 - 2/d)C_F$

rather than the zero value assumed for supercritical and shallower flows. In no instance is the stress within a discontinuity allowed to drop below the effective stress computed for a higher level of cover uniformity. Additional data may result in modification of this adjustment in the future.

(e) Hydraulic jump

Although construction of spillways with concave slope is discouraged, the earth spillway erosion model includes a component to estimate boundary stresses when a hydraulic jump is generated because of slope flattening. The stress estimate is based on a simplification of horizontal bed conditions with no attempt made to track the effects of jump formation on the shape of local scour. For phase 2 and the condition of a major discontinuity in phase 1, the approach and exit conditions are based on roughnesses and flow depths associated with conditions in the discontinuity, but original surface slopes are assumed to be retained.

When a jump is indicated by a change in normal flow conditions from supercritical upstream to subcritical downstream, the jump is assumed to take place at the change in slope with the downstream depth being the normal depth in the downstream reach or the sequent depth of the approach flow, whichever is less. Energy loss through the jump is computed and averaged over the length of the jump to obtain an effective energy slope. The fitted equation describing this averaged effective energy slope is:

$$[51–19]$$

$$\frac{\Delta E}{L} = \frac{\left[2.819(F_1 - 1) + 0.4134(F_1 - 1)^2 - 0.0201(F_1 - 1)^3 \right]}{100}$$

where:
$\Delta E/L$ = effective energy slope
F_1 = Froude number on entry to the jump

This energy slope is then combined with the jump sequent depth in equation 51–3 to provide an estimate of the gross stress generated by the jump. The greater of this gross stress, or that associated with the approach flow from the upstream reach, is then used in equation 51–1 or 51–6 for the appropriate computation of effective stress for the time increment.

(f) Headcut selection

In an actual spillway, a headcut may form anywhere along the flow path resulting in an infinite number of possible headcuts to be evaluated. It is therefore necessary to develop criteria for selecting a subset of these for evaluation. This subset is based on the location of changes in surface and subsurface conditions.

As previously indicated, each spillway reach is evaluated separately for the time of headcut formation. If phase 1 failure is indicated, it is assumed to occur at the upstream end of the reach, so that the headcut evaluated is the one with the least advance distance required to generate spillway breach. Because a reach is defined by constant slope and surface conditions, it is possible for the user to force evaluation of a potential headcut at any location along the spillway by entering a perturbation in surface conditions so that the desired point becomes the beginning of an evaluation reach.

Because the advance of a headcut may be accelerated by the exposure of a subsurface material, the horizontal coordinate of the downstream endpoint of all materials is located and the time to formation of a headcut above that location evaluated. Once formed, the location of the beginning of the headcut is advanced downstream as the headcut is deepened to locate the starting location that would expose the endpoint of the material the most rapidly. The means by which the starting location of the headcut is advanced downstream is strictly correct only when all other exposed material interfaces parallel the spillway surface in the region of downstream advance. However, a reasonable estimate is obtained for most practical subsurface configurations. This same approach is used when, during headcut advance computations, a headcut passes above the end of a material. The conditions at the time the headcut passes are saved for use as the starting point for additional computations in which the location of headcut initiation is projected in the downstream direction.

During headcut advance computations involving multiple materials, the potential for a more shallow headcut following a material interface to advance at a more rapid rate is evaluated at each time step. If this is found to occur, conditions are again stored for subsequent evaluation of an additional headcut following the material interface. Computations are allowed to

bifurcate in this fashion as many times as necessary to account for all material interfaces, but may bifurcate only once per interface per initial formation point.

Following the described logic, a large number of headcuts may be evaluated for a spillway depending on the complexity of the surface and subsurface conditions. The deepest headcut and the one progressing the furthest upstream are described in the output from the model. The output also describes the potential eroded surface developed from the composite of all headcuts evaluated. However, despite the large number of conditions evaluated, it is possible that the model will fail to evaluate the worst case for unusual and complex conditions. The user retains responsibility for evaluating input and output to determine whether additional surface reaches are needed to guarantee worst case identification. A more rigorous mathematical analysis guaranteeing identification of worst case conditions is an area for future model refinement.

628.5103 Model I/O summary

Input requirements for the model are a description of spillway surface conditions and the properties and location of geologic materials that may be exposed during erosion. Although conditions are required for erosion calculation only from the upstream end of the crest to the location of tailwater at low spillway flow, it is usually appropriate to input conditions from the inlet to the intersection of the flow-path with the valley floor. This allows computations, such as the determination of spillway rating, to be carried out using the same data entry.

The required surface parameters are a flow resistance parameter (either Manning's *n* or vegetal retardance curve index), vegetal cover factor, cover maintenance factor (uniform, minor discontinuities, or major discontinuities), potential vegetal rooting depth, and the representative diameter of the surface material. A maximum of 20 reaches may be defined by varying these parameters and bed slope. Surface conditions are used in determining the time of phase 1 failure for the reach and in determining the flow depth for phase 2 flow concentration computations.

The geologic material parameters required for each material that may be exposed are the plasticity index, the representative particle diameter, percent clay, bulk dry density, and the headcut erodibility index. The plasticity index is used in determination of time of phase 1 failure for materials exposed at the spillway surface. The representative particle diameter (d_{75} for fine grained material) is used in the computations of erodible particle roughness, n_s, and in the determination of the critical stress, τ_c, for surface detachment computations in phases 2 and 3. The percent clay and bulk dry density are used to determine the detachment rate coefficient, k_d, and may be replaced by direct input of that parameter. k_d is used in surface detachment (erosion depth) computations for phases 2 and 3. The headcut erodibility index K_h is used in the determination of the headcut advance threshold and rate for phase 3.

Output from the model includes the time of phase 1 failure (or the attack experienced expressed as a percent of that required to generate phase 1 failure) for each reach, a description of the headcut predicted to penetrate the furthest upstream, and a description of the deepest headcut evaluated. A potential eroded surface resulting from a composite of all headcuts evaluated is also generated.

The earth spillway erosion model provides a physically based means of estimating the performance of vegetated earth spillways subjected to flood flows. The model is based on relations developed from physical principles and laboratory experiment, and calibrated using data gathered from field spillways. Because of the complexity of the physical phenomena involved, the mathematical representation is necessarily simplified. Model input and output should, therefore, be examined critically. In applying the model, it should be recognized that the flow is able to search out the weakest surface and material conditions in the profile. The weakest conditions should be those reflected in the data input rather than the average conditions. Because the model necessarily incorporates simplifications as described herein, the user retains the responsibility of determining applicability of the model to his/her specific problem.

628.5104   References

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