Technical Supplement 14B

Scour Calculations

Issued August 2007

Cover photo: Where sediment transport capacity of a stream exceeds the sediment supply, scour may occur. Scour may be estimated using analytical tools and accommodated or re-mediated by the design.

Advisory Note

Techniques and approaches contained in this handbook are not all-inclusive, nor universally applicable. Designing stream restorations requires appropriate training and experience, especially to identify conditions where various approaches, tools, and techniques are most applicable, as well as their limitations for design. Note also that product names are included only to show type and availability and do not constitute endorsement for their specific use.

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Purpose

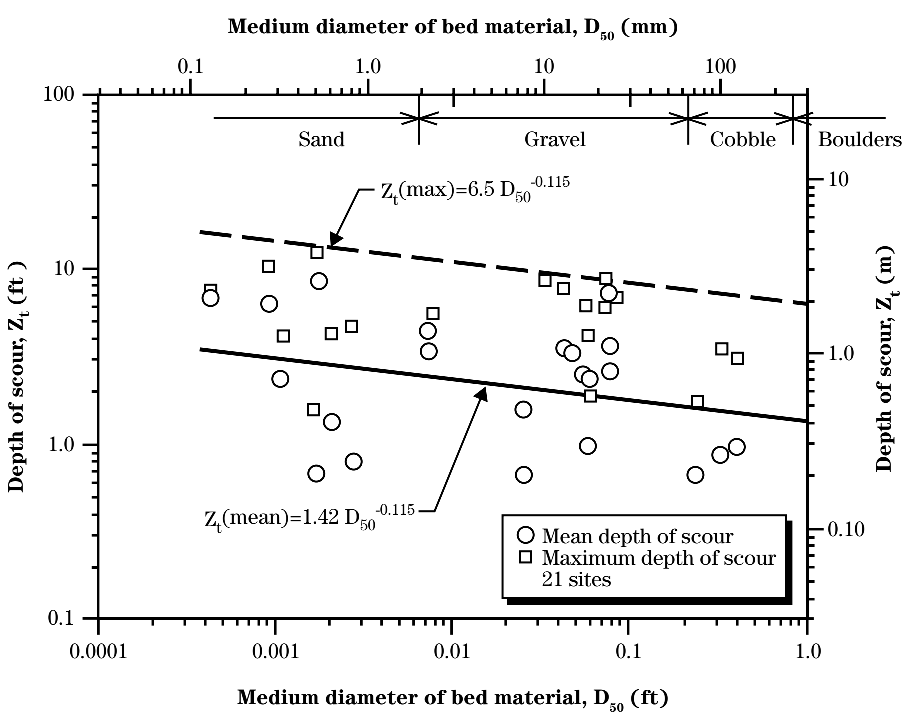
Scour is one of the major causes of failure for stream and river projects. It is important to adequately assess and predict scour for any stream or river design. Designers of treatments such as barbs, revetments, or weirs (that are placed on or adjacent to streambeds) must estimate the probable maximum scour during the design life of the structure to ensure that it can adjust for this potential change. This technical supplement provides guidance useful in performing scour depth computations.

Introduction

Streams continually mold and remold their streambeds by eroding and depositing sediments. Scour and fill of alluvial channels not undergoing long-term aggradation or degradation occur as fluctuations about some average condition. Blodgett (1986) presented information regarding bed elevation fluctuations from 21 sites on streams with a range of bed material sizes. Monthly or annual measurements were made at the same location within generally straight reaches, free of features like bedrock, bridge piers, or large boulders that might cause local scour. Mean and maximum scour depths are plotted in figure TS14B–1 (Blodgett 1986) as a function of median bed-material size. In this figure, the scour depth is defined as the depth of scour below a reference plane, which was set at the highest thalweg elevation measured during the period of observation. Clearly, scour depths can be quite significant.

Scour is perhaps the primary cause of failure of riverine hydraulic structures, and failure to adequately assess and predict scour hazard represents a major design flaw. For example, most failures of continuous bank protection projects like revetments are due to toe scour. The most spectacular examples of structural failure due to scour involve bridges, such as those

Figure TS14B–1 Scour observations from typical reaches of alluvial rivers



summarized in table TS14B–1 (Lagasse and Richardson 2001). Less well known, but also important, scour problems account for high failure rates sometimes reported for stream habitat structures and modified or realigned stream channels (Frissell and Nawa 1992). A survey of U.S. Army Corps of Engineers (USACE) flood control channel projects found that most reported problems were linked to some form of bed or bank instability, including local scour and vertical instability (McCarley, Ingram, and Brown 1990).

An analysis of potential scour is required for all types of streambank protection and stabilization projects. In addition, scour analysis should be a part of the design of any hard structure placed within the channel. Scour and deposition, of course, are processes that affect any movable bed channel design as described in NEH654.09. An analytical approach is needed because many streams tend to scour during high flows and fill during hydrograph recession. Therefore, severe scour can occur during periods when the bed is obscured, only to refill and appear completely different at baseflow.

Although the term scour includes all erosive action of running water in streams, including bed and bank erosion, the emphasis in this technical supplement is on erosion that acts mainly downward or vertically, such as bed erosion at the toe of a revetment or adjacent to a bank barb. Designers of objects placed on or adjacent to streambeds such as bridge piers, revetments, spurs, barbs, deflectors, weirs, sills, or grade control structures must estimate the probable maximum scour during the design life of the structure and ensure that the structure extends below maximum scour depth. This technical supplement provides guidance on scour depth computations.

Processes

Scour occurs due to several related processes, and estimated maximum scour is typically computed by summing the scour due to each individual process. Terms used to describe bed erosion processes include degradation, local scour, contraction scour, bend scour and others, and these are related as shown in table TS14B–2. Aggradation and degradation refer to an increase or decrease, respectively, in bed elevation over a long reach, through sediment deposition or erosion. Aggradation and degradation are major adjustments of a fluvial system due to watershed changes. In contrast, scour is erosion of the streambed that, except locally, does not influence the longitudinal profile or gradient of the stream. Scour may also be of a temporary, cyclic nature, with significant local erosion occurring during high flows, and refilling during the receding portion of the flow.

All types of scour are loosely categorized as either general or local scour (Brice et al. 1978). Local scour refers to erosion of the streambed that is immediately adjacent to (and apparently caused by) some obstruction to flow (fig. TS14B–2) (Brice et al. 1978)). General scour commonly affects the entire channel cross section, but it may affect one side or reach more than another (fig. TS14B–3) (Brice et al. 1978)). Both types

Table TS14B–1 Examples of bridge failures associated with scour

Date	Event	Stream	Conditions at time of event	Consequences	Cause
April 1987	NY State Thruway bridge collapses	Schoharie Creek, NY	Near-record flood	10 deaths	Cumulative effect of local scour over 10 yr
1989	U.S. 51 bridge collapses	Hatchie River, TN	—	8 deaths	Northward migration of the main river channel
March 10, 1995	Interstate 5 bridges collapse	Los Gatos Creek, CA	Large flood	7 deaths	Stream channel degradation combined with local scour

Table TS14B–2 Streambed erosion and deposition processes

General process	Specific process	Description and subtypes	
Aggradation or degradation		An increase or decrease in bed elevation over a long reach through sediment deposition or erosion	
Scour	General scour	Longitudinally local erosion that affects the entire channel cross section	Contraction scour Bend scour
	Bedform scour	Formation of troughs between crests of bedforms, usually in sand-bed streams	Dunes Antidunes
	Local scour	Erosion of the streambed that is immediately adjacent to (and apparently caused by) some obstruction to flow	Bridge pier and abutment scour Scour at structures that span the channel, such as weirs and sills Scour at structures that do not fully span the channel

Figure TS14B–2 Examples of local scour

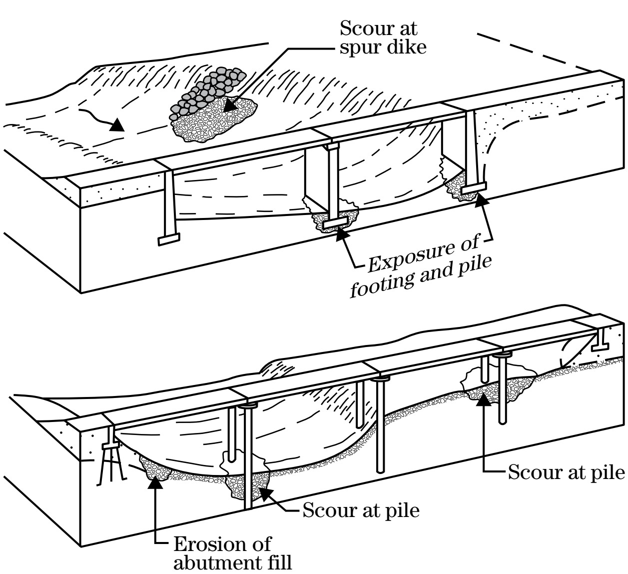
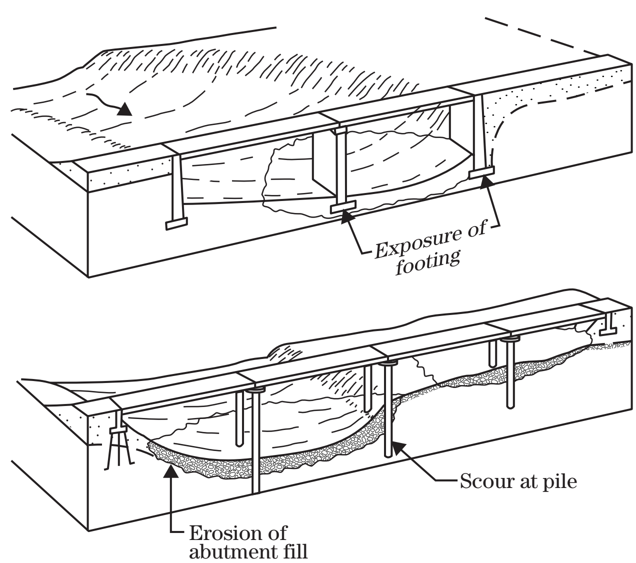


Figure TS14B–3 Examples of general scour



of scour occur discontinuously in the longitudinal direction along a reach, and both types can be cyclic in time. Note that spatially continuous vertical displacement of the streambed is referred to as either aggradation or degradation.

In many cases, physical deficiencies in streams with degraded habitat are addressed by inducing scour with structures that create pool habitat and cover (Brookes, Knight, and Shields 1996; Shields, Knight, and Cooper 1998; Lenzi, Comiti, and Marion 2004). In such cases, the designer seeks to maximize scour hole depth and volume subject to channel and structural stability constraints. Procedures for estimating this type of scour are presented below.

Scour is difficult to accurately measure in the field, and most design equations are based on theory supported by laboratory data. However, the following qualitative principles are useful in understanding scour processes (Laursen 1952; Vanoni 1975):

- The rate of scour is equal to the difference between the capacity for transport out of the scoured area and the rate of transport into the scoured area.
- Scour rates decline as scour progresses and enlarges the flow area.
- Scour asymptotically approaches a limiting extent (volume or depth) for a given set of initial conditions.

Effects of stream types

Flow regime—Stream channels may be classified as perennial, intermittent, or ephemeral based on their flow regime, as described in NEH654.07. Similar approaches are used to analyze and predict scour depths in all three types of channels. However, application of the three qualitative principles outlined indicates that extreme flow variations can lead to extreme variations in scour depths and patterns. Since scour asymptotically approaches a limiting depth for a given hydraulic condition, if flows are flashy, the limiting depth for a given flow may never be reached.

Bed material and sediment transport regime—Alluvial and threshold channels are fully described in NEH654.01 and NEH654.07. Scour in alluvial channels is usually live-bed scour, which implies that there is significant transport of sediment from upstream reaches into the reach in question. Scour occurs when transport capacity exceeds supply from upstream, and cyclic scour behavior is normal. Deposition or fill during waning stages of floods restores the scoured bed to near its preflood position, although scour holes may persist during oscillating flow conditions in gravel-bed streams (Neill 1973).

Scour in threshold channels tends to be clear-water scour unless flows become high enough that the threshold of bed sediment motion is exceeded. Clear-water scour implies that there is little or no movement of bed material from upstream reaches into the design reach. Clear-water scour is typically associated with coarse beds, flat gradient streams at low flow, local deposits of bed materials larger than the largest size being transported by the flow, armored streambeds, and vegetated channels or overbank areas where flow forces are less than those required to remove sediments protected by the vegetation.

Due to the complexity of scour, many of the studies used to support the equations presented below were conducted in flumes under clear-water conditions. The flow strength or bed shear stress was just lower than needed to erode and transport sediments from the bed, but adequate to trigger scour at a model contraction, spur, bridge pier, or other flow obstruction. The user must be aware that these equations probably will yield conservative results when applied to alluvial channels. Clear-water scour in coarse-bed streams reaches a maximum over a longer period of time than live-bed scour, but is about 10 percent greater than the equilibrium live-bed scour (Richardson and Davis 2001). During a flood event, streams with coarse-bed material may experience clear-water scour at low discharges during rising and falling stages, and live-bed scour at the higher discharges.

Different materials scour at different rates—Noncohesive silts and sands scour rapidly, while cohesive or cemented soils are much more scour resistant and erode relatively slowly. However, ultimate scour in cohesive or cemented soils may be just as great, even though the ultimate scour depth is reached more slowly. Under constant flow conditions, scour reaches maximum depth in sands within hours; in cohesive bed materials in days; in months in glacial till, sandstones, and shale; in years in limestone; and in centuries in dense granite. These are generalities; actual measurements show additional complexity. For example, tests of cohesive streambeds showed that erodibility varied across four orders of magnitude in a single north Mississippi watershed (Simon and Thomas 2002).

Planform—Channel planform interacts with scour processes (USACE 1991b). During high flows, the bed scours in meander bends and builds up in the shallower reaches (thalweg crossings) between the bends. On the recession side of a flood, the process is reversed. Some observers note that braided channels experience greatest scour during intermediate flows, when current vectors attack bank lines at sharp, impinging angles. In meandering channels, the thalweg in bends often moves toward the outer (concave) bank following placement of revetment or other types of direct erosion protection. The amount of additional bend scour is related to the relative erodibility of the bed and banks. Channels with highly erodible bed and banks experience most significant additional scour at the toe of a newly placed revetment.

Scour computations for design

The total scour depth needed for design of key-in or toe-down elevations may be computed by summing all the components of vertical bed change:

$$z_t = FS\left[z_{ad} + z_c + z_b + z_{bf} + z_s\right] \qquad \text{(eq. TS14B–1)}$$

where:
z_t = total scour depth, ft (m)
FS = factor of safety
z_{ad} = bed elevation changes due to reach-scale deposition (aggradation) or bed erosion (degradation), ft (m)
z_c = contraction scour, ft (m)
z_b = scour on the outside of bend, ft (m)
z_{bf} = bedform trough depth, ft (m)
z_s = local scour depth associated with a structure, ft (m)

Guidance for computing each component of scour is provided in table TS14B–3.

An overview of the analyses presented in this technical supplement, organized by channel bed type, is presented in a summary table later in this technical supplement.

Table TS14B–3 Types of scour analyses

Type of scour or process	Symbol	Type of analysis
Long-term bed elevation change	z_{ad}	Armoring, equilibrium slope, or sediment continuity
Total general scour		Empirical equations or regime equations
Contraction scour	z_c	Live-bed or clear-water contraction scour
Bend scour	z_b	Bend scour formulas, most include all types of scour
Bridge scour	Not treated herein	Guidance provided by Richardson and Davis (2001)
Bedform scour	z_{bf}	Formulas for dunes or antidunes. Select type of bedform using bedform predictor equation.
Local scour	z_s	Empirical relations for each major type of structure

Long-term bed elevation change

Although any given point in a stream is constantly changing, a stable stream maintains the same average vertical position for its bed when viewed over a long reach (>20 channel widths) over a long period of time (several decades to a few centuries). Such stable streams are rare, however, because disturbances in the form of human activities or natural events (rare flood events, volcanic eruptions, earthquakes, landslides, fires) are common. Human activities are the most common cause of vertical instability, and among these are urbanization, dam construction, channelization, streambed mining, and land use changes. The effect of each of these activities on fluvial systems is described in NEH654.01 and in USACE (1994a). In general, these activities change the supply of sediment or water to a reach (for example, a dam) or increase the sediment transport capacity of a reach (for example, channel straightening, which increases channel slope). Vertical (and often lateral) instability occurs as the channel degrades or aggrades in response to the imbalance between supply and transport (fig. TS14B–4).

For long-term scour estimation, the designer must compute the long-term bed elevation change required to produce an equilibrium slope. If coarse materials are present in the bed and are not mobilized by a design event, the designer should also compute the depth of scour needed to produce an armor layer. The correct scour depth to use in design (eq. TS14B–1) will be the lesser of the two depths. In general, armoring limits degradation in beds with gravels and cobbles, while beds of finer material degrade until they reach an equilibrium slope.

Armoring

Streambeds often feature a layer of coarse particles at the surface that overlies a heterogeneous mixture containing a wide range of sediment sizes. This layer, which is usually only one or two particle diameters thick, is referred to as the armor layer. Formation and destruction of armor layers on streambeds is described in NEH654.07. When a streambed contains at least some sediment that is too large to be transported by the imposed hydraulic conditions, finer particles are selectively removed. The layer of coarser materials left behind forms an armor layer that limits further scour unless and until higher levels of shear stress destroy the armor layer. For example, armor layer formation is often observed downstream from dams. Borah (1989) proposed the following relationship to compute the scour depth below a dam in a channel with a well-mixed bed comprised of particles with the same specific gravity (fig. TS14B–5):

Figure TS14B–4 Conceptual representation of the relationship between long-term average vertical stability and sediment transport

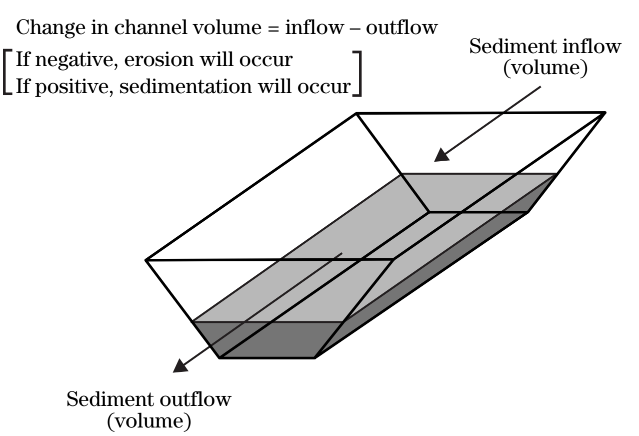
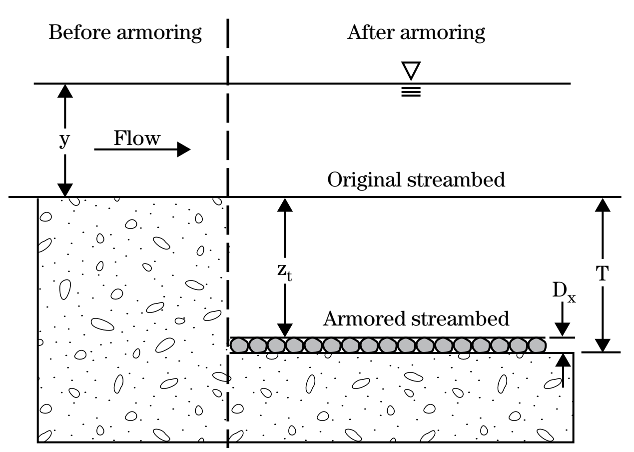


Figure TS14B–5 Definition of terms for armor limited scour



$$z_t = T - D_x \quad \text{(eq. TS14B–2)}$$

where:
T = thickness of the active layer of the bed, ft (m)
D_x = smallest armor size or the size of the smallest nontransportable particle present in the bed material, ft (m)

T is related to D_x as follows:

$$T = \frac{D_x}{(1-e)P_x} \quad \text{(eq. TS14B–3)}$$

where:
e = porosity of the bed material
P_x = fraction of bed material (expressed as a decimal) of a size equal to or coarser than D_x

Various approaches may be used to compute the smallest armor particle size, D_x. Borah (1989) suggested using relations based on the Shields curve for the initiation of motion. These relations take the form:

$$D_x = K\left(\frac{yS_e}{\Delta S_g}\right)^a \left(\frac{U_*}{\nu}\right)^b \quad \text{(eq. TS14B–4)}$$

where:
y = flow depth, ft (m)
S_e = energy slope
ΔS_g = relative submerged density of bed-material sediments ≅ 1.65
U_* = shear velocity = $(gyS_e)^{0.5}$

where:
g = acceleration of gravity, 32.2 ft/s^2 (9.81 m/s^2)
ν = kinematic viscosity of water, ft^2/s (m^2/s)

K, a, and b are constants based on the particle Reynolds number as shown in table TS14B–4.

where:
D_{50} = median grain size of sediment mixture in ft (m). The bed porosity e, D_{50}, and P_a are all estimated from analyses of bed-material samples. Bed porosity may also be estimated using a formula (Komura and Simons 1967):

$$e = 0.245 + \frac{0.0864}{(0.1D_{50})^{0.21}} \quad \text{(eq. TS14B–5)}$$

where:
D_{50} = median grain size in mm

Equilibrium slope

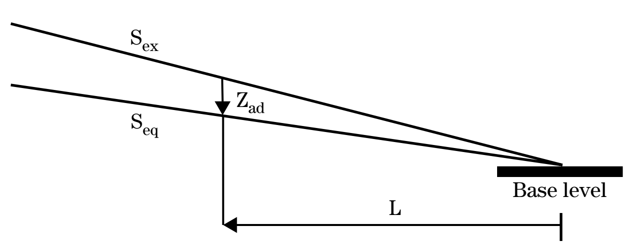
When sediment transport capacity exceeds sediment supply, channel bed degradation occurs until an armor layer forms that limits further degradation or until the channel bed slope is reduced so much that the boundary shear stress is less than a critical level needed to entrain the bed material. This new, lower slope may be called the equilibrium slope, S_{eq}. Slope adjustment in a sediment deficient reach occurs by degradation, proceeding from the upstream end to the downstream, and the downstream extent of degradation is often limited by a base level control. Figure TS14B–6 illustrates the relationship between existing slope, S_{ex}, equilibrium slope and ultimate general scour due to bed degradation, Z_{ad}, for a reach of length L with base level control).

For example, the reach downstream from a reservoir without major tributaries may degrade first just below the dam, and a wave of bed degradation will proceed downstream, gradually tapering off as a base level control (a culvert or a downstream reservoir) is reached. Without downstream control, degradation will continue until halted by channel bed armoring, or until the

Table TS14B–4 Constants for computation of minimum armor particle size

Particle Reynolds number $\frac{U_* D_{50}}{\nu}$	K	a	B
≤10	68	1.67	0.67
Between 10 and 500	27	0.86	–0.14
≥500	17	1.0	0.0

Figure TS14B–6 Definition of equilibrium slope, S_{eq}



entire profile reaches equilibrium slope. The amount of ultimate degradation at a given location upstream from the base level control may be estimated by:

$$z_{ad} = L\left(S_{ex} - S_{eq}\right) \qquad \text{(eq. TS14B–6)}$$

where:
L = distance upstream of the base level control

Equilibrium slope is a function of the contributing drainage area. Equilibrium slopes are greater for smaller drainage areas, and therefore equilibrium slope and ultimate degradation must be computed reach by reach.

Several approaches for computing equilibrium slope are presented below (Lagasse, Schall, and Richardson 2001), as outlined in table TS14B–5. If the computed stable slope is greater than the existing slope, the risk of additional degradation is low, and the streambed may already be armored.

Use of the relationships below is complicated by channel response. If bed degradation is associated with bank failure, sediment supply may be replenished from the eroding banks, at least temporarily. A rough technique for computing sediment supply from banks is described by Pemberton and Lara (1984), and more detailed computations are contained in the ARS bank stability model, available at the following Web site:

http://www.ars.usda.gov/Business/docs.htm?docid=5044

Channel incision may also lead to narrowing, which affects discharge. It is also difficult to select a single discharge for use with the above relationships. See the discussion in NEH654.05 and NEH654.09 regarding channel-forming discharges.

When a base level control is lowered or removed (the downstream bed elevation is lowered due to channel change), channel degradation will proceed upstream, migrating up each of the tributaries to the watershed divide. Watershedwide consequences can be severe (Simon and Thomas 2002), with sediment yield increasing by an order of magnitude due to enlargement of the channel. Ultimate degradation may again be computed based on equilibrium slope. Critical shear stresses are very low for sands, and the associated equilibrium slopes are so flat that the amount of potential degradation is quite large.

Calculation of equilibrium slope as a stability assessment is also described in NEH654.13.

Table TS14B–5 Approaches for determining equilibrium slope

Bed type	Sediment supply from upstream	Approach for equilibrium slope	Equation(s)
Cohesive silt or clay	Any	Watershed-specific regression	n/a
Sand to fine gravel $0.1 \le D_{50} \le 5.0$ mm	Drastically reduced or none	Back calculation based on critical shear stress	TS14B–7
	Reduced	Back calculation based on sediment supply	TS14B–12 or TS14B–13
Coarser than sand	Drastically reduced or none	Manning and Shields	TS14B–14
		Meyer-Peter and Müller	TS14B–15
		Schoklitsch	TS14B–16
		Henderson	TS14B–17
Sand or gravel	Any	Sediment continuity	TS14B–18

Cohesive beds

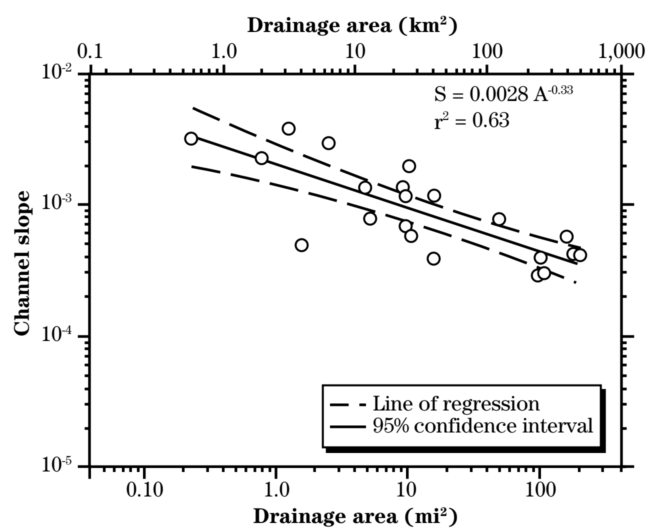
Nickpoint (or knickpoint) migration is a dramatic form of vertical instability that occurs in cohesive soils (fig. TS14B–7). A nickpoint (or headcut) is an abrupt change or inflection in the longitudinal profile of a cohesive streambed. In noncohesive materials, analogous features are manifest as short, steep reaches known as nickzones (or knickzones). Both types of features tend to migrate upstream, particularly during high flows. Bed degradation in the immediate vicinity of a migrating nickpoint can be dramatic, as the bed may be lowered or degraded up to several meters in a single flow event.

The sequence of changes that typically occurs in channels when a headcut passes through have been described in a conceptual model known as the channel evolution model (CEM), or incised channel evolution model (ICEM) (Simon 1989) as described in NEH654.01, NEH654.03, and NEH654.13. Due to the complexities of cohesive bed erosion, it is difficult to predict the rate of nickpoint migration, even given the hydrograph. However, the ultimate amount of degradation may be estimated by extending a thalweg profile from a fixed downstream base level upstream at a slope equal to the equilibrium slope, S_{eq}, determined as described (fig. TS14B–6).

Some investigators have developed watershed-specific regressions for predicting S_{eq} for watersheds with beds of sand and consolidated cohesive outcrops. These formulas may be used to predict S_{eq} from the contributing drainage area. The regressions are based on bed slope and drainage area for reaches that have undergone enough degradation to attain equilibrium (fig. TS14B–8 (Simon and Thomas 2002)). This approach may be sufficient for estimation purposes, but it ignores the unsteady nature of sediment supply and resultant complex response. The scatter in predicted values is large (fig. TS14B–8). A similar alternative approach involves fitting an exponential function to plots of thalweg elevation at a given cross section versus time to predict future bed elevations (Simon 1992).

Figure TS14B–7 Headcut migrating upstream through cohesive streambed toward bridge in north-central MS. Headcut was triggered by downstream channelization.

Figure TS14B–8 Empirical equilibrium slope–drainage area relationship for Yalobusha River watershed in northern MS



Sand and fine gravel—no bed-material sediment supplied from upstream

For channels with bed material coarser than sand, armoring and slope reduction processes may occur simultaneously. Both types of analyses must be performed to determine which will provide the limiting factor. Pemberton and Lara (1984) also suggest that stable slopes may be computed for channels with noncohesive beds with sediment sizes between 0.1 millimeter and 5 millimeters by obtaining a critical shear stress value from the graphical compilation published by Lane (1952) (fig. TS14B–9).

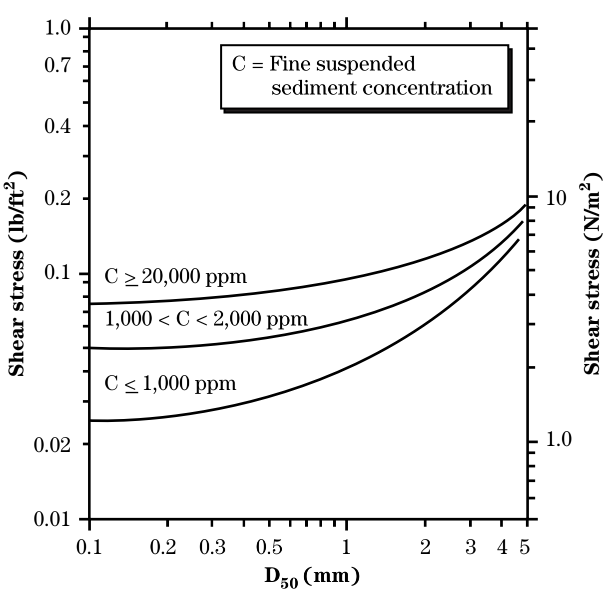
$$S_{eq} = \left(\frac{\tau_c}{\gamma_w y}\right) \qquad \text{(eq. TS14B–7)}$$

where:

τ_c = critical shear stress from the curve in figure TS14B–9, lb/ft^2 (N/m^2)

y = mean flow depth, ft (m)

Figure TS14B–9 Critical shear stress for channels with boundaries of noncohesive material. Critical shear stress increases with increasing fine suspended sediment concentration.



Sand and fine gravel—reduced sediment supply from upstream

It is not uncommon to have the sediment supply reduced to a stream reach. This occurs when a watershed is reforested, in later stages of urbanization, bed material is mined, diversions are constructed, or when reservoirs are placed in one or more subwatersheds. The concept of equilibrium slope remains valid for these conditions. Use observed bed-material sediment discharge data to fit a regression function of the form:

$$q_s = au^b y^c \qquad \text{(eq. TS14B–8)}$$

where:

q_s = sediment transport capacity in dimensions of volume per unit width per unit time, ft^2/s (m^2/s)

u = mean streamwise velocity, ft/s (m/s)

y = mean flow depth, ft (m)

a, b, c = coefficients and exponents from regression

The sediment transport capacity may be converted to dimensions of weight per unit width per unit time (tons/d) by multiplying by 7,144 (228,960 to convert m^2/s to metric tons/d).

For purposes of equilibrium slope computation, q_s, should be computed using the mean velocity and flow depth corresponding to the channel-forming discharge as defined in NEH654.05. Since sediment supply and sediment transport capacity are determined for the same water discharge, computation of equilibrium slope is not very sensitive to errors in determining effective discharge. If available sediment data are inadequate to generate a reliable regression, a sediment transport relationship may be used to synthesize coefficients. For sand streambeds, the following formulas are available for the coefficients a, b, and c in equation TS14B–8 (Yang 1996):

$$a = 0.025n^{(2.39-0.8\log D_{50})}\left(D_{50}-0.07\right)^{-0.14} \qquad \text{(eq. TS14B–9)}$$

$$b = 4.93 - 0.74\log D_{50} \qquad \text{(eq. TS14B–10)}$$

$$c = -0.46 + 0.65\log D_{50} \qquad \text{(eq. TS14B–11)}$$

D_{50} has units of millimeters.

When SI units are used in the equation for q_s coefficients b and c are unchanged, and the coefficient, a, must be multiplied by a factor of $0.3048^{(2\text{-}b\text{-}c)}$. These formulas are based on regression of a large data set with ranges given in table TS14B–6.

Similar regression coefficients for sediment transport under conditions outside these ranges (0.1 mm < D_{50} < 5.0 mm) are provided by Richardson, Simons, and Lagasse (2001). If it is assumed that bed-material size and channel width do not change as the channel degrades, the equilibrium slope may be computed by:

$$S_{eq} = \left(\frac{a}{q_s}\right)^{\frac{10}{3(c-b)}} q^{\frac{2(2b+3c)}{3(c-b)}} \left(\frac{n}{K}\right)^2 \qquad \text{(eq. TS14B–12)}$$

where:
K = 1.486 (1.0 for SI units), and other variables are as previously defined

For a reduction in sediment supply to a reach in which all other characteristics remain unchanged (water discharge, roughness, and channel width), the equilibrium slope may be computed by:

$$S_{eq} = S_{ex}\left(\frac{Q_{s\,(future)}}{Q_{s\,(existing)}}\right)^{\frac{10}{3(b-c)}} \qquad \text{(eq. TS14B–13)}$$

where:
S_{ex} = existing channel slope
Q_s = sediment supply, ft^3/s (m^3/s)

Table TS14B–6 Ranges for data set underlying the Yang sediment transport relation (eqs. TS14B–8 through TS14B–11)

Quantity	Range (SI units)
D_{50}, mm	0.1–2.0
u, velocity, ft/s (m/s)	2.0–8.0 (0.6–2.4)
y, depth, ft (m)	2.0–25 (0.6–7.6)
S, slope	0.00005–0.002
Manning *n*	0.015–0.045
Froude No.	0.07–0.70
q, unit discharge of water, ft^3/s (m^3/s)	4.0–200 (0.37–18.6)

The sediment supply for existing conditions may be measured or computed, while the supply for future conditions must be computed using an appropriate sediment transport relation. In both cases, the sediment transport rate must correspond to the channel-forming discharge.

Beds coarser than sand—no sediment supplied from upstream

When a reservoir with long storage time is placed on a river or stream, bed-material sediment supply to downstream reaches is drastically reduced and can be cut off entirely. A similar reduction occurs in the latter stages of urbanization when construction sites and other disturbed areas are covered with impervious surfaces or vegetation. Four equations are presented for S_{eq}, the equilibrium slope, in conditions where sediment transport rates are negligibly small. Variable definitions follow the fourth equation. Equilibrium slope may be selected as an average of that provided by the four relations, or the most applicable relationship may be selected for use based on a study of the original references.

- Simultaneous solution of the Manning and Shields equations (for D_{50} > 6 mm):

$$S_{eq} = \left[\theta_c D_c \Delta S_g\right]^{\frac{10}{7}} \left(\frac{K}{qn}\right)^{\frac{6}{7}} \qquad \text{(eq. TS14B–14)}$$

- Based on Meyer-Peter and Müller sediment transport relationship for material coarser than sand:

$$S_{eq} = K\frac{\left(D_{50}\right)^{\frac{10}{7}} n^{\frac{9}{7}}}{\left(D_{90}\right)^{\frac{5}{14}} q^{\frac{6}{7}}} \qquad \text{(eq. TS14B–15)}$$

- Based on the Schoklitsch equation for coarse sand or gravel:

$$S_{eq} = K\left(\frac{D_m}{q}\right)^{\frac{3}{4}} \qquad \text{(eq. TS14B–16)}$$

- Based on the Henderson formula for materials larger than 6 mm:

$$S_{eq} = K\, Q_d^{-0.46} D_{50}^{1.15} \qquad \text{(eq. TS14B–17)}$$

where:
K = constants given as shown in table TS14B–7
S_{eq} = equilibrium channel slope at which sediment particles of size D_c and larger will no longer move
ΔS_g = relative submerged density of bed-material sediments ≅ 1.65
q = channel-forming discharge per unit width, ft^3/s/ft (m^3/s/m)
n = Manning's roughness coefficient
D_{90} = sediment size for which 90% by weight of bed material is finer, m (ft)
D_{50} = median sediment size, ft (m) (**Note units**)
D_m = mean bed-material particle size, mm
Q_d = design discharge, ft^3/s (m^3/s)
Q_b = discharge over bed of channel, ft^3/s (m^3/s). Normally Q_d/Q_b = 1 for wide channels
y = mean flow depth, ft (m)
n_b = Manning's roughness coefficient for streambed

Sediment continuity analysis

In theory, sediment continuity analysis may be used for channels with any type of bed material. In practice, the lack of reliable sediment transport relations for channels with bed material finer than sand or coarser than cobbles makes such analysis difficult. In continuity analysis, the volume of sediment deposited in or eroded from a reach during a given period of time is computed as the difference between the volumes of sediment entering and leaving the reach:

$$\Delta V = V_{s_{in}} - V_{s_{out}} \qquad \text{(eq. TS14B–18)}$$

where:
ΔV = volume of bed-material sediment stored or eroded, ft^3 (m^3)
$V_{s_{in}}$ = volume of bed-material sediment supplied to the reach, ft^3 (m^3)
$V_{s_{out}}$ = volume of bed-material sediment transported out of the reach, ft^3 (m^3)

From equation TS14B–18, the average amount of bed level change may be computed by:

$$z_{ad} = \frac{\Delta V}{W_c L_r} \qquad \text{(eq. TS14B–19)}$$

where:
W_c = average channel width, ft (m)
L_r = reach length, ft (m)

Values of V_s may be computed using appropriate sediment transport relationships if bed-material sediment grain size distribution, design discharge, and reach hydraulics are known. Selection and use of sediment transport relationships are described in NEH654.09 and Richardson, Simons, and Lagasse (2001). Lagasse, Schall, and Richardson (2001) demonstrate the use of the Yang equations for sand and gravel for this purpose. Normally, sediment concentrations are computed only for the design discharge and converted to volume by multiplying by the water discharge and a time $\Delta\tau$ corresponding to the duration of the design discharge. For a more complete analysis, sediment concentration may be computed for a range of water discharges and combined with a flow-duration curve to obtain long-term values of ΔV. Alternatively, the design event hydrograph may be expressed as a series of

Table TS14B–7 Constants for equilibrium slope formulas for coarse bed channels with little or no sediment load input

Relationship	U.S. units	SI units	Reference
Manning and Shields	1.486	1.0	Lagasse, Schall, and Richardson (2001)
Meyer-Peter and Müller	60.1	28.0	Lagasse, Schall, and Richardson (2001)
Schoklitsch	0.00174	0.000293	Pemberton and Lara (1984)
Henderson	0.44 (D_{50} in ft)	0.33 (D_{50} in m)	Henderson (1966)

n discrete time intervals, and ΔV values computed for the average discharge occurring during each interval. Numerical integration is then used to obtain the total ΔV for the event:

$$\Delta V = \sum_{i=1}^{n} \left(V_{s_{in}\,i} - V_{s_{out}\,i} \right) \Delta t_i \qquad \text{(eq. TS14B–20)}$$

This type of analysis may be laborious if several events are simulated, and changes in reach hydraulics due to changes in bed-material gradation may be hard to track. More sophisticated methods are described in the following section.

More complex approaches for long-term aggradation or degradation

Detailed assessment of scour and deposition in a channel reach under natural (unsteady) inputs of water and sediment require numerical (computer) simulation modeling. Since flow records are input as hydrographs, it is not necessary to select a single design flow. Primary types of simulation models include one-dimensional models, which simulate changes in bed elevation with streamwise distance, but ignore variations from one side of the channel to the other, and two-dimensional models, which represent the channel bed as a mosaic of rectangular areas, but do not simulate variations in velocity in the vertical direction. One-dimensional models have limited capability to simulate local scour.

The models route sediment down a channel and adjust the channel geometry (usually bed elevation, but not bank position) to reflect imbalances in sediment supply and transport capacity. The BRI–STARS (Molinas 1990) and HEC–6 (USACE 1993c) models are examples of sediment transport models that can be used for single event or long-term degradation and aggradation estimates. HEC–6 is introduced in NEH654.13, and simulates only changes in bed elevation, while BRI–STARS has an option for predicting width adjustment. The USDA ARS model CONCEPTS includes a full suite of routines for assessing the geotechnical stability of channel banks and erosion of bank material through both hydraulic and geotechnical processes, as well as one-dimensional flow modeling (Langendoen 2000). The information needed to run these models includes (Lagasse, Schall, and Richardson 2001):

- channel and flood plain geometry
- structure geometry
- hydraulic roughness
- geologic or structural vertical controls
- downstream water surface relationship
- event or long-term inflow hydrographs
- tributary inflow hydrographs
- bed-material gradations
- upstream sediment supply
- tributary sediment supply
- selection of appropriate sediment transport relationship
- depth of alluvium

CONCEPTS also requires data describing geotechnical properties of bank soils. None of the models can predict the formation of nickpoints or their migration rates. Modeling movable-bed channels requires specialized training and experience. A description of how models should be used is presented by USACE (1993c).

General scour

Process description

General scour refers to all types of scour that are not local (fig. TS14B–3). General scour commonly, but not necessarily, occurs over the entire cross section, and may involve reaches of varying length depending on the type of scour and site-specific conditions. General scour includes contraction scour and bend scour. Presumably, most of the scour measured at the 21 sites observed by Blodgett (1986) was general scour. He noted that:

$$z_t\left(\text{mean}\right) = K\,D_{50}^{\;-0.11} \qquad \text{(eq. TS14B–21)}$$

and

$$z_t\left(\text{max}\right) = K\,D_{50}^{\;-0.11} \qquad \text{(eq. TS14B–22)}$$

where:

z_t (mean) = best fit curve (fig. TS14B–1) for observed scour depth, ft

z_t (max) = enveloping curve (fig. TS14B–1) for maximum scour depth, ft

K = coefficient = 1.42 and 6.5 for z_t mean and z_t max, respectively (0.84 and 3.8 for SI units), and D_{50} is the median size of the bed material, ft (mm)

Pemberton and Lara (1984) suggested that regime equations provided by Blench (1970) and Lacey (1931) could be used to predict general scour in natural channels. A designer may compute scour depth using both formulas, and average the outcome or take the largest value.

These regime relationships may be expressed as:

$$z_t = KQ_d^a W_f^b D_{50}^{\ c} \qquad \text{(eq. TS14B–23)}$$

where:

z_t = maximum scour depth at the cross section or reach in question, ft (m)

K = coefficient (table TS14B–8)

Q_d = design discharge, ft^3/s (m^3/s)

W_f = flow width at design discharge, ft (m)

D_{50} = median size of bed material (mm)

a, b, c = exponents (table TS14B–8)

Values for the coefficient and exponents are as shown in table TS14B–8 when U.S. units are used for Q_d, and W_f and D_{50} is in millimeters.

Values for the exponents, a, b, and c are unchanged when SI units are used, but values for the coefficient K when SI units are used for Q and W_f, and D_{50} is in millimeters (table TS14B–9).

Contraction scour

Contraction scour occurs when the flow cross section is reduced by natural features, such as stone outcrops, ice jams, or debris accumulations, or by constructed features such as bridge abutments. Contraction scour is most often observed when bridge approaches force flood plain flow back into the main channel to pass under the bridge. According to the law of continuity, a decrease in flow area requires an increase in the mean velocity component normal to the area, which produces an attendant increase in boundary shear stresses and bed-material transport, assuming the boundary is erodible. As erosion progresses, area increases and velocity decreases, leading to an equilibrium condition in which the rate of bed material transported into the contracted reach is equivalent to the rate of transport out of the contracted reach. Contraction scour is a

Table TS14B–8 Constants for Lacey and Blench relations, U.S. units (D_{50} in mm)

Condition	Lacey				Blench			
	K	**a**	**b**	**c**	**K**	**a**	**b**	**c**
Straight reach	0.097	1/3	0	–1/6	0.530	2/3	–2/3	–0.1092
Moderate bend	0.195	1/3	0	–1/6	0.530	2/3	–2/3	–0.1092
Severe bend	0.292	1/3	0	–1/6	0.530	2/3	–2/3	–0.1092
Right angle bend	0.389	1/3	0	–1/6	1.105	2/3	–2/3	–0.1092
Vertical rock wall	0.487	1/3	0	–1/6				

Table TS14B–9 Constant K for Lacey and Blench relations, SI units (D_{50} in mm)

Condition	Lacey	Blench
Straight reach	0.030	0.162
Moderate bend	0.059	0.162
Severe bend	0.089	0.162
Right angle bend	0.119	0.337
Vertical rock wall	0.148	0.000

form of general scour because material is removed from all, or almost all, of the wetted perimeter of the contracted section.

Live-bed contraction scour

Live-bed conditions may be assumed at a site if the mean velocity upstream exceeds the critical velocity for the beginning of motion, V_c, for the median size of bed material available for transport, D_{50}. When the velocity falls below the critical level, clear-water scour dominates. Both types of scour may occur during a given hydrologic event. The critical velocity may be estimated by:

$$V_c = Ky^{\frac{1}{6}} D_{50}^{\frac{1}{3}} \qquad \text{(eq. TS14B–24)}$$

where:
V_c = critical velocity, ft/s (m/s)
y = average flow depth in the reach in question, ft (m)
D_{50} = median bed particle size, ft (m) (note units)
K = a constant which is 11.17 for U.S. units or 6.19 for SI units

Richardson and Davis (2001) provide guidance for estimating contraction scour associated with bridges. In general, the procedure consists of using the following equations for estimating contraction scour depth under live-bed conditions:

$$z_c = y_2 - y_o \qquad \text{(eq. TS14B–25)}$$

and

$$\frac{y_2}{y_1} = \left(\frac{Q_2}{Q_1}\right)^{\frac{6}{7}} \left(\frac{W_{b1}}{W_{b2}}\right)^{a} \qquad \text{(eq. TS14B–26)}$$

where:
z_c = contraction scour
y_o = average initial depth in the contracted section
y_1 = average depth in the upstream channel
y_2 = average ultimate depth in the contracted section
Q_1 = flow rate in upstream channel, ft^3/s (m^3/s)
Q_2 = flow rate in the contracted section, ft^3/s (m^3/s)
W_{b1} = bottom width of the upstream channel, ft (m)
W_{b2} = bottom width of the contracted section, ft (m)
a = empirical exponent based on ratio of shear velocity to fall velocity of bed material determined (table TS14B–10):

Table TS14B–10 Exponent a for contraction scour relation

U_*/ω	a	Mode of bed-material transport
<0.50	0.59	Mostly contact bed-material discharge
0.50 to 2.0	0.64	Some suspended bed-material discharge
>2.0	0.69	Mostly suspended bed-material discharge

$$U_* = \left(\frac{\tau_o}{\rho}\right)^{\frac{1}{2}} = \left(gy_1S_e\right)^{\frac{1}{2}}$$

where:
U_* = $(\tau_o/\rho)^{1/2} = (gy_1S_e)^{1/2}$, shear velocity in the upstream section, ft/s (m/s)
g = acceleration of gravity, 32.2 ft/s^2, (9.81 m/s^2)
S_e = slope of energy grade line of main channel, ft/ft (m/m)
τ_o = average bed shear stress in the upstream section, lb/ft^2 (N/m^2), given by:

$$\tau_o = \gamma_\omega RS_e$$

where:
R = hydraulic radius, ft (m)
S_e = energy slope
ρ = density of water, 1.94 $slugs/ft^3$ (1,000 kg/m^3)
ω = fall velocity of bed material based on the D_{50}, ft/s (m/s)

Fall velocity for sand-sized particles may be read from the curves in figure TS14B–10 (Richardson and Davis 2001) or computed from formulas provided by Ahrens (2000).

$$\omega = \frac{K_1 \Delta S_g g D_s^2}{\nu} + K_2 \sqrt{\Delta S_g g D_s} \qquad \text{(eq. TS14B–27)}$$

where:

$$K_1 = 0.055 \tanh\left[12A^{-0.59} \exp\left(-0.0004A\right)\right] \qquad \text{(eq. TS14B–28)}$$

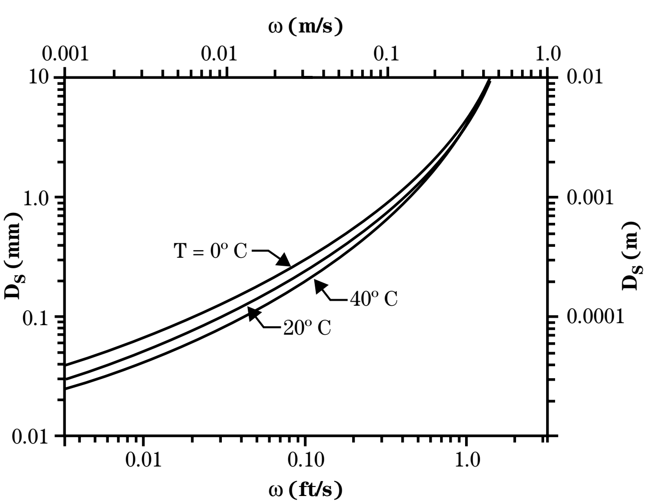
$$K_2 = 1.06 \tanh\left[0.016A^{0.50} \exp\left(-\frac{120}{A}\right)\right] \qquad \text{(eq. TS14B–29)}$$

ΔS_g = relative submerged density of bed-material sediments $\simeq$ 1.65
g = acceleration of gravity, 32.2 ft/s^2 (9.81 m/s^2)
ν = kinematic viscosity of water, ft^2/s (m^2/s)
D_s = a characteristic sediment diameter, ft (m)

$$A = \frac{\Delta S_g g D_s^3}{\nu^2} \qquad \text{(eq. TS14B–30)}$$

If bottom width is not easily defined, it is permissible to use top width for W_{b1} and W_{b2}, but it is important to use a consistent definition of width for both quantities. In sand-bed streams, a contraction scour zone is often formed during high flows and refilled during falling stages. In such a case, y_o may be approximated by y_1. Live-bed scour depths are sometimes limited by coarse sediments within the sediment mixture that form an armor layer. When gravel or larger sized material is present, it is recommended that scour depths be calculated using both live-bed and clear-water approaches, and that the smaller of the two scour depths be used. The procedure is a simplified version of one described in greater detail in Petersen (1986). An alternative approach for gravel-bed contraction scour is presented by Wallerstein (2003) that is based on sediment continuity.

Figure TS14B–10 Fall velocity for sand-sized particles with a specific gravity of 2.65



Clear-water contraction scour

Clear-water scour occurs when there is insignificant transport of bed-material sediment from the upstream into the contracted section. In some cases, a channel constriction creates enough of a backwater condition to induce sediment deposition upstream. Scour in the contracted section normally increases as the flow velocity increases. Live-bed scour becomes clear-water scour in the contracted section. The magnitude of clear-water contraction scour may be computed as follows (Richardson and Davis 2001):

$$z_c = y_2 - y_o \qquad \text{(eq. TS14B–31)}$$

and

$$y_2 = \left[\frac{KQ_2^2}{D_s^{\frac{2}{3}} W_{b2}^2}\right]^{\frac{3}{7}} \qquad \text{(eq. TS14B–32)}$$

where:
- K = 0.0077 for U.S. units and 0.025 for SI units
- Q_2 = discharge through the contracted section, ft^3/s (m^3/s)
- D_s = diameter of the smallest nontransportable particle in the bed material. Assumed = $1.25D_{50}$, ft (m), (note units)
- W_{b2} = bottom width of the contracted section, ft (m)

The assumption that $D_m = 1.25D_{50}$ implies that some armoring takes place as scour occurs. If the bed material is stratified, the ultimate scour depth may be determined by using the clear-water scour equation sequentially with successive D_s of the bed-material layers.

Bridge scour

Flow under bridges often produces local scour around bridge piers. Contraction of the floodway by bridge abutments and approaches also causes contraction scour across the cross section. Due to the economic and safety implications of bridge scour, it has received more study than any other type of scour, with extensive analyses of the effects of pier and abutment geometry, flow regime, sediment load, and bedforms. Scour at bridges is intensified when debris becomes trapped against the upstream side of piers (fig. TS14B–11 (Huizinga and Rydlund 2001). When the water surface upstream from a bridge opening is higher than downstream, a special condition known as pressure flow occurs. Pressure flow scour may be two to three times as great as normal contraction scour.

NEH654 TS14Q provides guidance for the analysis and design of small bridge abutments. The reader is also referred to Richardson and Davis (2001) for further design guidance.

Bend scour

Flow through channel meander bends results in water moving in a corkscrew or helical pattern that moves sediment from the outside (concave bank) to the inside of the bend, which is often a point bar. Velocity components not in the streamwise direction are referred to as secondary currents, and the secondary currents that occur in meander bends, though often quite complex, generally have the effect of eroding outer banks. The bank toe is often the locus of maximum shear and erosion, particularly if the bank is armored or otherwise resistant to erosion. Empirical relationships between the maximum depth of scour in a bend and the average depth in a bend have been developed using much of the field data as described in

Figure TS14B–11 Downstream face of Horse Island Chute bridge near Chester, IL, as viewed from left (north) embankment. Note debris trapped on upstream face of bents.

NEH654.09. Briefly, the field data lead to a conservative formula for bend scour, $z_b = y_{mean} - y_{max}$.

$$z_b = y\left(\frac{y_{max}}{y} - 1\right) \qquad \text{(eq. TS14B–33)}$$

where:
y = average flow depth in the bend, ft (m)
y_{max} = maximum flow depth in the bend, ft (m)

$$\frac{y_{max}}{y} = 1.5 + 4.5\left(\frac{W_i}{Rc}\right) \qquad \text{(eq. TS14B–34)}$$

where:
W_i = channel width at bend inflection point, ft (m)
Rc = bend radius of curvature, ft (m)

This equation is an asymptotic relationship with a theoretical minimum y_{max}/y_{mean} of 1.5 representing pool scour depths expected in a straight channel with a pool-riffle bed topography. From this upper-bound relationship, y_{max}/y_{mean} ranges from 4 to 3 for W_i/Rc between 0.33 and 0.56. For channels with W_i /Rc > 0.56, y_{max} is independent of bend curvature, and it is recommended that a value of 4 be used for y_{max}/y_{mean}. Consult NEH654.09 for additional detail.

Relations for predicting the location of maximum depth are also provided in NEH654.09. The length of the scoured zone may be approximated using a relationship by Chen and Cotton (1988):

$$\frac{L_p}{R} = 0.0604\left(\frac{R^{\frac{1}{6}}}{n}\right) \qquad \text{(eq. TS14B–35)}$$

where:
L_p = recommended length of protection, ft (m), measured downstream from the bend apex (fig. TS14B–12)
R = hydraulic radius = flow area/wetted perimeter, ft (m)
n = Manning n value for the bend

This relationship (eq. TS14B–35 and fig. TS14B–13) is only approximate, and scour locations vary considerably from bend to bend and with time in a given bend. NEH654.09 presents information regarding the observed distribution of scour locations (referred to as the pool-offset ratio) in a study of bends along the Red River.

Scour depths at bank toes on the outside of bends usually increase after construction of armored bank revetments. Increased resistance to bank erosion must intensify stresses acting at the bank toe, causing deeper scour. Maynord (1996) suggested the following empirical relationship for estimating toe scour in such a situation. This equation is embedded in the CHANLPRO software (Maynord, Hebler, and Knight 1998):

$$\frac{y_{max}}{y_c} = FS\left[1.8 - 0.051\left(\frac{Rc}{W_i}\right) + 0.0084\left(\frac{W_i}{y_c}\right)\right] \qquad \text{(eq. TS14B–36)}$$

where:
y_{max} = maximum water depth in the bend, ft (m)
y_c = mean water depth in the crossing upstream from the bend, ft (m)
FS = a factor of safety defined below

This relationship is limited to situations where (1.5 < Rc/W_1 < 10) and (20 < W_1/y < 125). The factor of safety, FS, may vary from 1.00 to 1.10.

The relationship above was developed using 215 data points from several rivers. When FS = 1.00, 25 percent of the observed values of y_{max}/y_c were underpredicted by more than 5 percent. When FS = 1.19, only 2 percent of the observed values of y_{max}/y_c were underpredicted by more than 5 percent (Maynord 1996). The above equation is similar to the one recommended in NEH654.09 for bend scour. In fact, the values of y_{max}/y_c predicted by these relationships vary by less than 25 percent for FS = 1.19 and 5 < Rc/W_1 < 10. The bend scour equation from NEH654.09 is slightly more conservative than the Maynord (1996) equation. Only one of the two equations should be used, even if the outside of the bend is protected.

Figure TS14B–12 Definition of recommended length for protection downstream from a bend apex, L_p

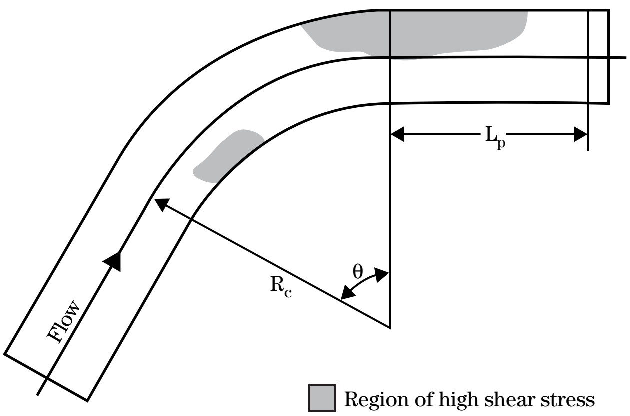
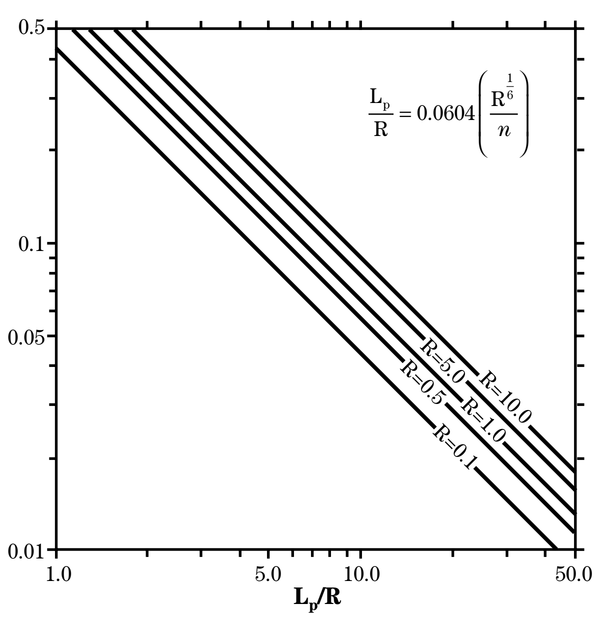


Figure TS14B–13 Recommended length of protection divided by hydraulic radius, L_p/R, as a function of Manning roughness coefficient for the bend, n, and hydraulic radius, R



$$\frac{L_p}{R} = 0.0604\left(\frac{R^{\frac{1}{6}}}{n}\right)$$

Bedform scour

Process description

Mobile riverbeds deform to produce ripples, dunes, and antidunes at specific levels of shear stress for a given sediment size (fig. TS14B–14). Most textbooks also recognize large bars (forms having length equal to the channel width or greater) as a type of bedform, but reliable predictors for bars have not been developed. In practice, bedforms other than bars are uncommon in channels dominated by sediments coarser than sand. Dunes and antidunes in sand beds can result in additional scour, since they migrate by a systematic process of erosion and deposition (ripples are too small to be significant), controlled by flow velocities. The passage of a large dune may increase local scour depths as much as 30 percent.

Many attempts have been made to develop relationships to predict the type and dimensions of bedforms based on the bed sediment gradation and the imposed flow. In general, scour analysis involves the use of a bedform predictor that is related to bedform type and amplitude. Half of the amplitude is then assumed to contribute to total scour. Some types of bedforms, however, often occur side-by-side in a cross section or reach of a natural stream. Nonetheless, scour computations normally focus on either dunes or antidunes, which have the greatest amplitude.

Bedform predictors

Water flowing over an erodible bed can produce a variety of configurations. Van Rijn (1984) suggested that dunes would form when:

$$D_* = D_{50}\left(\frac{\Delta S_g g}{\nu^2}\right)^{\frac{1}{3}} > 10 \qquad \text{(eq. TS14B–37)}$$

and

$$3 < T_{ts} < 15 \qquad \text{(eq. TS14B–38)}$$

where:

$$T_{ts} = \frac{\tau_s^* - \tau_c^*}{\tau_c^*} \qquad \text{(eq. TS14B–39)}$$

D_* = dimensionless sediment size
D_{50} = median grain size, ft (m) (note units)
g = acceleration of gravity, 32.2 ft/s^2 (9.81 m/s^2)
ν = kinematic viscosity of water, ft^2/s (m^2/s)
T_{ts} = dimensionless transport-stage parameter
τ_s^* = bed shear stress due to skin or grain friction, lb/ft^2 (N/m^2), which may be computed by

$$\tau_s^* = \frac{\rho g u^2}{\left[18\log\left(\frac{12R}{3D_{90}}\right)\right]^2} \qquad \text{(eq. TS14B–40)}$$

where:
u = mean flow velocity, ft/s (m/s)
R = hydraulic radius, ft (m)
D_{90} = size larger than 90 percent of the bed material by weight, ft (m) (note units)
τ_c^* = critical shear stress for motion from the Shields diagram, which may be computed by τ_c^* = 103.0 θD_{50} for D_{50} in ft and τ_c^* in lb/ft^2 (τ_c^*=16,187 θD_{50} for D_{50} in m and τ_c^* in N/m^2)

where:
θ = dimensionless Shields stress ranging from 0.02 to 0.10 for sands and larger sediments. (See compilation of values by Buffington and Montgomery (1997) for appropriate value or use the following equation to compute a value.):

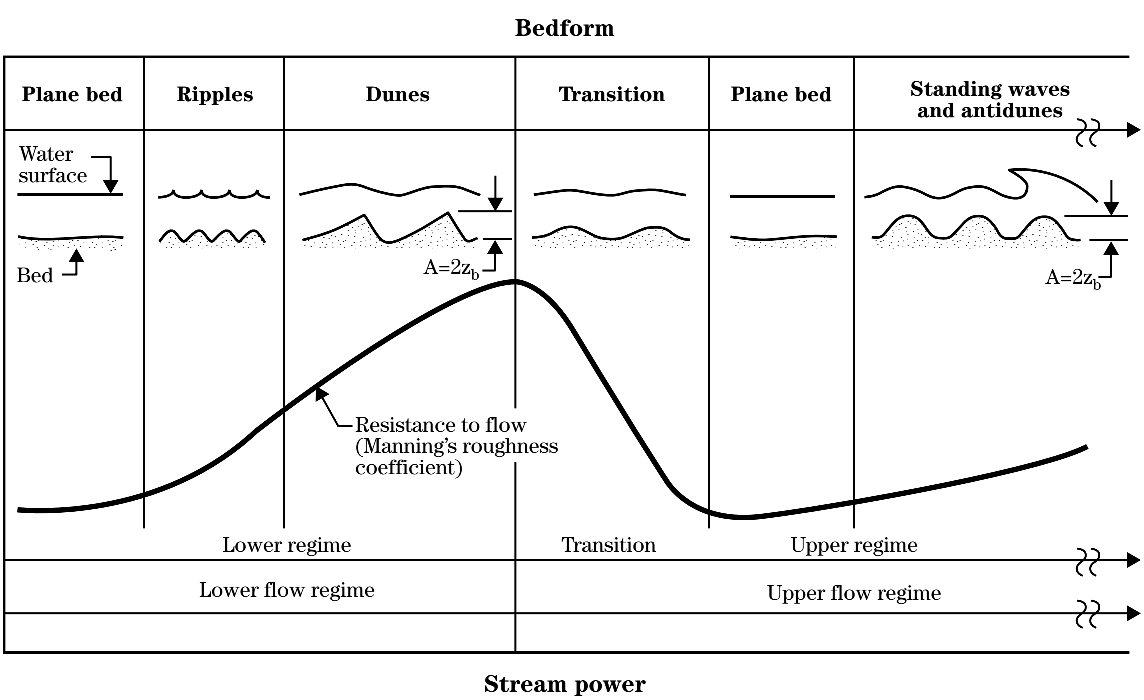
$$\theta_c = \frac{0.24}{D_*} + 0.055\left[1-\exp\left(-0.02D_*\right)\right] \qquad \text{(eq. TS14B–41)}$$

where:

$$D_* = D_{50}\left(\frac{1.65g}{\nu^2}\right)^{\frac{1}{3}} \qquad \text{(eq. TS14B–42)}$$

where:
D_* = dimensionless sediment size
D_{50} = median grain size in ft (m)
g = acceleration of gravity, 32.2 ft/s^2 (9.81 m/s^2)
ν = kinematic viscosity of water, ft^2/s (m^2/s)

Figure TS14B–14 Relative relationships between progression of alluvial bedforms and flow intensity



Transitional bedforms occur for $15 < T_{ts} < 25$, and antidunes occur when $T_{ts} > 25$. These relationships may be used to determine what type of bedform will occur, given design conditions. Additional complexities arise due to the influence of water temperature and suspended sediment concentration on viscosity, blanketing of coarse sediment beds by sands during certain events, and the fact that mean flow velocity is governed by total flow resistance, which itself is a function of bedform type and geometry.

If the above analysis indicates that dunes will occur, dune height may be computed by:

$$\Delta = 0.11 D_{50}^{0.3} y^{0.7} \left(1 - \exp\left(-0.5T_{ts}\right)\right)\left(25 - T_{ts}\right) \quad \text{(eq. TS14B–43)}$$

where:
Δ = dune height, ft (m)
y = mean flow depth, ft (m)

Similar relationships for antidunes are not available, but the flow depth may be used as a conservative estimator of maximum antidune height. For either dunes or antidunes, the scour depth is assumed to be equal to half of the bedform height:

$$z_{bf} = \frac{\Delta}{2} \quad \text{(eq. TS14B–44)}$$

Some empirical formulas (eq. TS14B–35) that are based on data sets from sand-bed streams implicitly include bedform scour. Usually bedform scour is much smaller in magnitude than other types of scour in sand-bed rivers.

Consult reviews by Garcia (in press) and Yang (1996) for more information on bedforms.

Scour associated with structures

Structures that span the full width of the channel

Structures that span the full width of the channel include sills, grade control structures, and structures comprised of boulders. The latter are intended to create step-pool morphology in steep, coarse-bed streams. Sills may be thought of as very low weirs, and grade control structures are higher weirs with associated appurtenances. These are used for bed erosion control and pool habitat development (fig. TS14B–15). Figure TS14B–15(a) shows a weir immediately after construction in a sand- and gravel-bed stream. The view shown is facing upstream. Central notch was constructed with invert at existing streambed elevation, and figure TS14B–15(b) is facing downstream from the notch about 6 months later.

Figure TS14B–15 Scour associated with low stone weir

(a)

(b)

Predicting scour depths downstream from weirs and grade control structures is too complex for theoretical calculations. Empirical formulas are used and are based on laboratory flume tests and field data. The scour equations are intended to allow prediction of scour depths in unprotected noncohesive alluvial beds. Commonly, grade control structures are built with preformed, stone-protected downstream scour holes (also called stilling basins). In some cases, these basins are sized using scour prediction equations. Since the equations provided below are empirical formulas, the engineer should become familiar with the original references and apply the formulas with care if the project falls outside the ranges of parameters used to generate them. A more comprehensive treatment of this topic is provided by Simons and Sentürk (1992).

Sills

Series of relatively low weirs (sills) are often used to develop pool habitats and to prevent mild to moderate bed degradation. Often these structures are installed by excavating a trench in the bed perpendicular to the flow and placing the structure into the trench so that the initial crest elevation is at bed elevation. Subsequent erosion produces a pool-and-riffle profile. Lenzi et al. (2002) reviewed work by others and conducted a series of flume experiments, resulting in different formulas for low gradient (slope ≤ 0.02) and high gradient (S ≥ 0.08) mountain streams.

For low gradient streams, the scour depth, z_s (fig. TS14B–16), is given by:

$$\frac{z_s}{H_s} = 0.180\frac{a_1}{\Delta S_g D_{50}} + 0.369 \qquad \text{(eq. TS14B–45)}$$

and the length of the scour pool, l_p (based only on tests with gravel sediments) is given by:

$$\frac{l_p}{H_s} = 1.87\frac{a_1}{\Delta S_g D_{50}} + 4.02 \qquad \text{(eq. TS14B–46)}$$

while for high gradient streams, z_s is given by:

$$\frac{z_s}{H_s} = 0.4359 + 1.4525\left(\frac{a_1}{H_s}\right)^{0.8626} + 0.0599\left(\frac{a_1}{\Delta S_g D_{95}}\right)^{1.4908} \qquad \text{(eq. TS14B–47)}$$

and the length of the scour pool is given by:

$$\frac{l_p}{H_s} = 4.479 + 0.023\left(\frac{a_1}{H_s}\right)^{-1.808} + 2.524\left(\frac{a_1}{\Delta S_g D_{95}}\right)^{1.129} \qquad \text{(eq. TS14B–48)}$$

where:

z_s = depth of scour downstream from structure, ft (m), measured from the crest of the structure to the lowest point within the scour pool

H_s = specific energy of critical flow over the sill, ft (m), where:

$$H_s = 1.5 \times \sqrt[3]{\frac{q^2}{g}} \qquad \text{(eq. TS14B–49)}$$

where:

q = flow per unit width over the sill at design discharge, ft³/s/ft (m³/s/m)

g = acceleration of gravity, 32.2 ft/s² (9.81 m/s²)

a_1 = the “morphological jump” =

$$a_1 = \left(S_o - S_{eq}\right)L_s$$

where:

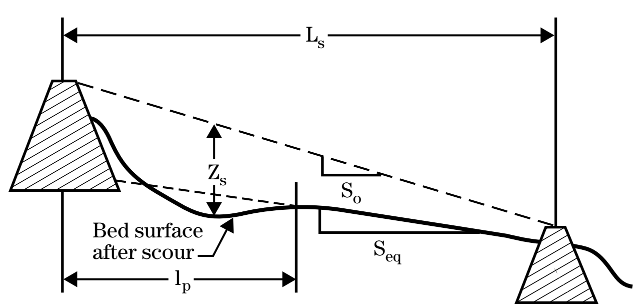
S_o = initial longitudinal bed slope

L_s = horizontal distance between sills, ft (m) (fig. TS14B–16)

S_{eq} = equilibrium bed slope after scour, which may be estimated by:

$$S_{eq} = \frac{\theta_c \Delta S_g D_{50}}{y} \qquad \text{(eq. TS14B–50)}$$

Figure TS14B–16 Definition sketch for computing scour associated with sills



where:
θ_c = critical dimensionless shear stress or-Shields constant, which may be computed for a given sediment size using equation TS14B–41
ΔS_g = relative submerged density of bed-material sediments
D_{50} = median size of bed-material sediments, ft (m) (note units)
l_p = length of scour pool, ft (m)

The formulas were based on data with $0.225 < a_1/H_s < 1.872$ and $0.161 < a_1/\Delta D_{95} < 1.150$, and any application outside these ranges should be done with greatest care. Subsequent application by Lenzi, Comiti, and Marion (2004) to a mountain river predicted scour hole depth below 26 bed sills accurately, but overpredicted scour hole length.

Step-pool structures

Thomas et al. (2000) studied natural step-pools in eight coarse grained mountain streams in Colorado and developed regression equations for design of step-pool structures in steep, boulder-bed streams (fig. TS14B–17):

$$\frac{z_s}{W} = -0.0118 + 1.394\left(\frac{h_d}{W}\right) + 5.514\left(\frac{Sq_{25}}{W^{\frac{3}{2}}\sqrt{g}}\right) \qquad \text{(eq. TS14B–51)}$$

and

$$\frac{l_p}{W} = 0.409 + 4.211\left(\frac{h_d}{W}\right) + 87.341\left(\frac{Sq_{25}}{W^{\frac{3}{2}}\sqrt{g}}\right) \qquad \text{(eq. TS14B–52)}$$

where:
z_s = depth of scour downstream from structure, ft (m), measured from the crest of the structure to the lowest point within the scour pool
W = average active channel width, ft (m)
h_d = height of step crest above controlling bed elevation at downstream end of pool, ft (m)
S = average channel bed slope
q = flow per unit width over the sill at design discharge (q_{25} is for 25-yr discharge), ft^3/s/ft (m^3/s/m)
g = acceleration of gravity, 32.2 ft/s^2 (9.81 m/s^2)
l_p = length of scour pool, ft (m)

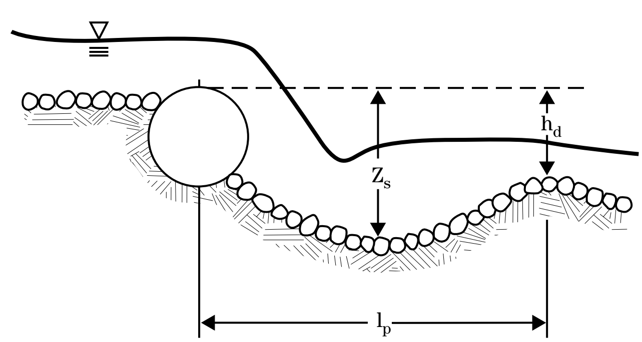
Grade control structures and weirs

Weirs such as grade control structures (fig. TS14B–18) differ from sills in that they are built with crest elevations well above the existing bed. They normally produce backwater effects at baseflow. Several empirical approaches are available for computing the depth of scour in unprotected, noncohesive beds downstream from a vertical weir. Most of these equations were originally developed to compute the depth of scour downstream from dams. The Veronese (1937) equation yields an estimate of erosion measured from the tailwater surface to the bottom of the scour hole:

$$y + z_s = K h_d^{0.225} q^{0.54} \qquad \text{(eq. TS14B–53)}$$

where:
y = average depth of flow in channel downstream from scour hole, ft (m)
z_s = depth of scour, ft (m)
K = a coefficient = 1.32 for U.S. units (1.90 for SI units)
h_d = vertical distance between water surface elevation upstream and downstream from the weir, ft (m)
q = discharge per unit width, ft^3/s/ft (m^3/s/m)

Figure TS14B–17 Definition sketch for computing scour associated with step-pool structures



The Veronese (1937) equation was modified by Yildiz and Üzücek (1994) to include the angle of the weir overfall jet, α

$$y + z_s = K h_d^{0.225} q^{0.54} \cos\alpha \qquad \text{(eq. TS14B–54)}$$

where:

α = angle the incident jet makes with the vertical. A vertical overfall of water from a cantilevered pipe or sharp-crested weir would have α=0.

Neither version of the Veronese equation contains any expression that reflects the erodibility of the bed, which intuitively seems to be a major deficiency. A more recent formula for scour produced by a free falling jet addresses this issue (Mason and Arumugam 1985). The form is limited to SI units:

$$z_s = K \frac{q^a h_d^b y_t^{0.15}}{g^{0.30} D_m^{0.10}} \qquad \text{(eq. TS14B–55)}$$

where:

z_s = depth of scour (m)
K = $6.42 - 3.2h_d^{0.10}$ (eq. TS14B–56)
q = discharge per unit width (m^3/s/m)
h_d = vertical distance between water surface elevation upstream and downstream from the weir (m)
y_t = tailwater depth above original ground surface (m)
a = $0.6–h_d/300$ (eq. TS14B–57)
b = $0.15\ h_d/200$ (eq. TS14B–58)
g = acceleration of gravity, 9.81 m/s^2
D_m = mean bed-material particle size, m (note units). In the case of beds made of rock, a value of 0.25 meter is used.

Figure TS14B–18 (a) Low- and (b) high-drop grade control structures

(a)

(b)

D'Agostino and Ferro (2004) presented a review of previous work dealing with prediction of scour downstream from grade control structures. In addition, they compiled available data sets and analyzed them using stepwise regression to produce a function of dimensionless variables that were formed, using dimensional analysis. They proposed the following relationship for computing the maximum scour depth (fig. TS14B–19):

$$\frac{z_s}{y_w} = 0.540\left(\frac{W_w}{y_w}\right)^{0.593}\left(\frac{y_t}{h_d}\right)^{-0.126} A_{50}^{0.544}\left(\frac{D_{90}}{D_{50}}\right)^{-0.856}\left(\frac{W_w}{W}\right)^{-0.751}$$

(eq. TS14B–59)

where:

y_w = vertical distance between weir crest and upstream channel bed, ft (m)
W_w = width of weir crest, ft (m)
y_t = tailwater depth above original ground surface, m
h_d = difference in water surface elevation upstream of weir and downstream from weir, ft (m)
A_{50} = a dimensionless quantity defined below
D_{50} = median size of bed material, ft (m) (note units)
D_{90} = size of bed material larger than 90 percent of the bed by weight, ft (m) (note units)
W = flow width at design discharge, ft (m)

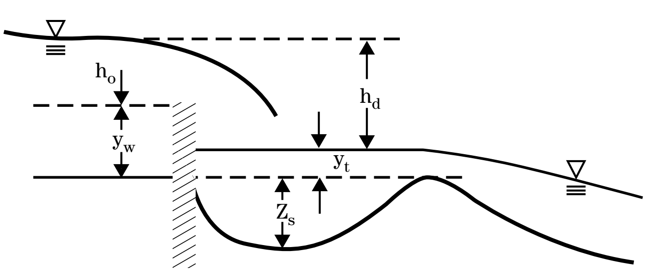
The quantity A_{50} is given by:

$$A_{50} = \frac{Q_d}{W_w y_w \sqrt{g D_{50} \Delta S_g}} \qquad \text{(eq. TS14B–60)}$$

where:

ΔS_g = relative submerged density of bed-material sediments ≅ 1.65
Q_d = design discharge, ft^3/s (m^3/s)

Figure TS14B–19 Definition sketch for computing scour associated with grade control structures



The following relationship was recommended for estimating the horizontal distance between the weir and the deepest point in the scour hole:

$$\frac{L_s \max}{y_w} = 1.616\left(\frac{W_w}{y_w}\right)^{0.662}\left(\frac{y_t}{h_d}\right)^{-0.117} A_{90}^{0.455}\left(\frac{W_w}{W}\right)^{-0.478}$$

(eq. TS14B–61)

The quantity A_{90} is similar to A_{50}:

$$A_{90} = \frac{Q}{W_w y_w \sqrt{g D_{90} \Delta S_g}} \qquad \text{(eq. TS14B–62)}$$

Sloping drop structures such as rock ramps or Newbury riffles may be attractive options in some stream restoration projects, particularly from an aesthetic and fish passage standpoint. Laursen, Flick, and Ehlers (1986) ran a limited number of flume experiments with sloping sills with slopes of 4H:1V and produced the following relationship:

$$\frac{y_2}{y_c} = 4\left(\frac{y_c}{D_s}\right)^{0.2} - 3\left(\frac{D_r}{y_c}\right)^{0.1} \qquad \text{(eq. TS14B–63)}$$

where:

y_2 = depth of water in downstream channel after scour, ft (m)
y_c = critical flow depth for the design unit discharge, ft (m)
D_s = characteristic bed sediment size, assumed to be median D_{50}, ft (m) (note units)
D_r = characteristic size of rock or riprap used to build the sloping structure, assumed to be median D_{50}, ft (m) (note units).

The depth of scour, z_s, is given by:

$$z_s = y_2 - y_1 \qquad \text{(eq. TS14B–64)}$$

where:

y_1 = depth of water in downstream channel before scour, ft (m)

The analysis and design of grade control structures is also described in NEH654 TS14G.

Structures that partially span the channel

Structures that protrude from one bank into the channel include groins (groynes), spur dikes (spurs), deflectors, bank barbs, and bendway weirs. Kuhnle, Alonso, and Shields (1999, 2002) conducted a series of clear-water, steady-flow, movable-bed flume studies using various spur dike geometries and measured the depth and volume of scour adjacent to the spurs. Empirical formulas for scour depth were developed based on earlier work by Melville (1992). The Melville formulas produce scour depth predictions that are likely conservatively large for prototype conditions. Kuhnle also developed a formula for scour hole volume, and both of his formulas produced acceptable estimates for models of paired current deflectors (Biron et al. 2004). Scour volume is of interest if spurs or deflectors are being used to create pool habitats. Figure TS14B–20(a) shows flags delineating scour hole of short spur, and (b) shows a scour hole downstream from a similar spur in the same reach 1 year after a low extension was added (project described by Shields, Bowie, and Cooper 1995).

Figure TS14B–20 Scour associated with stone spur dike

(a)

(b)

The Kuhnle formulas are:

$$\frac{z_s}{y} = K_1\left(\frac{L_c}{y}\right)^a \qquad \text{(eq. TS14B–65)}$$

and

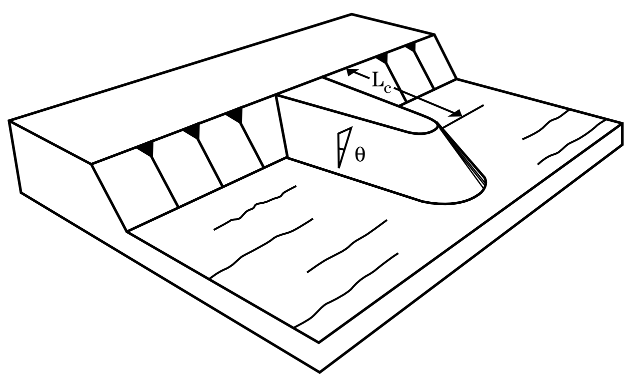
$$\frac{V_s}{z_s^3} = K_2\left(\frac{L_c}{y}\right)^b \qquad \text{(eq. TS14B–66)}$$

where:

z_s = maximum depth of local scour associated with spur dike, ft (m)
y = mean flow depth in approaching flow, ft (m)
L_c = length of spur crest measured perpendicular to flow direction, ft (m) (fig. TS14B–21)
V_s = volume of scour hole, ft^3 (m^3)

The coefficient K_1 is a dimensionless constant reflecting the effect of flow intensity, flow depth, sediment size, sediment gradation, and channel and spur geometry. Kuhnle suggested a value of $K_1 = 2$ when the water surface elevation is below the spur crest and $K_1 = 1.41$ when the spur is submerged.

Figure TS14B–21 Definition sketch showing crest length, L_c, and side slope angle, θ, for spur dikes



The exponent a is a dimensionless exponent that varies with L_c/y. It has a value of a = 1 for $L_c/y < 1$, a = ½ for $1 < L_c/y < 25$, and a = 0 for $L_c/y > 25$.

K_2 = dimensionless coefficient that varies with the angle the spur crest makes with the approach flow

K_2 = 17.106 for perpendicular spurs, and K_2 = 12.11 for spurs that are at a nonperpendicular angle (45° or 135°)

b = dimensionless exponent that varies with spur crest angle. b = –0.781 for perpendicular spurs, and b = 0 in other cases

Rahman and Haque (2004) suggested that K_1 be modified to reflect the shape of the spur cross section for shorter spurs ($L_c/y < 10$):

$$K_1 = 0.75\left(1+\frac{2\tan\phi}{\tan\theta}\right)^{-\frac{1}{2}} \qquad \text{(eq. TS14B–67)}$$

where:

ϕ = angle of repose of bed sediment

θ = side slope of spur structure (fig. TS14B–21)

These formulas produce large scour depths for long spurs. Richardson and Davis (2001) suggest an alternative approach that may be used for such cases.

The analysis and design of spurs and deflectors is presented in more detail in NEH654 TS14H.

Table TS14B–11 presents a summary of scour analyses and applicability to various bed types.

Table TS14B–11 Summary of scour analyses and applicability to various bed types

Predominant bed material	Type of analysis								
	Long-term bed elevation change		**General scour**			**Local scour**			**All types of scour**
	Armoring analysis	**Equilibrium slope**	**Contraction scour**	**Bend scour**	**Bedform scour**	**Bridge pier and abutment scour**	**Structures that span the channel**	**Structures that do not fully span the channel**	**Numerical modeling**
Clay or silt, cohesive	X	✓ Regional regressions (fig. 8)	O	O	X	O	O	O	O
Sand	X	✓No change in bed-material sediment supply—(12)	✓ Empirical relations (21–23)						✓
Fine gravel <6 mm	✓ (2–4)	✓ Reduction in bed sediment supply—(13) ✓ Elimination of bed-material sediment supply—(7)	✓ Live-bed conditions (25–26) ✓ Clear-water conditions (31–32)	✓ (33–36)	✓ Dunes (43–44) ✓ Antidunes (43, assume Δ = y)	✓Richardson and Davis (2001)	✓ Vertical drops (53–55, 59) ✓ Ramps or sloping drops (63–64)	✓ (65, 67)	✓
Gravel >6 mm, cobble		✓ (14–17)		X	X		✓ Sills (45, 47)	O	✓
Boulders	O	O	O	X	X	O	✓ Step-pool structures (51)	O	O

✓ = applicable, X = process not generally observed in this environment, O = process may occur, but analysis is beyond the state of the art. Numbers in parentheses refer to equations in the text. Gray shading indicates techniques with low precision and high uncertainty.

Example computations

Sand-bed reach

A stream restoration project is planned for a sand-bed channel that is currently straight and extremely wide due to historic channelization and straightening. The channel will be narrowed by 30 percent, and stone spur dikes (also known as bank barbs) will be added for stabilization and scour pool development. Side slope of spurs will be 2H:1V, and crests will be submerged at design discharge. Sediment supply from upstream is expected to be unchanged during the life of the project.

Given:

ρ water	1,000 kg/m^3	1.94 slug/ft^3
ρ solids	2,630	5.10 slug/ft^3
relative submerged density, ΔSg	1.63	1.63 constant in
Manning's equation	1	1.486 C in Strickler
equation	0.034	
Shields constant θ_c	0.038215	0.038215 grain
roughness k_s	2.8 mm	0.009 ft
D_{95}	1.5 mm	0.005 ft
D_{90}	0.96 mm	0.003 ft
D_{84}	0.8 mm	0.003 ft
D_{mean}	0.28 mm	0.001 ft
D_{50}	0.3 mm	0.0010 ft
bed sediment internal friction angle	45 deg	0.785 rad
distance to downstream base level control	2,000 m	6,562 ft
Manning's *n*	0.027 s/m$^{(1/3)}$	0.027 s/ft$^{(1/3)}$
design discharge, Q_d	392.2 m^3/s	13849 ft^3/s
flow width	60 m	197 ft
channel width, W	70 m	230 ft
mean flow depth, y	3.0 m	9.8 ft
hydraulic radius, R	3.0 m	9.8 ft
mean streamwise velocity, u	2.2 m/s	7.1 ft/s
unit discharge, q	6.5 m^3/s/m	70 ft^3/s/ft
unit discharge, 25 yr, q_{25}	30 m^3/s/m	0.86 ft^3/s/ft
existing bed slope, S	0.0008 m/m	0.0008 ft/ft
bend radius of curvature, Rc	1,000 m	3,281 ft
L, length of spur crest	20 m	65.6 ft
spur side slope	2H:1V	0.46 rad
spur crest above water surface?	N	

Find:
Total predicted scour depth

Step 1 Compute bed elevation change due to reach-scale degradation based on equilibrium slope.

a. Compute the smallest armor particle size, D_x using equation TS14B–4.

$$D_x = K\left(\frac{yS_e}{\Delta S_g}\right)^a \left(\frac{U_*}{\nu}\right)^b$$

$$U_* = (gyS_e)^{0.5}$$

Assume $S_e = S_o$

$$U_* = \sqrt{32.2 \times 9.8 \times 0.0008} = 0.50 \text{ ft/s}$$

$$\text{Particle Re} = \frac{U_* D_{50}}{\nu} = \frac{0.50 \times 0.001}{1.05 \times 10^{-5}} = 48$$

Therefore, K = 27, a = 0.86, b = –0.14.

$$D_x = 27\left(\frac{9.8 \times 0.0008}{1.63}\right)^{0.86} \left(\frac{0.50 \times}{1.05 \times 10^{-5}}\right)^{-0.14}$$
$$= 0.06 \text{ ft}$$
$$= 18 \text{ mm}$$

The bed does not contain material large enough to form an armor layer.

Step 2 Compute depth of scour needed to produce an equilibrium slope assuming no change in sediment discharge into the reach.

a. Using Yang (1996) regression equation, compute sediment discharge (eqs. TS14B–8 to TS14B–11).

$$a = 0.025 n^{(2.39 - 0.8 \log D_{50})} \left(D_{50} - 0.07\right)^{-0.14}$$
$$= 0.025 \left(0.027\right)^{(2.39 - 0.8 \log D_{50})} \left(0.3 - 0.07\right)^{-0.14}$$
$$= 1.21 \times 10^{-6}$$

$$b = 4.93 - 0.74 \log D_{50}$$
$$= 4.93 - 0.74 \log\left(0.3\right)$$
$$= 5.32$$

$$c = -0.46 + 0.65 \log D_{50}$$
$$= -0.46 + 0.65 \log\left(0.3\right)$$
$$= -0.80$$

$$q_s = au^b y^c$$

$$q_s = \left(1.21 \times 10^{-6}\right)\left(7.1\right)^{5.32}\left(9.8\right)^{-0.80} = 0.0066 \text{ ft}^2\text{/s}$$

b. Compute equilibrium slope (eq. TS14B–12).

$$S_{eq} = \left(\frac{a}{q_s}\right)^{\frac{10}{3(c-b)}} q^{\frac{2(2b+3c)}{3(c-b)}} \left(\frac{n}{K}\right)^2$$

$$S_{eq} = \left(\frac{1.21 \times 10^{-6}}{0.066}\right)^{-0.54} 70^{-0.89} \left(\frac{0.027}{1.486}\right)^2 = 0.00083$$

Since the existing channel slope is approximately equal to the equilibrium slope, long-term degradation should be minimal.

Step 3 Compute contraction scour.

a. Check for live bed conditions using equation TS14B–24.

$$V_c = Ky^{\frac{1}{6}} D_{50}^{\frac{1}{3}}$$

$$V_c = 11.17\left(9.8\right)^{\frac{1}{6}}\left(0.001\right)^{\frac{1}{3}} = 1.60 \text{ ft/s}$$

Since u = 7.1 ft/s > 1.6 ft/s, live bed conditions occur.

b. Compute fall velocity with equations TS14B–27 to TS14B–30.

$$A = \frac{\Delta S_g g D_s^3}{\nu^2}$$

$$A = \frac{1.63 \times 32.2 \times \left(0.001\right)^3}{\left(1.05 \times 10^{-5}\right)^2} = 500$$

$$K_1 = 0.055 \tanh\left[12 A^{-0.59} \exp\left(-0.0004 A\right)\right]$$

$$K_1 = 0.055 \tanh\left[12\left(500\right)^{-0.59} \exp\left(-0.0004\left(500\right)\right)\right] = 0.014$$

$$K_2 = 1.06 \tanh\left[0.016 A^{0.50} \exp\left(-\frac{120}{A}\right)\right]$$

$$K_2 = 1.06 \tanh\left[0.016\left(500\right)^{0.50} \exp\left(-\frac{120}{500}\right)\right] = 0.291$$

$$\omega = \frac{K_1 \Delta S_g g D_s^2}{\nu} + K_2 \sqrt{\Delta S_g g D_s}$$

$$\omega = \frac{0.014 \times 1.63 \times 32.2\left(0.001\right)^2}{\left(1.05 \times 10^{-5}\right)} + \left(0.291\right)\sqrt{1.63 \times 32.2 \times 0.001} = 0.135 \text{ ft/s}$$

c. Compute U_*/ω.

$$U_* = \sqrt{32.2 \times 9.8 \times 0.0008} = 0.50 \text{ ft/s}$$

$$\frac{U_*}{\omega} = \frac{0.50}{0.135} = 3.72 \text{ ft/s}$$

d. Using U_*/ω, look up a → a = 0.69 (table TS14B–10).

e. Compute y_2 with equation TS14B–26, assuming $y_1 = y_0 = 9.8$ ft, and since $Q_1 = Q_2$.

$$\frac{y_2}{y_1} = \left(\frac{Q_2}{Q_1}\right)^{\frac{6}{7}}\left(\frac{W_{b1}}{W_{b2}}\right)^{a}$$

$$\frac{y_2}{9.8} = \left(\frac{60}{40}\right)^{0.69} = 1.32$$

$$y_2 = 1.32 \times 9.8 = 13.0 \text{ ft}$$
$$z_c = y_2 - y_o = 13.0 - 9.8 = 3.2 \text{ ft}$$

Step 4 Compute bedform scour.
For dunes to form (eq. TS14B–37):

$$D_* = D_{50}\left(\frac{\Delta S_g g}{\nu^2}\right)^{\frac{1}{3}} > 10$$

$$D_* = 0.001\left(\frac{1.63 \times 32.2}{\left(1.05 \times 10^{-5}\right)^2}\right)^{\frac{1}{3}} = 7.8 < 10$$

Dunes should not form.

Step 5 Compute local scour at spur dikes using equations TS14B–65 and TS14B–67.

$$K_1 = 0.75\left(1 + \frac{2\tan\phi}{\tan\phi}\right)^{-\frac{1}{2}}$$

$$K_1 = 0.75\left(1 + \frac{2\tan(45)}{\tan(27)}\right)^{-\frac{1}{2}} = 0.34$$

$$\frac{z_s}{y} = K_1\left(\frac{L_c}{y}\right)^{a}$$

since 1< L/y < 25, a = 0.5

$$\frac{z_s}{y} = 0.34\left(\frac{65.6}{9.8}\right)^{0.5} = 0.88$$

$$\begin{aligned} z_s &= 0.88y \\ &= 0.88 \times 9.8 \\ &= 8.6 \text{ ft} \end{aligned}$$

Step 6 Compute total scour (eq. TS14B–1).

$$z_t = FS\left[z_{ad} + z_c + z_b + z_{bf} + z_s\right]$$

$$z_t = 1.3\left[0 + 3.2 + 0 + 0 + 8.6\right] = 15.4 \text{ ft}$$

Compare with Blodgett (1986) using equation TS14B–21.

$$z_t \max = KD_{50}^{-0.115}$$

$$z_t \max = 6.5\left(0.001\right)^{-0.115} = 14.4 \text{ ft}$$

The predicted z_t value is close to this value. Values of z_t predicted using the Lacey and Blench formulas are somewhat smaller, 5.7 feet and 10.3 feet, respectively.

Gravel-bed reach

Scour analysis is needed to support design of instream habitat structures for a gravel-bed river with a single-thread, nearly straight channel. Low weirs will be placed in a shallow reach to develop pool habitats.

The reach appears to be actively degrading, with a base level control (confluence with larger river) 6,562 feet (2,000 m) downstream. Sediment supply from upstream has been greatly reduced due to advanced urban development.

Given:

relative submerged density, delta S_g	1.63	1.63
angle of repose of sediment, ϕ	45 deg	0.79 rad
constant in Manning's equation	1	1.486
C in Strickler equation	0.034	
Shields constant θ_c	0.056	0.055653
grain roughness k_s	87.5 mm	0.287 ft
D_{95}	175 mm	0.574 ft
D_{90}	65 mm	0.213 ft
D_{84}	25 mm	0.082 ft
D_{mean}	15 mm	0.049 ft
D_{50}	13 mm	0.043 ft
bed sediment internal friction angle	45 deg	0.785 rad
distance to downstream base level control	2,000 m	6,562 ft
Manning's *n*	0.030 s/m$^{(1/3)}$	0.030 s/ft$^{(1/3)}$
design discharge, Q_d	40.5 m^3/s	1,430 ft^3/s
flow width	18 m	60 ft
channel width, W	19 m	62 ft
mean flow depth, y	1.2 m	3.9 ft
hydraulic radius, R	1.2 m	3.9 ft
mean streamwise velocity, u	1.8 m/s	6.1 ft/s
unit discharge, q	2.2 m^3/s/m	24 ft^3/s/ft
existing bed slope, S	0.0024 m/m	0.0024 ft/ft
bend radius of curvature, Rc	1,000 m	3,281 ft
distance between weirs, L	250 m	820.3 ft
weir height above up streambed, y_w	0.3 m	1.0 ft
weir width, W_w	15 m	49.2 ft
difference in upstream and downstream water surface	0.5 m	1.6 ft
water depth above uneroded bed, y_d	0.7 m	2.3 ft
angle of the overfall jet with the vertical	0 Deg	0.0 rad

Find:
Total predicted scour depth

Step 1 Compute bed elevation change due to reach-scale degradation based on equilibrium slope.

a. Compute the smallest armor particle size, D_x using equation TS14B–4.

$$D_x = K\left(\frac{yS_e}{\Delta S_g}\right)^a \left(\frac{U_*}{\nu}\right)^b$$

$U_* = (gyS_e)^{0.5}$. Assume $S_e = S_o$

$$U_* = \sqrt{32.2 \times 3.9 \times 0.0024} = 0.55 \text{ ft/s}$$

$$\text{Particle Re} = \frac{U_* D_{50}}{\nu} = \frac{0.55 \times 0.043}{1.05 \times 10^{-5}} = 2{,}252$$

Therefore, K = 17, a = 1.0, b = 0

$$D_x = 17\left(\frac{3.9 \times 0.0024}{1.63}\right) = 0.0976 \text{ ft} = 30 \text{ mm}$$

Particles of this size and larger are present in the bed, so an armor layer can form.

b. Compute T, the active bed layer thickness using equation TS14B–3.

$$T = \frac{D_x}{(1-e)P_x}$$

where the bed porosity given by equation TS14B–5 is:

$$e = 0.245 + \frac{0.0864}{(0.1D_{50})^{0.21}} = 0.245 + \frac{0.0864}{(0.1 \times 13)^{0.21}} = 0.327$$

since D_{84} = 30 mm, P_a = 0.16. Therefore,

$$T = \frac{0.0976}{(1-0.327)(0.16)} = 0.91 \text{ ft}$$

c. Compute maximum scour depth limited by armoring, z_x.

$$z_x = T - D_x = 0.91 - 0.098 = 0.81 \text{ ft}$$

Step 2 Compute the depth of scour needed to produce an equilibrium slope. First, find the equilibrium slope.

a. Manning and Shields relation (eq. TS14B–14)

$$S_{eq} = \left[\theta_c D_c \Delta S_g\right]^{\frac{10}{7}} \left(\frac{K}{qn}\right)^{\frac{6}{7}} \quad \text{Let } D_c = D_{50}$$

$$S_{eq} = \left[0.056 \times 0.043 \times 1.63\right]^{\frac{10}{7}} \left(\frac{1.486}{24 \times 0.03}\right)^{\frac{6}{7}} = 0.00068$$

b. Meyer-Peter and Müller (eq. TS14B–15)

$$S_{eq} = K\frac{(D_{50})^{\frac{10}{7}} n^{\frac{9}{7}}}{(D_{90})^{\frac{5}{14}} q^{\frac{6}{7}}}$$

$$S_{eq} = 60.1\frac{(0.043)^{\frac{10}{7}} (0.03)^{\frac{9}{7}}}{(0.065)^{\frac{5}{14}} (24)^{\frac{6}{7}}} = 0.0013$$

c. Schoklitsch (eq. TS14B–16)

$$S_{eq} = K\left(\frac{D_m}{q}\right)^{\frac{3}{4}}$$

$$S_{eq} = 0.00174\left(\frac{15}{24}\right)^{\frac{3}{4}} = 0.0012$$

d. Henderson (eq. TS14B–17)

$$S_{eq} = K\, Q_d^{-0.46} D_{50}^{1.15}$$

$$S_{eq} = 0.44(1{,}430)^{-0.46}(0.043)^{1.15} = 0.00042$$

e. Compute bed degradation using equation TS14B–6. Use the average of the first three S_{eq} values computed above = 0.0011.

$$z_{ad} = L\left(S_{ex} - S_{eq}\right)$$

$$z_{ad} = (6{,}562)(0.0024 - 0.0011) = 8.5 \text{ ft}$$

Since the armor layer is formed after 0.81 ft of degradation, armoring controls.

Step 3 Compute scour downstream from weirs.

a. Using Veronese (1937) formula (eq. TS14B–53):

$$y + z_s = Kh_d^{0.225} q^{0.54}$$

$$y + z_s = 1.32(1.6)^{0.225}(23)^{0.54} = 8.2 \text{ ft}$$

$$\begin{aligned} z_s &= 8.2 - y \\ &= 8.2 - 3.9 \\ &= 4.3 \text{ ft} \end{aligned}$$

b. Using formula of Mason and Arumugam (1985) (eq. TS14B–55):

$$z_s = K \frac{q^a h_d^b y_t^{0.15}}{g^{0.30} D_m^{0.10}}$$

$$z_s = 3.4 \frac{(2.2)^{0.6}(0.5)^{0.15}(0.7)^{0.15}}{(9.8)^{0.30}(0.25)^{0.10}} = 2.7 \text{ m}$$

c. Using formula of D'Agostino and Ferro (2004) (eqs. TS14B–59 and TS14B–60):

$$A_{50} = \frac{Q_d}{W_w y_w \sqrt{g D_{50} \Delta S_g}}$$

$$A_{50} = \frac{1,430}{(49.2)(1.0)\sqrt{32.2 \times 0.043 \times 1.63}} = 19.3$$

$$\frac{z_s}{y_w} = 0.540 \left(\frac{W_w}{y_w}\right)^{0.593} \left(\frac{y_t}{h_d}\right)^{-0.126} A_{50}^{0.544} \left(\frac{D_{90}}{D_{50}}\right)^{-0.856} \left(\frac{W_w}{W}\right)^{-0.751}$$

$$\frac{z_s}{y_w} = 0.540 \left(\frac{15}{0.3}\right)^{0.593} \left(\frac{0.7}{0.5}\right)^{-0.126} (19.3)^{0.544} \left(\frac{0.213}{0.043}\right)^{-0.856} \left(\frac{49.2}{62}\right)^{-0.751} = 7.96$$

$$z_s = 7.96\, y_w = 7.96 \times 0.3 = 2.4 \text{ m}$$

Estimate depth of scour pools below weirs as the maximum of the above three results.

$$z_s = 2.7 \text{ m} = 8.9 \text{ ft}$$

Step 4 Compute total scour for design using equation TS14B–1.

$$z_t = FS\left[z_{ad} + z_c + z_b + z_{bf} + z_s\right]$$

Use a factor of safety of 1.3.

$$z_t = 1.3\left[0.8 + 0.0 + 0.0 + 0.0 + 8.9\right] = 12.6 \text{ ft}$$

Compare with Blodgett (1986) (eq. TS14B–22).

$$z_t(\max) = K D_{50}^{-0.11}$$

$$z_t \max = 6.5(0.043)^{-0.115} = 9.3 \text{ ft}$$

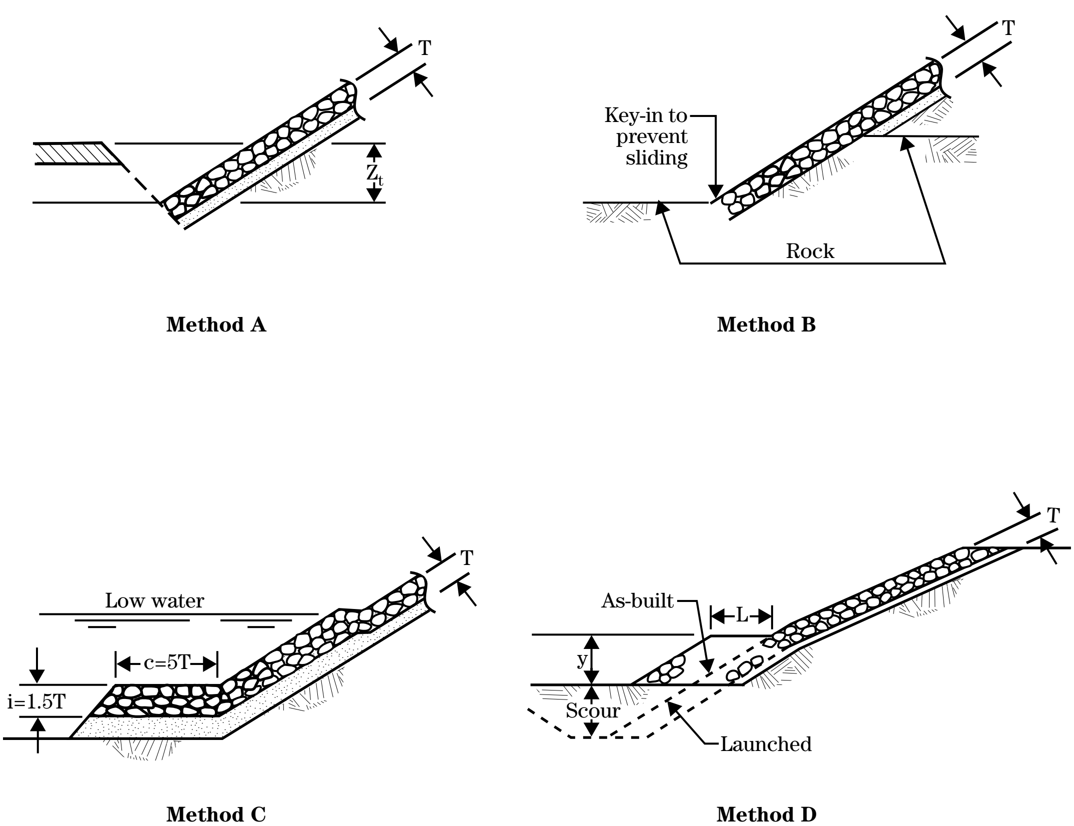
The predicted value of 12.6 feet is well within the scatter about Blodgett's relationship shown in figure TS14B–1. Predicted values of z_t, using the Lacey (1931) and Blench (1970) formulas, are much smaller: 1.4 feet and 3.3 feet, respectively. However, these values are close to the value of z_t mean (fig. TS14B–1) of 2 feet from Blodgett's formula.

Design features and measures to address scour

Structures may be designed to withstand scour in either of two ways (fig. TS14B–22 (USACE 1991b)). They may be extended down into the bed a sufficient distance (dig it in or key it in) to be beneath the projected total scour depth (method A, fig. TS14B–22) or until contact is made with a nonerodible material (method B, fig. TS14B–22). The key-it-in approach (method A) is most often used with armor revetment (Biedenharn, Elliott, and Watson 1997), but is difficult and costly to do in a flowing stream. Conventional excavation is usually not feasible in water depths >10 feet (3 m). Greater water depths usually require dredging or dewatering for construction.

Alternatively, additional loose material (stone) may be incorporated into the structure so that it will self-launch into the scour zone as scour occurs and inhibit deeper scour that would endanger the bank and the rest of the structure (methods C and D, fig. TS14B–22). Method C is recommended for situations where little scour is expected such as in straight, nonbraided reaches that are not immediately downstream from bends. Method D is more robust and is useful when water depths prohibit excavation for a method A type design. No excavation is needed for method D, as toe scour is a substitute for mechanical excavation when this method is used. The self-launching approach (method D) offers the advantage that it provides a built-in indicator of scour as it occurs. However, a self-launching toe requires more material.

Figure TS14B–22 Four methods for designing stone structures to resist failure due to bed scour



These approaches may be used with any type of stone structure. The volume of additional stone required at the toe of a revetment for method D is computed as follows (USACE 1991b):

Assume launch slope = 1V:2H
Revetment toe thickness after launching = 1.5 T_r, where T_r is the thickness of the bank revetment, feet (m), and therefore,

$$V_{stone} = 3.35 T_r z_t \qquad \text{(eq. TS14B–68)}$$

where:
V_{stone} = additional volume of stone added to toe for launching per unit streamwise length of revetment, ft^3/ft (m^3/m)
z_t = total projected scour depth, as before, ft (m)

Variations on the self-launching toe approach include windrow revetments (linear piles of riprap placed along the top bank) and trenchfill revetments (trenches excavated at the low water level and filled with stone).

With several possible choices of structures to counteract scour, designers should select a scour control strategy based on careful consideration of the possible modes of failure, their likelihood, the consequences of each failure mode, and the difficulty of detecting failures in time to correct them. A quantitative strategy for selecting scour control measures based on this approach is described by Johnson and Niezgoda (2004).

List of symbols

a_1 = morphological jump = $(S_o - S_{eq})L_s$ ft (m)
Δ = dune height, ft (m)
D_* = dimensionless sediment size
D_{50} = median bed-material size, mm or ft (m)
D_{90} = size larger than 90 percent of the bed material by weight, mm or ft (m)
D_{90} = size of bed material larger than 90% of the bed by weight, ft (m) (note units)
D_x = the smallest armor size or the size of the smallest nontransportable particle present in the bed material, ft (m)
D_c = diameter of the sediment particle, mm or ft (m)
D_m = mean bed-material particle size, mm or ft (m)
D_s = a characteristic sediment diameter, ft (m)
ΔS_g = change in relative submerged density of bed-material sediments ≅1.65
ΔV = change in volume of bed-material sediment stored or eroded, ft^3 (m^3)
e = porosity of the bed material
ϕ = angle of repose of bed sediment
FS = factor of safety
g = acceleration of gravity, 32.2 ft/s^2 (9.81 m/s^2)
γ_s = specific weight of sediment particles lb/ft^3 (N/m^3)
γ_w = specific weight of water, lb/ft^3 (N/m^3)
h_d = height of step crest above controlling bed elevation at downstream end of pool, ft (m)
h_d = vertical distance between water surface elevation upstream and downstream from the weir, ft (m)
H_s = specific energy of critical flow over the sill, ft (m)
φ = side slope of spur structure
L_r = reach length, ft (m)
L_c = length of spur crest measured perpendicular to flow direction, ft (m)
L_p = recommended length of protection, ft (m)
L_s = horizontal distance between sills, ft (m)
l_p = length of scour pool, ft (m)
$L_s max$ = horizontal distance between weir and deepest point of downstream scour hole, ft (m)
n = Manning's roughness coefficient
ν = kinematic viscosity of water, ft^2/s (m^2/s)
P_x = the fraction of bed material comprised of particles size D_a or larger
q = channel-forming or design discharge per unit width, $ft^3/s/ft$ ($m^3/s/m$)
θ = dimensionless Shields stress
θ_c = critical dimensionless shear stress or Shields constant
Q_d = design discharge, ft^3/s (m^3/s)
Q_s = sediment supply, ft^3/s (m^3/s)
q_s = sediment transport capacity in dimensions of volume per unit width per unit time, ft^2/s (m^2/s)
ρ = Density of water, 1.94 $slugs/ft^3$ (1,000 kg/m^3)
R = hydraulic radius, ft (m)
Rc = bend radius of curvature, ft (m)
S = average channel bed slope
S_e = energy slope
S_e = Slope of energy grade line of main channel, ft/ft (m/m)
S_e = energy slope, then the following analyses may be used to find S_{eq}
S_{eq} = equilibrium channel slope at which sediment particles of size D_c and larger will no longer move
S_{ex} = existing channel slope
S_g = specific gravity of the sediment
T = thickness of the active layer of the bed, ft (m)
τ_c = critical boundary shear stress, lb/ft^2 (N/m^2)
τ_c^* = critical shear stress for motion from the Shields diagram, lb/ft^2 (N/m^2)
τ_o = average bed shear stress, lb/ft^2 (N/m^2)
T_r = thickness of the bank revetment, ft (m)
τ_s^* = bed shear stress due to skin or grain friction, lb/ft^2 (N/m^2)
T_{ts} = dimensionless transport-stage parameter
u = mean streamwise velocity, ft/s (m/s)
U_* = shear velocity = $(gyS_e)^{0.5}$

V_c = critical velocity, ft/s (m/s)

V_{stone} = additional volume of stone added to toe for launching per unit streamwise length of revetment, ft^3/ft (m^3/m)

V_s = volume of scour hole, ft^3 (m^3)

Vs_{in} = volume of bed-material sediment supplied to the reach, ft^3 (m^3)

Vs_{out} = volume of bed-material sediment transported out of the reach, ft^3 (m^3)

W = average active channel width, ft (m)

ω = fall velocity of bed material based on the D_{50}, ft/s (m/s)

W = flow width at design discharge, ft (m)

W_{b1} = bottom width of the upstream channel, ft (m)

W_{b2} = bottom width of the contracted section, ft (m)

W_c = average channel width, ft (m)

W_f = flow width at design discharge, ft (m)

W_i = channel width at bend inflection point, ft (m)

W_w = width of weir crest, ft (m)

y = flow depth, ft (m)

y_c = mean water depth in the crossing upstream from the bend, ft (m)

y_{max} = maximum flow depth in the bend, ft (m)

y_t = tailwater depth above original ground surface, m

y_w = vertical distance between weir crest and upstream channel bed, ft(m)

z_{ad} = bed elevation changes due to reach-scale deposition (aggradation) or general scour (degradation), ft (m)

z_b = scour on the outside of bend, ft (m)

z_{bf} = bedform trough depth, ft (m)

z_c = clear-water contraction scour, ft (m)

z_s = depth of scour downstream from structure, ft (m), measured from the crest of the structure to the lowest point within the scour pool

z_s = local scour depth associated with a structure, ft (m)

z_t = total scour depth, ft (m)