Part 630
National Engineering Handbook

Chapter 19
Transmission Losses

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# Chapter 19

## Transmission Losses

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Chapter 19

Transmission Losses

630.1900 Introduction

Natural stream channels in arid and semiarid regions are generally ephemeral. Flow is occasional and follows storms, which are infrequent. When flows occur in normally dry stream channels, the volume of flow is reduced by infiltration into the bed, the banks, and possibly the flood plain. These losses to infiltration, called transmission losses, reduce not only the volume of the hydrograph, but also the peak discharge.

This chapter describes a procedure for estimating the volume of runoff and peak discharge for ephemeral streams; it can be used with or without observed inflow-outflow data. If available, observed inflow-outflow data can be used to derive regression equations for the particular channel reach. Procedures based on the derived regression equations enable a user to determine prediction equations for similar channels of arbitrary length and width.

Chapter 19 also gives the procedures for estimating parameters of the prediction equations in the absence of observed inflow-outflow data. These procedures are based on characteristics of the bed and bank material. Approximations for lateral inflow and out-of-bank flow are also presented.

630.1901 Assumptions and limitations

(a) Assumptions

The methods described in this chapter are based on the following assumptions:

- Water is lost in the channel; no streams gain water.
- Infiltration characteristics and other channel properties are uniform with distance and width.
- Sediment concentration, temperature, and antecedent flow affect transmission losses, but the equations represent the average conditions.
- The channel reach is short enough that an average width and an average duration represent the width and duration of flow for the entire channel reach.
- Once a threshold volume has been satisfied, outflow volumes are linearly proportional to inflow volumes.
- Once an average loss rate is subtracted and the inflow volume exceeds the threshold volume, peak rates of outflow are linearly proportional to peak rates of inflow. Moreover, the rate of change in outflow peak discharge with changing inflow peak discharge is the same as the rate of change in outflow volume with changing inflow volume.
- Lateral inflow can be either lumped at points of tributary inflow or uniform with distance along the channel.
- For volume and peak discharge calculations, lateral inflow is assumed to occur during the same time as the upstream inflow.

(b) Limitations

The main limitations of the procedures are:

- Hydrographs are not specifically routed along the stream channels; predictions are made for volume and peak discharge.
- Peak flow equations do not consider storage attenuation effects or steepening of the hydrograph rise.
- Analyses on which the procedures are based represent average conditions or overall trends.
- Influences of antecedent flow and sediment concentration in the streamflow have not been quantified.
- Estimates of effective hydraulic conductivity in the streambed are empirically based and represent average rates.
- Peak discharge of outflow is decreased by the average loss rate for the duration of flow.
- Procedures for out-of-bank flow are based on the assumption of a weighted average for the effective hydraulic conductivity.

### 630.1902 Symbols and notation

#### Upstream inflow

\[ P = \text{inflow volume (acre-feet)} \]
\[ p = \text{peak rate of inflow (cubic feet per second)} \]

#### Lateral inflow

\[ Q_L = \text{lateral inflow volume (acre-feet per mile)} \]
\[ q_L = \text{peak rate of lateral inflow (cubic feet per second per foot)} \]

#### Outflow

\[ Q(x,w) = \text{outflow volume (acre-feet)} \]
\[ q(x,w) = \text{peak rate of outflow (cubic feet per second)} \]

#### Channel reach

\[ D = \text{duration of flow (hours)} \]
\[ K = \text{effective hydraulic conductivity (inches per hour)} \]
\[ V = \text{total available storage volume of alluvium in the channel reach (acre-feet)} \]
\[ w = \text{average width of flow (feet)} \]
\[ x = \text{length of reach (miles)} \]

#### Prediction equations (parameters)

\[ a = \text{regression intercept for unit channel (acre-feet)} \]
\[ a(D) = \text{regression intercept for unit channel with a flow of duration } D \text{ (acre-feet)} \]
\[ a(x,w) = \text{regression intercept for a channel reach of length } x \text{ and width } w \text{ (acre-feet)} \]
\[ b = \text{regression slope for unit channel} \]
\[ b(x,w) = \text{regression slope for a channel reach of length } x \text{ and width } w \]
\[ k = \text{decay factor (foot-miles)}^{1} \]
\[ k(D,P) = \text{decay factor for unit channel with a flow duration } D \text{ and volume } P \text{ (foot-miles)}^{1} \]
\[ P_0 = \text{threshold volume or amount of inchannel loss above which channel outflow occurs for a unit channel (acre-feet). Channel outflow is 0.0 until the threshold volume is achieved.} \]
\[ P_0(x,w) = \text{threshold volume or amount of inchannel loss above which channel outflow occurs for a channel reach of length } x \text{ and width } w \text{ (acre-feet). Channel outflow is 0.0 until the threshold volume is achieved.} \]
630.1903 Applications

The simplified procedures are summarized here; additional details and derivations are given in the appendices. Methods have been developed for two situations—when observed inflow-outflow data are available and when no observed data are available.

(a) Summary of procedure

The prediction equation for outflow volume, without lateral inflow, is

\[
Q(x, w) = \begin{cases} 
 0 & P \leq P_o(x, w) \\
 0 & P > P_o(x, w)
\end{cases}
\]

(eq. 19–1)

where the threshold volume is

\[
P_o(x, w) = \frac{-a(x, w)}{b(x, w)}
\]

(eq. 19–2)

The corresponding equation for peak discharge is shown in equation 19–3 below. In this equation, 12.1 converts from acre-feet per hour to cubic feet per second.

If lateral inflow is uniform, the volume equation is that shown in equation 19–4 at the bottom of the page.

The corresponding equation for peak discharge is shown in equation 19–5 at the bottom of this page. In this equation the factor 5,280 converts cubic feet per second per foot to cubic feet per second per mile. Derivations and background information are in appendix 19A.

For a channel reach with only tributary lateral inflow, equations 19–1 and 19–3 would be applied on the tributary channel and the main channel to the point of tributary inflow. Then the sum of the outflows from these two channel reaches would be the inflow to the lower reach of the main channel.

The procedures described by equations 19–1, 19–3, 19–4, and 19–5 require that the upstream inflow volumes and lateral inflow volumes along the channel reach be estimated using the procedures described in National Engineering Handbook, part 630 (NEH 630), chapter 10. Peak flow rates and flow durations are estimated by use of procedures described in NEH 630, chapter 16.

\[
q(x, w) = \begin{cases} 
 0 & Q(x, w) = 0 \\
 \frac{12.1}{D} \left\{ a(x, w) - \left[ 1 - b(x, w) \right] P \right\} + b(x, w)p & Q(x, w) > 0
\end{cases}
\]

(eq. 19–3)

\[
Q(x, w) = \begin{cases} 
 0 & P \leq P_o(x, w) \\
 a(x, w) + b(x, w)p + \frac{Q_o(x, w)}{kw} \left[ 1 - b(x, w) \right] & P > P_o(x, w)
\end{cases}
\]

(eq. 19–4)

\[
q(x, w) = \begin{cases} 
 0 & Q(x, w) = 0 \\
 \frac{12.1}{D} \left\{ a(x, w) - \left[ 1 - b(x, w) \right] P \right\} + b(x, w)p + \frac{q_L(5,280)}{kw} \left[ 1 - b(x, w) \right] & Q(x, w) > 0
\end{cases}
\]

(eq. 19–5)
(b) Estimating parameters from observed inflow-outflow data

If a channel reach has an assumed length \( x \) and average width \( w \), then \( n \) observations on \( P_i \) and \( Q_i \) (without lateral inflow) can be used to estimate the parameters in equation 19–1. Parameters of the linear regression equation can be estimated as

\[
b(x, w) = \frac{\sum (Q_i - \bar{Q})(P_i - \bar{P})}{\sum (P_i - \bar{P})^2} \quad \text{(eq. 19–6)}
\]

and

\[
a(x, w) = \bar{Q} - b(x, w)\bar{P} \quad \text{(eq. 19–7)}
\]

where:

\[
\bar{Q} = \text{mean outflow volume}
\]
\[
\bar{P} = \text{mean inflow volume}
\]
\[
n = \text{number of observations on } P_i \text{ and } Q_i
\]

Alternative formulas recommended for computation are

\[
\sum_{i=1}^{n} (Q_i - \bar{Q})(P_i - \bar{P}) = \frac{n \sum P_i Q_i - \left( \sum P_i \right) \left( \sum Q_i \right)}{n} \quad \text{(eq. 19–8)}
\]

and

\[
\sum_{i=1}^{n} (P_i - \bar{P})^2 = \frac{n \sum P_i^2 - \left( \sum P_i \right)^2}{n} \quad \text{(eq. 19–9)}
\]

Linear regression procedures are available on most computer systems and on many handheld calculators.

Constraints on the parameters are

\[
a(x, w) < 0 \quad \text{and} \quad 0 \leq b(x, w) \leq 1
\]

When one or both of the constraints are not met, the following procedure is suggested:

1. Plot the observed data on rectangular coordinate paper: \( P_i \) on the X-axis and \( Q_i \) on the Y-axis.
2. Plot the derived regression equation on the graph with the data.
3. Check the data for errors (such as events with lateral inflow or computational errors). Pay particular attention to any data points far from the regression line, especially those points that may be strongly influencing the slope or intercept.
4. Correct data points that are in error; remove points that are not representative.
5. Recompute the regression slope and intercept using equations 19–6 to 19–9 and the corrected data.

A great deal of care and engineering judgment must be exercised in finding and eliminating errors from the set of observed inflow-outflow observations.

(1) Unit channels

A unit channel is defined as a channel of length \( x = 1 \) mile and width \( w = 1 \) foot. Parameters for the unit channel are required to compute parameters for channel reaches with arbitrary length and width. The unit channel parameters are computed by the following equations:

\[
k = \frac{-\ln b(x, w)}{x w} \quad \text{(eq. 19–10)}
\]

\[
b = e^{-k} \quad \text{(eq. 19–11)}
\]

\[
a = \frac{a(x, w) (1 - b)}{1 - b(x, w)} \quad \text{(eq. 19–12)}
\]

where \( a(x, w) \) and \( b(x, w) \) are the regression parameters derived from the observed data. In this case the length \( x \) and width \( w \) are fixed known values. Particular care must be taken to maintain the maximum number of significant digits in determining \( k \), \( b \), and \( a \). Otherwise, significant round-off errors can result.

(2) Reaches of arbitrary length and width

Given parameters for a unit channel, parameters for a channel reach of arbitrary length \( x \) and arbitrary width \( w \) are computed by the following equations:

\[
b(x, w) = e^{-kxw} \quad \text{(eq. 19–13)}
\]

\[
a(x, w) = \frac{a(x, w)}{1 - b(x, w)} \quad \text{(eq. 19–14)}
\]

\[
P_o(x, w) = \frac{-a(x, w)}{b(x, w)} \quad \text{(eq. 19–2)}
\]
(c) Estimating parameters in the absence of observed inflow-outflow data

When inflow-outflow data are not available, an estimate of effective hydraulic conductivity is needed to predict transmission losses. Effective hydraulic conductivity, \( K \), is the infiltration rate averaged over the total area wetted by the flow and over the total duration of flow. Because effective hydraulic conductivity represents a space-time average infiltration rate, it incorporates the influence of temperature, sediment concentration, flow irregularities, errors in the data, and variations in wetted area. For this reason it is not the same as the saturated hydraulic conductivity for clear water under steady-state conditions. Analysis of observed data results in equations for the unit channel intercept

\[
a(D) = -0.00465KD \quad \text{(eq. 19–15)}
\]

and for the decay factor on ungaged reaches

\[
k(D, P) = -1.09 \ln \left[ 1.0 - 0.00545 \frac{KD}{P} \right] \quad \text{(eq. 19–16)}
\]

Given values of \( a \) and \( k \) from equations 19–15 and 19–16, equations 19–13, 19–14, and 19–2 are used to compute parameters for a particular \( x \) and \( w \). Derived relationships between bed material characteristics, effective hydraulic conductivity, and the unit channel parameters \( a \) and \( k \) are shown in table 19–1. These data can be used to estimate parameters for ungaged channel reaches.

<table>
<thead>
<tr>
<th>Bed material group</th>
<th>Bed material characteristics</th>
<th>Effective hydraulic conductivity ( K ) (in/h)</th>
<th>Intercept ( a ) (acre-ft)</th>
<th>Decay factor ( k ) (ft-mi)(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Very high loss rate</td>
<td>Very clean gravel and large sand</td>
<td>&gt;5</td>
<td>&lt;- 0.023</td>
<td>&gt;0.030</td>
</tr>
<tr>
<td>2: High loss rate</td>
<td>Clean sand and gravel, field conditions</td>
<td>2.0 to 5.0</td>
<td>-0.0093 to -0.023</td>
<td>0.0120 to 0.030</td>
</tr>
<tr>
<td>3: Moderately high loss rate</td>
<td>Sand and gravel mixture with low silt-clay content</td>
<td>1.0 to 3.0</td>
<td>-0.0047 to -0.014</td>
<td>0.0060 to 0.018</td>
</tr>
<tr>
<td>4: Moderate loss rate</td>
<td>Sand and gravel mixture with high silt-clay content</td>
<td>0.25 to 1.0</td>
<td>-0.0012 to -0.0047</td>
<td>0.0015 to 0.0060</td>
</tr>
<tr>
<td>5: Insignificant to low loss rate</td>
<td>Consolidated bed material; high silt-clay content</td>
<td>0.001 to 0.10</td>
<td>-5 x 10(^{-6}) to -5 x 10(^{-4})</td>
<td>6 x 10(^{-6}) to 6 x 10(^{-4})</td>
</tr>
</tbody>
</table>

1/ See appendix 19C for sources of basic data.
2/ Values are for unit duration, \( D = 1 \) hour. For other durations, \( a(D) = -0.00465KD \).
3/ Values are for unit duration and volume, \( D/P = 1 \). For other durations and volumes, use:

\[
k(D, P) = -1.09 \ln \left[ 1.0 - 0.00545 \frac{KD}{P} \right]
\]
(d) Summary of parameter estimation techniques

Suggested procedures for use when observed data are available are summarized in table 19–2. Procedures for use on ungaged channel reaches are summarized in table 19–3. Again, whatever procedure is used, the parameter estimates must satisfy the constraints $a(x,w) < 0$ and $0 \leq b(x,w) \leq 1$.

### Table 19–2

<table>
<thead>
<tr>
<th>Step</th>
<th>Source</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Perform regression analysis</td>
<td>Eqs. 19–6, 19–7, 19–2</td>
<td>Prediction equations for the particular reach</td>
</tr>
<tr>
<td>2. Derive unit channel parameters</td>
<td>Eqs. 19–10 to 19–12</td>
<td>Unit channel parameters</td>
</tr>
<tr>
<td>3. Calculate parameters</td>
<td>Eqs. 19–13, 19–14, 19–2</td>
<td>Parameters of the prediction equations for arbitrary x and w</td>
</tr>
</tbody>
</table>

### Table 19–3

<table>
<thead>
<tr>
<th>Step</th>
<th>Source</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Estimate inflow</td>
<td>Hydrologic analysis</td>
<td>Mean duration of flow, $D$, and volume of inflow $P$</td>
</tr>
<tr>
<td>2. Identify bed material</td>
<td>Table 19–1</td>
<td>Effective hydraulic conductivity, $K$</td>
</tr>
<tr>
<td>3. Derive unit channel parameters</td>
<td>Eqs. 19–15, 19–16, 19–11</td>
<td>Unit channel parameters</td>
</tr>
<tr>
<td>4. Calculate parameters</td>
<td>Eqs. 19–13, 19–14, 19–2</td>
<td>Parameters of the prediction equations for arbitrary x and w</td>
</tr>
</tbody>
</table>

### 630.1904 Examples

The following examples illustrate application of the procedures for several cases under a variety of circumstances. As in any analysis, all possible combinations of circumstances are impossible to consider, but the examples presented here should provide an overview of useful applications of the procedures. Use of these procedures requires judgment and experience. At each step of the process, care should be taken to ensure that the results are reasonable and consistent with sound engineering practice.

Example 19–1 illustrates application of the procedures with and without observed data when flow is within the channel banks and there is no lateral inflow. Example 19–2 is for the same channel reach, but is based on assumption of uniform lateral inflow between the inflow and outflow stations. Approximations for out-of-bank flow are described in example 19–3.
Example 19–1

Given: A channel reach of length \( x = 5.0 \) miles, average width \( w = 70 \) feet.
Bed material consists of sand and gravel with a small percentage of silt and clay.
Assume a mean flow duration \( D = 4 \) hours and a mean inflow volume of \( P = 34 \) acre-feet.

Find: The prediction equations for the channel reach. Estimate the outflow volume and peak for an inflow \( P = 50 \) acre-feet and \( p = 1,000 \) cubic feet per second.

Case 1 Observed inflow-outflow data

\[ \begin{array}{cccccc}
P_i & 20.0 & 100.0 & 25.0 & 10.0 & 15.0 \\
Q_i & 6.0 & 75.0 & 9.0 & 0.1 & 2.5 \\
\end{array} \]

\( P = 34 \)
\( Q = 18.52 \)

Solution: Follow the procedure outlined in table 19–2, step 1, for \( x = 5.0 \) miles and \( w = 70 \) feet.

\[ b(x, w) = \frac{\sum (Q_i - \bar{Q})(P_i - \bar{P})}{\sum (P_i - \bar{P})^2} = 0.850 \]

\[ a(x, w) = \bar{Q} - b(x, w)\bar{P} = 18.52 - 0.850(34) = -10.38 \text{ acre-ft} \]

\[ P_o(x, w) = \frac{-a(x, w)}{b(x, w)} = \frac{10.38}{0.850} = 12.21 \text{ acre-ft} \]

Substituting these values in equation 19–1, the prediction equation for volume is

\[ Q(x, w) = \begin{cases} 0 & \text{if } P \leq 12.21 \\ -10.38 + 0.850P & \text{if } P > 12.21 \end{cases} \]

and the prediction equation (from equation 19–3) for peak discharge is

\[ q(x, w) = \begin{cases} 0 & \text{if } Q(x, w) = 0 \\ -31.4 - 0.454P + 0.850p & \text{if } Q(x, w) > 0 \end{cases} \]

For an inflow volume \( P = 50 \) acre-feet and an inflow peak rate \( p = 1,000 \) cubic feet per second, the predicted outflow volume is

\[ Q(x, w) = -10.38 + 0.850(50) = 32.1 \text{ acre-ft} \]
and the predicted peak rate of outflow is

\[ q(x, w) = -31.4 - 0.454(50) + 0.850(1,000) = 796 \text{ ft}^3/\text{s} \]

**Case 2  No observed inflow-outflow data**

**Solution:** Follow the procedures outlined in table 19–3. From table 19–1, estimate \( K = 1.0 \) inch per hour, with \( D = 4.0 \) hour, \( P = 34 \) acre-feet, so:

\[
\begin{align*}
    a &= -0.00465KD = -0.01860 \text{ acre-ft} \\
    k &= -1.09 \ln \left(1.0 - 0.00545 \frac{KD}{P}\right) \\
    &= 0.000699 \text{ (ft-mi)}^{-1}
\end{align*}
\]

and

\[ b = e^{-k} = e^{-0.000699} = 0.999301 \]

are the unit channel parameters. From equations 19–13, 19–14, and 19–2, the parameters for the given reach with \( x = 5.0 \) miles and \( w = 70 \) feet are

\[
\begin{align*}
    b(x, w) &= e^{-kxw} = e^{-(0.000699)(5.0)(70)} \\
    &= 0.783 \\
    a(x, w) &= \frac{a}{1-b} \left[1-b(x, w)\right] \\
    &= \frac{-0.01860}{(1-0.999301)} (1-0.783) \\
    &= -5.78 \text{ acre-ft} \\
    &\text{and} \\
    P_o(x, w) &= -\frac{a(x, w)}{b(x, w)} \\
    &= \frac{(-5.78)}{0.783} = 7.38 \text{ acre-ft}
\end{align*}
\]

The prediction equation for the volume is

\[
Q(x, w) = \begin{cases} 
0 & P \leq 7.38 \\
-5.78 + 0.783P & P > 7.38
\end{cases}
\]
Example 19–1  No lateral inflow or out-of-bank flow—Continued

and the prediction equation for peak discharge is

\[
q(x, w) = \begin{cases} 
0 & Q(x, w) = 0 \\
-17.5 - 0.656P + 0.783p & Q(x, w) > 0
\end{cases}
\]

For an inflow volume of \( P = 50 \) acre-feet and an inflow peak rate of \( p = 1,000 \) cubic feet per second, the predicted outflow volume is

\[
Q(x, w) = -5.78 + 0.783(50) = 33.4 \text{ acre-ft}
\]

and the predicted peak rate of outflow is

\[
q(x, w) = -17.5 - 0.656(50) + 0.783(1,000)
= 733 \text{ ft}^3/\text{s}
\]
Example 19–2  Uniform lateral inflow

**Given:** The channel reach parameters from example 19–1 and a lateral inflow of 21.3 acre-feet at a peak rate of 500 cubic feet per second. Assume the lateral inflow is uniformly distributed.

**Find:** The volume of outflow and peak rate of outflow if P = 50 acre-feet and p = 1,000 cubic feet per second.

**Solution:** Compute the lateral rates as follows:

\[
Q_L = \frac{21.3 \text{ acre-ft}}{5.0 \text{ mi}} = 4.26 \text{ acre-ft/mi}
\]

and

\[
q_L = \frac{500 \text{ ft}^3/\text{s}}{(5.0 \text{ mi})(5.280 \text{ ft/mi})} = 0.0189 \text{ ft}^3/\text{ft}
\]

Using \(a(x,w) = -5.78\), \(b(x,w) = 0.783\), \(k = 0.000699\), and \(w = 70\) from case 2 of example 19–1 in equation 19–4, the result is

\[
Q(x,w) = -5.78 + 0.783P + \frac{Q_L}{kw}(1 - 0.783) = 52.3 \text{ acre-ft}
\]

The corresponding calculations for peak discharge of the outflow hydrograph (eq. 19–5) are

\[
q(x,w) = -17.5 - 0.656P + 0.783p + \frac{q_L(5.280)}{kw}(1 - 0.783) = 1,175 \text{ ft}^3/\text{s}
\]
Example 19–3  Approximations for out-of-bank flow

Given: A channel reach of length x = 10 miles and an average width of inbank flow \( w_1 = 150 \) feet with inbank flow up to a discharge of 3,000 cubic feet per second. Once the flow exceeds 3,000 cubic feet per second, out-of-bank flow rapidly covers wide areas. The bed material consists of clean sand and gravel, and the out-of-bank material is sandy with significant amounts of silt-clay.

Find: Determine the outflow if the inflow is \( P = 700 \) acre-feet with a peak rate of \( p = 4,000 \) cubic feet per second. Assume the mean duration of flow is 12 hours and the total average width of out-of-bank flow is 400 feet. Also, estimate the distance downstream before the flow is back within the channel banks.

Solution: Using the procedures outlined in table 19–3, make the following calculations:

Inbank flow:

\[
\begin{align*}
  w_1 &= 150 \text{ ft} \\
  K_1 &= 3.0 \text{ in/h (average hydraulic conductivity from table 19–1)}
\end{align*}
\]

Out-of-bank flow:

\[
\begin{align*}
  w_2 &= 400 \text{ ft (includes width } w_1) \\
  K_2 &= 0.5 \text{ in/h for width } w_2 - w_1 \text{ (average hydraulic conductivity from table 19–1)}
\end{align*}
\]

The weighted average for effective hydraulic conductivity is

\[
K = \frac{w_1 K_1 + (w_2 - w_1) K_2}{w_2} \quad \text{(eq. 19–17)}
\]

\[
K = 1.44 \text{ in/h}
\]

Using this average value of \( K \), \( D = 12 \) hours, and \( P = 700 \) acre-feet in equations 19–15 and 19–16, the unit channel parameters are

\[
\begin{align*}
  a &= -0.00465 KD = -0.08035 \text{ acre-ft} \\
  k &= -1.09 \ln \left( 1.0 - 0.00545 \frac{KD}{P} \right) \\
  &= 0.000147 \text{ (ft-mi)}^{-1}
\end{align*}
\]

and

\[
b = e^{-k} = e^{-0.000147} = 0.99985
\]

Given the unit channel parameters and \( w_2 = 400 \) feet, the parameters for the channel reach are

\[
b(x, w_2) = e^{-k w_2} = e^{-(0.000147)(400)x} = e^{-0.0588x}
\]

and

\[
a(x, w_2) = \frac{a}{1 - b(x, w_2)} = \frac{-0.08035}{(1 - e^{-0.0588x})}
\]
Example 19–3  Approximations for out-of-bank flow—Continued

Now, estimate the distance downstream until flow is contained within the banks (from equation 19–3) as

\[ q(x, w) = \frac{12.1}{D} \left[ a(x, w) - \left[ 1 - b(x, w) \right] P \right] + b(x, w) P \]

Use an upper limit as

\[ q(x, w) = 3,000 \text{ ft}^3/\text{s} \leq b(x, w) P = e^{-0.0588x} (4,000) \]

which means

\[ e^{-0.0588x} \geq \frac{3,000}{4,000} = 0.75 \]

\[ x \leq -\frac{1.0}{0.0588} \ln 0.75 = 4.89 \text{ mi} \]

Then a trial-and-error solution of the volume and peak discharge equations for various values of \( x < 4.89 \) miles produces a best estimate of \( x = 3.6 \) miles. Based on this value, the parameters are

\[ b(3.6, w_2) = 0.809 \]

and

\[ a(3.6, w_2) = -102.3 \text{ acre-ft} \]

Therefore, the predictions for \( x = 3.6 \) miles are

\[ Q(3.6, w_2) = -102.3 + 0.809(700) = 464.0 \text{ acre-ft} \]

for the volume, and

\[ q(3.6, w_2) = -238.0 + 0.809(4,000) = 2,998 \text{ ft}^3/\text{s} \]

for the peak rate.

For distances beyond this point, the flow will be contained in the channel banks. Using \( K = 3.0 \), \( D = 12 \), and \( P = 464.0 \text{ acre-feet} \) (the inflow from the upstream reach), the parameters for inbank flow with a distance of \( x = 10.0 - 3.6 = 6.4 \) miles are

\[ a = -0.00465 KD = -0.1674 \text{ acre-ft} \]

\[ k = -1.09 \ln \left( 1.0 - 0.00545 \frac{KD}{P} \right) \]

\[ = 0.000461 \text{ (ft-mi)}^{-1} \]

and

\[ b = e^{-k} = e^{-0.000461} = 0.99954 \]
With these unit channel parameters, the parameters for inbank flow are

\[ b(6.4, w_1) = e^{-kxw_1} = e^{-(0.000461)(6.4)(150)} = 0.642 \]

and

\[ a(6.4, w_1) = \frac{a}{1-b} \left[ 1 - b(x, w_1) \right] \]

\[ = \frac{-0.1674}{1-0.99954} [1 - 0.642] \]

\[ = -130.3 \text{ acre-ft} \]

The predicted outflow is

\[ Q(6.4, w_1) = -130.3 + 0.642(464.0) \]

\[ = 167.6 \text{ acre-ft} \]

for the volume and

\[ q(6.4, w_1) = -298.9 + 0.642(2,998) \]

\[ = 1,626 \text{ ft}^3/\text{s} \]

for the peak discharge. Therefore, the prediction is out-of-bank flow for about 3.6 miles and inbank flow for 6.4 miles, with an outflow volume of 168 acre-feet and a peak discharge of 1,626 cubic feet per second.
Example 19–3 illustrates the need for judgment in applying the procedure for estimating losses in out-of-bank flow. Care must be taken to ensure that transmission losses do not reduce the flow volume and peak to the point where flow is entirely within the channel banks. If this occurs, then the reach length must be broken into subreaches, as illustrated in this example.

In some circumstances, an alluvial channel could be underlain by nearly impervious material that might limit the potential storage volume in the alluvium (V) and thereby limit the potential transmission losses. Once the transmission losses fill the available storage, nearly all additional inflow becomes outflow. The procedure as shown in example 19–4 is modified to predict and apply this secondary threshold volume, \( P_1 \).

**Example 19–4**  
Transmission losses limited by available storage

**Given:** The channel reach in example 19–1 with total available storage (maximum potential transmission loss) of \( V = 30 \) acre-feet.

**Find:** Given the volume equation from case 1 of example 19–1, compute equations to apply after the potential losses are satisfied. From example 19–1, \( a(x,w) = -10.38 \) acre-feet, \( b(x,w) = 0.850 \), and \( P_o(X,W) = 12.21 \) acre-feet.

**Solution:** The total losses are \( P - Q(x,w) \) computed as

\[
P - [a(x,w) + b(x,w)P] = -a(x,w) + [1 - b(x,w)]P
\]

Equating this computed loss to \( V \) and solving for the inflow volume predicts the inflow volume above which only the maximum alluvial storage is subtracted:

\[
P_1 = \frac{V + a(x,w)}{1 - b(x,w)}
\]

For this example, this threshold inflow volume is 130.8 acre-feet. With this additional threshold, the prediction equation for outflow volume is modified to

\[
\begin{cases} 
0 & P \leq P_o(x,w) \\
\left( a(x,w) + b(x,w)P \right) & P_o(x,w) \leq P \leq P_1 \\
\left( P - V \right) & P > P_1
\end{cases}
\]

(eq. 19–18)

The solution to this general equation is

\[
\begin{cases} 
0 & P \leq 12.21 \\
-10.38 + 0.850P & 12.21 \leq P \leq 130.8 \\
\left( P - 30 \right) & P > 130.8
\end{cases}
\]
The slope of the regression line is equal to

\[
\frac{Q(x, w)}{P - P_0(x, w)}
\]

so an equivalent slope, once the available storage is filled, is

\[
b_{eq} = \frac{(P - V)}{P - P_0(x, w)}
\]

which for this example is

\[
b_{eq} = \frac{(P - 30)}{(P - 12.21)}
\]

For an inflow volume of \( P = 300 \) acre-feet and peak rate of inflow \( p = 3,000 \) cubic feet per second, the equivalent slope is \( b_{eq} = 0.938 \). Using the equivalent slope, the peak equation is

\[
q(x, w) = -\frac{12.1}{D} [P - Q(x, w)] + b_{eq} p
\]

\[
= -90.75 + 0.938(3,000) = 2,723 \text{ ft}^3/\text{s}
\]

Therefore, the predicted outflow is \( Q(x, w) = 270 \) acre-feet and \( q(x, w) = 2,723 \) cubic feet per second.

If the storage limitation had been ignored, the original equations would have predicted an outflow volume of 245 acre-feet and a peak rate of outflow of 2,384 cubic feet per second. If a channel reach has limited available storage, the procedure should be modified, as it was in this example, to compute losses that do not exceed the available storage.
630.1905 Summary

The examples presented illustrate the wide range of applications of the transmission loss procedures described in this chapter. They were chosen to emphasize some limitations and the need for sound engineering judgment. These concepts are summarized in table 19–4.

### Table 19–4 Outline of examples and comments on their applications

<table>
<thead>
<tr>
<th>Example</th>
<th>Procedure</th>
<th>Special circumstances</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>19–1</td>
<td>Table 19–2</td>
<td>Observed data available</td>
<td>Slope and intercept must satisfy the constraints</td>
</tr>
<tr>
<td></td>
<td>(case 1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19–1</td>
<td>Table 19–3</td>
<td>No observed data</td>
<td>Typical application</td>
</tr>
<tr>
<td></td>
<td>(case 2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19–2</td>
<td>Table 19–3,</td>
<td>Uniform lateral inflow</td>
<td>Importance of lateral inflow demonstrated</td>
</tr>
<tr>
<td></td>
<td>Eqs. 19–4, 19–5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19–3</td>
<td>Table 19–3,</td>
<td>Out-of-bank flow</td>
<td>Judgment required to interpret results</td>
</tr>
<tr>
<td></td>
<td>Eq. 19–17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19–4</td>
<td>Table 19–2,</td>
<td>Limited available storage</td>
<td>Concept of equivalent slope used</td>
</tr>
<tr>
<td></td>
<td>Eq. 19–18</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
630.1906 References


Appendix 19A

Derivation of Procedures for Estimating Transmission Losses When Observed Data are Available

In much of the southwestern United States, watersheds are characterized as semiarid with broad alluvium-filled channels that abstract large quantities of streamflow (Babcock and Cushing 1942; Burkham 1970a, 1970b; Renard 1970). These abstractions or transmission losses are important because streamflow is lost as the flood wave travels downstream, and thus, runoff volumes are reduced. Although these abstractions are referred to as losses, they are an important part of the water balance. They diminish streamflow, support riparian vegetation, and recharge local aquifers and regional ground water (Renard 1970).

Simplified procedures have been developed to estimate transmission losses in ephemeral streams. These procedures include simple regression equations to estimate outflow volumes (Lane, Diskin, and Renard 1971) and simplified differential equations for loss rate as a function of channel length (Jordan 1977). Other, more complicated methods have also been used (Lane 1972; Wu 1972; Smith 1972; Peebles 1975).

Lane, Ferreira, and Shirley (1980) developed a procedure to relate parameters of the linear regression equations (Lane, Diskin, and Renard 1971) to a differential equation coefficient and the decay factor proposed by Jordan (1977). This linkage between the regression and differential equations provides the basis of the applications described in this chapter.

Empirical basis of the regression equation

When observed inflow-outflow data for a channel reach of an ephemeral stream with no lateral inflow are plotted on rectangular coordinate paper, the result is often no outflow for small inflow events, with outflow increasing as inflow increases. When data are fitted with a straight-line relationship, the intercept on the X axis represents an initial abstraction. Graphs of this type suggest equations of the form

\[ Q(x, w) = \begin{cases} 0 & P \leq P_0(x, w) \\ a(x, w) + b(x, w)P & P > P_0(x, w) \end{cases} \]  

(eq. 19–1)

By setting \( Q(x, w) = 0.0 \) and solving for \( P \), the threshold volume, the volume of losses that occur before outflow begins is

\[ P_0(x, w) = \frac{-a(x, w)}{b(x, w)} \]  

(eq. 19–2)

Differential equation for changes in volume

Linkage with the regression model

Differential equations can be used to approximate the influence of transmission losses on runoff volumes. Because the solutions to these equations can be expressed in the same form as the regression equations, least-squares analysis can be used to estimate parameters in the transmission loss equations.

Unit channel

The rate of change in volume, \( Q \) (as a function of arbitrary distance), with changing inflow volume, \( P \), can be approximated as

\[ \frac{dQ}{dx} = -c-k \, Q(x) \]  

(eq. 19–19)

Substituting the initial condition and defining \( P = Q(x = 0) \), the solution of equation 19–19 is

\[ Q(x) = ce^{-kx} + Pe^{-kx} \]  

(eq. 19–20)

For a unit channel, equation 19–20 becomes

\[ Q = c(1-e^{-k}) + Pe^{-k} \]  

(eq. 19–21)

which corresponds to the regression equation

\[ Q = a + bP \]  

(eq. 19–22)

Equating equations 19–21 and 19–22, it follows that

\[ b = e^{-k} \]  

(eq. 19–11)

and

\[ a = -\frac{c}{k} \left(1-e^{-k}\right) = -\frac{1}{k} \left(1-b\right) \]  

(eq. 19–23)

are the linkage equations.
Equation 19–23 can be solved for \( c \) as

\[
    c = -k \frac{a}{1-b}
\]

**Channel of arbitrary length and width**

For a channel of width \( w \) and length \( x \),

\[
    \frac{dQ}{dx} = -wc - wkQ(x, w)
\]

where: \( c = -k \frac{a}{1-b} \) so that the differential equation is

\[
    \frac{dQ}{dx} = wk \frac{a}{1-b} - wkQ(x, w)
\]

Defining \( P \) as \( Q(x = 0) \) and substituting this initial condition, the solution is

\[
    Q(x, w) = \frac{a}{1-b} \left(1 - e^{-kxw}\right) + Pe^{-kxw}
\]

From the linkage

\[
    b(x, w) = e^{-kxw} \quad (eq. \, 19–13)
\]

and

\[
    a(x, w) = \frac{a}{1-b} \left[1 - b(x, w)\right] \quad (eq. \, 19–14)
\]

where:

- \( a \) and \( b \) = unit channel parameters
- \( k \) = decay factor

**Influence of uniform lateral inflow**

If \( Q_L \) is the uniform lateral inflow (acre-feet/mile), this inflow becomes an additional term in the differential equation

\[
    \frac{dQ}{dx} = wk \frac{a}{1-b} - wkQ(x, w) + Q_L
\]

The solution is

\[
    Q(x, w) = \frac{a}{1-b} \left(1 - e^{-kxw}\right) + Pe^{-kxw} + \frac{Q_L}{kw} \left(1 - e^{-kxw}\right)
\]

and through the linkage, the outflow volume equation for upstream inflow augmented by uniform lateral inflow is

\[
    Q(x, w) = a(x, w) + b(x, w)P + \frac{Q_L}{kw} \left[1 - b(x, w)\right]
\]

**Approximations for peak discharge**

The basic assumption for peak discharge, \( q(x, w) \), is that the outflow peak, once an average loss rate has been subtracted, is equal to \( b(x, w) \) times the peak of the inflow hydrographs, \( p \). That is, assume that

\[
    q(x, w) = -\frac{P - Q(x, w)}{D} + b(x, w)p
\]

where:

\[
    P = Q(x, w) - a(x, w) + \left[1 - b(x, w)\right]P
\]

so that

\[
    q(x, w) = \frac{12.1}{D} \left\{a(x, w) - \left[1 - b(x, w)\right]P\right\} + b(x, w)p
\]

where:

- \( D \) = mean duration of flow, and
- 12.1 converts acre-feet per hour to cubic feet per second

For a peak lateral inflow rate of \( q_L \) (ft\(^3\)/s/ft), uniform along the reach, the peak discharge equation becomes

\[
    q(x, w) = \frac{12.1}{D} \left\{a(x, w) - \left[1 - b(x, w)\right]P\right\} + b(x, w)p + \frac{q_L(5,280)}{kw} \left[1 - b(x, w)\right]
\]

where 5,280 converts cubic feet per second per foot to cubic feet per second per mile.

For small inflows where the volume of transmission losses is about equal to the volume of inflow, the peak discharge equation, equation 19–3, overestimates the peak rate of outflow. The relation between peak rate of outflow observed and that computed from equation 19–3 is shown in figure 19A–1. The bias shown in figure 19A–1 is for small events and tends to overpredict.
but the equation does well for the larger events. The computed values shown in figure 19A–1 were based on the mean duration of flow for each channel reach. Better agreement of predicted and observed peak rates of outflow might be obtained by using actual flow durations.

Figure 19A–1  Observed versus computed peak discharge of the outflow hydrograph

![Graph showing observed versus computed peak discharge](image-url)
Appendix 19B

Analysis of Selected Data Used to Develop the Procedure for Estimating Transmission Losses

Selected data had to be analyzed so that parameters of the prediction equations could be related to hydrograph characteristics and to effective hydraulic conductivity. Events involving little or no lateral inflow were selected from channel reaches in Arizona, Kansas, Nebraska, and Texas (table 19B–1).

The data shown in table 19B–1 are not entirely consistent because the events were floods of different magnitudes. The Walnut Gulch data are from a series of small to moderate events representing inbank flow, whereas the Queen Creek data are for relatively larger floods and no doubt include some out-of-bank flow.

The Trinity River data represent pumping diversions entirely within the channel banks. Data for the Kansas-Nebraska streams represent floods of unknown size and may include out-of-bank flow.

Data summarized in table 19B–1 were subjected to linear regression analysis to estimate the parameters \(a(x,w), b(x,w), P_{ij}(x,w),\) and \(kxw\). These parameters are summarized in table 19B–2. Parameters for the unit channels were computed for 10 channel reaches and are shown in table 19B–3.

### Table 19B–1

Hydrologic data used in analyzing transmission losses (Lane, Ferreira, and Shirley 1980)

<table>
<thead>
<tr>
<th>Location</th>
<th>Reach identification</th>
<th>Length, x</th>
<th>Average Number</th>
<th>Number of</th>
<th>Inflow volume</th>
<th>Outflow volume</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mi</td>
<td>Average width, w</td>
<td>of events</td>
<td>Mean</td>
<td>Standard deviation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ft</td>
<td></td>
<td></td>
<td>acre-ft</td>
<td>acre-ft</td>
</tr>
<tr>
<td>Walnut Gulch, AZ</td>
<td>11-8</td>
<td>4.1</td>
<td>38</td>
<td>11</td>
<td>16.5</td>
<td>14.4</td>
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<tr>
<td></td>
<td>8-6</td>
<td>0.9</td>
<td>--</td>
<td>3</td>
<td>13.7</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>8-1</td>
<td>7.8</td>
<td>--</td>
<td>3</td>
<td>16.3</td>
<td>--</td>
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<td>121</td>
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<td>48.3</td>
<td>51.7</td>
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<td>4.2</td>
<td>132</td>
<td>32</td>
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<td>10</td>
<td>4,283</td>
<td>5,150</td>
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<td></td>
<td>gaging station</td>
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<tr>
<td>Elm Fork of Trinity River, TX</td>
<td>Elm Fork-1</td>
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<td>--</td>
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<tr>
<td></td>
<td>Elm Fork-2</td>
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<td>--</td>
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<td>--</td>
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<td>Elm Fork-3</td>
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<td>3</td>
<td>454</td>
<td>--</td>
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<td>72</td>
<td>4</td>
<td>1,217</td>
<td>663</td>
</tr>
</tbody>
</table>

1/ Data file at USDA-ARS, Southwest Rangeland Water Research Center, 442 E. 7th Street, Tucson, AZ 85705.
2/ Data from Babcock and Cushing (1942).
3/ Data from the Texas Board of Water Engineers (1960).
4/ Data from Jordan (1977).
### Table 19B–2
Parameters for regression model and differential equation model for selected channel reaches (Lane, Ferreira, and Shirley 1980)

<table>
<thead>
<tr>
<th>Location</th>
<th>Reach identification</th>
<th>Reach no.</th>
<th>Length, x mi</th>
<th>Average width, w ft</th>
<th>Regression intercept, a(x,w) / acre-ft</th>
<th>Model slope, b(x,w) / acre-ft</th>
<th>Threshold volume, P_o(x,w) / acre-ft</th>
<th>Decay factor, k_xw</th>
<th>R^2</th>
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</thead>
<tbody>
<tr>
<td>Walnut Gulch, AZ</td>
<td>11-8</td>
<td>1</td>
<td>4.1</td>
<td>38</td>
<td>-4.27</td>
<td>0.789</td>
<td>5.41</td>
<td>0.2370</td>
<td>.98</td>
</tr>
<tr>
<td></td>
<td>8-6</td>
<td>2</td>
<td>0.9</td>
<td>--</td>
<td>-0.34</td>
<td>0.860</td>
<td>0.40</td>
<td>0.1508</td>
<td>.99</td>
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<tr>
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<td>8-1</td>
<td>3</td>
<td>7.8</td>
<td>--</td>
<td>-2.38</td>
<td>0.245</td>
<td>9.71</td>
<td>1.4065</td>
<td>.84</td>
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<tr>
<td></td>
<td>6-2</td>
<td>4</td>
<td>2.7</td>
<td>107</td>
<td>-4.92</td>
<td>0.823</td>
<td>5.98</td>
<td>0.1948</td>
<td>.98</td>
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<tr>
<td></td>
<td>6-1</td>
<td>5</td>
<td>6.9</td>
<td>121</td>
<td>-5.56</td>
<td>0.469</td>
<td>11.86</td>
<td>0.7572</td>
<td>.84</td>
</tr>
<tr>
<td></td>
<td>2-1</td>
<td>6</td>
<td>4.2</td>
<td>132</td>
<td>-8.77</td>
<td>0.673</td>
<td>13.03</td>
<td>0.3960</td>
<td>.84</td>
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<tr>
<td>Queen Creek, AZ</td>
<td>Upper to lower station</td>
<td>7</td>
<td>20.0</td>
<td>277</td>
<td>-117.2</td>
<td>0.648</td>
<td>180.90</td>
<td>0.4339</td>
<td>.98</td>
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<tr>
<td>Elm Fork of Trinity River, TX</td>
<td>Elm Fork-1</td>
<td>8</td>
<td>9.6</td>
<td>--</td>
<td>-15.0</td>
<td>1.004</td>
<td>1/2</td>
<td>-</td>
<td>.99</td>
</tr>
<tr>
<td></td>
<td>Elm Fork-2</td>
<td>9</td>
<td>21.3</td>
<td>--</td>
<td>+7.6 /2</td>
<td>0.944</td>
<td>-</td>
<td>-</td>
<td>.99</td>
</tr>
<tr>
<td></td>
<td>Elm Fork-3</td>
<td>10</td>
<td>30.9</td>
<td>120</td>
<td>-8.7</td>
<td>0.952</td>
<td>9.14</td>
<td>0.0492</td>
<td>.99</td>
</tr>
<tr>
<td>Kansas-Nebraska</td>
<td>Prairie Dog</td>
<td>11</td>
<td>26.0</td>
<td>17</td>
<td>-353.1</td>
<td>0.896</td>
<td>394.10</td>
<td>0.1098</td>
<td>.95</td>
</tr>
<tr>
<td></td>
<td>Beaver</td>
<td>12</td>
<td>39.0</td>
<td>14</td>
<td>-157.3</td>
<td>0.646</td>
<td>243.50</td>
<td>0.4370</td>
<td>.99</td>
</tr>
<tr>
<td></td>
<td>Sappa</td>
<td>13</td>
<td>35.0</td>
<td>23</td>
<td>-1,076.3</td>
<td>0.796</td>
<td>1,352.10</td>
<td>0.2282</td>
<td>.98</td>
</tr>
<tr>
<td></td>
<td>Smokey Hills</td>
<td>14</td>
<td>47.0</td>
<td>72</td>
<td>-99.1</td>
<td>0.614</td>
<td>161.40</td>
<td>0.4878</td>
<td>.81</td>
</tr>
</tbody>
</table>

1/ Channel reaches where derived regression parameters did not satisfy the constraints.
<table>
<thead>
<tr>
<th>Location</th>
<th>Identification</th>
<th>Unit length parameters</th>
<th>Unit width parameters</th>
<th>Unit length and width parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(a(w))</td>
<td>(b(w))</td>
<td>(P_o(w))</td>
</tr>
<tr>
<td>Walnut</td>
<td>11–8</td>
<td>-1.13657</td>
<td>0.94384</td>
<td>1.2042</td>
</tr>
<tr>
<td></td>
<td>Gulch, AZ</td>
<td>-1.93484</td>
<td>0.93039</td>
<td>2.0796</td>
</tr>
<tr>
<td></td>
<td>6–1</td>
<td>-1.08819</td>
<td>0.89607</td>
<td>1.2144</td>
</tr>
<tr>
<td></td>
<td>2–1</td>
<td>-2.41320</td>
<td>0.91002</td>
<td>2.6518</td>
</tr>
<tr>
<td></td>
<td>Queen Creek, AZ</td>
<td>-7.14508</td>
<td>0.97854</td>
<td>7.3018</td>
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<tr>
<td></td>
<td>Upper to lower station</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Trinity River, TX</td>
<td>-0.28825</td>
<td>0.99841</td>
<td>0.2887</td>
</tr>
<tr>
<td></td>
<td>Kansas-Nebraska</td>
<td>-14.39986</td>
<td>0.99579</td>
<td>14.3705</td>
</tr>
<tr>
<td></td>
<td>Prairie Dog</td>
<td>-4.95071</td>
<td>0.98886</td>
<td>5.0065</td>
</tr>
<tr>
<td></td>
<td>Beaver</td>
<td>-34.28091</td>
<td>0.93505</td>
<td>34.5652</td>
</tr>
<tr>
<td></td>
<td>Sappa</td>
<td>-2.65060</td>
<td>0.98068</td>
<td>2.6782</td>
</tr>
</tbody>
</table>
Appendix 19C

Estimating Transmission Losses When No Observed Data are Available

Estimating transmission losses when observed inflow-outflow data are not available requires a technique for using effective hydraulic conductivity to develop parameters for the regression analysis.

**Estimating effective hydraulic conductivity**

The total volume of losses for a channel reach is \( KD \), where \( K \) is the effective hydraulic conductivity and \( D \) is the duration of flow. Also, the total losses are \( P - Q(x,w) \), so that:

\[
KD = 0.0275 [P - Q(x,w)]
\]

where 0.0275 converts acre-feet per foot-mile-hour to inches per hour. Or, solving for \( K \):

\[
K = \frac{0.0275 [P - Q(x,w)]}{D}
\]

But

\[
P - Q(x,w) = -a(x,w) + [1 - b(x,w)]P
\]

so that

\[
K = \frac{0.0275}{D} \left\{ -a(x,w) + [1 - b(x,w)]P \right\}
\]

(eq. 19–24)

is an expression for effective hydraulic conductivity. If mean values for \( D \) and \( P \) are used, then equation 19–24 estimates the mean value of the effective hydraulic conductivity.

**Effective hydraulic conductivity versus model parameters**

For a unit channel, outflow is the difference between inflow and transmission losses:

\[
Q = P - KD
\]

Because \( Q = a + bP \),

\[-a + (1 - b)P = KD\]

However, because \( a \) and \( (1 - b)P \) are in acre-feet and \( KD \), the product of conductivity and duration, is in inches, the dimensionally correct equation is

\[-a + (1 - b)P = 0.0101KD\]

where 0.0101 converts inches over a unit channel to acre-feet. Because this equation is in two unknowns \( (a \) and \( b) \), an additional relationship is required to solve it. As a first approximation, the total losses are partitioned between the two terms in the equation. That is, let

\[
a = -\alpha (0.0101KD)
\]

and

\[(1 - b) = (1 - \alpha) \left( \frac{0.0101KD}{P} \right)\]

Solving for \( b \),

\[
b = 1 - (1 - \alpha) \left( \frac{0.0101KD}{P} \right)
\]

where \( 0 \leq \alpha \leq 1 \) is a weighting factor. Solve for \( k \) by substituting \( b = e^k \) and taking the negative natural log of both sides; i.e.:

\[
k = -\ln \left[ 1 - (1 - \alpha) \left( \frac{0.0101KD}{P} \right) \right]
\]

The selected data were analyzed to determine \( \alpha \) by least-squares fitting as shown in table 19C–1. For the data shown in table 19C–1, the estimate of \( \alpha \) was 0.46. Figures 19C–1 and 19C–2 show the data in table 19C–1 plotted according to the equations

\[
a(D) = -0.00465KD \quad \text{(eq. 19–15)}
\]

and

\[
k(D,P) = -1.09 \ln \left[ 1.0 - 0.00545 \frac{KD}{P} \right] \quad \text{(eq. 19–16)}
\]

where for each channel reach, mean values were used for \( K \), \( D \), and \( P \). These relationships were used to calculate the values shown in table 19–1 of the main text.

Auxiliary data compiled in a report by Wilson, Decook, and Neuman (1980) are shown in table 19C–2. Although the estimates of infiltration rates were obtained by a variety of methods, most rates were based on streamflow data. Because these estimates generally involved longer periods of flow than in the smaller
Table 19C–1  Data for analysis of relations between effective hydraulic conductivity and model parameters (Lane, Ferreira, and Shirley 1980)

<table>
<thead>
<tr>
<th>Location</th>
<th>Unit channel intercept, a (acre-ft)</th>
<th>Decay factor, k (ft-mi)–1</th>
<th>K (in/h)</th>
<th>KD (in)</th>
<th>KD/P</th>
<th>–ln[1–0.00545(KD/P)]</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walnut Gulch</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11–8</td>
<td>−0.03076</td>
<td>0.001521</td>
<td>1.55</td>
<td>4.96</td>
<td>0.3010</td>
<td>0.001643</td>
<td>Inbank flow</td>
</tr>
<tr>
<td>6–2</td>
<td>−0.01874</td>
<td>0.000674</td>
<td>1.36</td>
<td>6.26</td>
<td>0.0834</td>
<td>0.000455</td>
<td></td>
</tr>
<tr>
<td>6–1</td>
<td>−0.00950</td>
<td>0.000907</td>
<td>1.03</td>
<td>3.71</td>
<td>0.0768</td>
<td>0.000419</td>
<td></td>
</tr>
<tr>
<td>2–1</td>
<td>−0.01915</td>
<td>0.000714</td>
<td>1.11</td>
<td>4.44</td>
<td>0.0901</td>
<td>0.000492</td>
<td></td>
</tr>
<tr>
<td>Queen Creek</td>
<td>−0.02597</td>
<td>0.0000783</td>
<td>0.54</td>
<td>29.16</td>
<td>0.0668</td>
<td>0.0000371</td>
<td>Mixed flow</td>
</tr>
<tr>
<td>Elm Fork</td>
<td>−0.00240</td>
<td>0.000133</td>
<td>0.01</td>
<td>0.84</td>
<td>0.0019</td>
<td>0.0000104</td>
<td>Inbank flow</td>
</tr>
<tr>
<td>Kansas-Nebraska</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prairie Dog</td>
<td>−0.84201</td>
<td>0.000248</td>
<td>1.28</td>
<td>122.9</td>
<td>0.0650</td>
<td>0.000355</td>
<td>Mixed flow:</td>
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<tr>
<td>Beaver</td>
<td>−0.35548</td>
<td>0.000800</td>
<td>1.38</td>
<td>199.7</td>
<td>0.0771</td>
<td>0.000421</td>
<td>average widths</td>
</tr>
<tr>
<td>Sappa</td>
<td>−1.49310</td>
<td>0.000283</td>
<td>2.57</td>
<td>287.8</td>
<td>0.0465</td>
<td>0.000254</td>
<td>may be under-</td>
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<tr>
<td>Smokey Hills</td>
<td>−0.03697</td>
<td>0.000144</td>
<td>0.17</td>
<td>16.3</td>
<td>0.0134</td>
<td>0.000073</td>
<td>estimated</td>
</tr>
</tbody>
</table>

Least-squares fit:

\[
a(D) = -0.00465 K D
\]

\[
k(D, P) = -1.09 \ln \left[ 1.0 - 0.00545 \frac{K D}{P} \right]
\]
ephemeral streams, they should be representative of what is called effective hydraulic conductivity. The data show the range of estimates of hydraulic conductivity for various streams within a river basin as estimated by several investigators. For this reason, they should be viewed as qualitative estimates. Improved estimates based on site-specific conditions were used in developing the prediction equations.

For comparison, seepage loss rates for unlined canals are shown in table 19C–3. Though these data are not strictly comparable with loss rates in natural channels, they do show the variation in infiltration rates with different soil characteristics. Infiltration rates varied by a factor of over 20 (0.12–3.0 in/h) from a clay loam soil to a very gravelly soil.

**Table 19C–2** Auxiliary transmission loss data for selected ephemeral streams in southern Arizona (Wilson, DeCook, and Neuman 1980)

<table>
<thead>
<tr>
<th>River basin</th>
<th>Stream reach</th>
<th>Estimation method</th>
<th>Effective hydraulic conductivity (in/h)</th>
<th>Source of estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Santa Cruz</td>
<td>Santa Cruz River, Tucson to Continental</td>
<td>Streamflow data ¹</td>
<td>1.5 – 3.4</td>
<td>Matlock (1965)</td>
</tr>
<tr>
<td></td>
<td>Santa Cruz River, Tucson to Cortero</td>
<td>Streamflow data</td>
<td>3.2 – 3.7</td>
<td>Matlock (1965)</td>
</tr>
<tr>
<td></td>
<td>Rillito Creek, Tucson</td>
<td>Streamflow data</td>
<td>0.5 – 3.3</td>
<td>Matlock (1965)</td>
</tr>
<tr>
<td></td>
<td>Rillito Creek, Cortero</td>
<td>Streamflow data</td>
<td>2.2 – 5.5</td>
<td>Matlock (1965)</td>
</tr>
<tr>
<td></td>
<td>Pantano Wash, Tucson</td>
<td>Streamflow data</td>
<td>1.6 – 2.0</td>
<td>Matlock (1965)</td>
</tr>
<tr>
<td></td>
<td>Average for Tucson area</td>
<td>—</td>
<td>1.65</td>
<td>Matlock (1965)</td>
</tr>
<tr>
<td>Gila</td>
<td>Queen Creek</td>
<td>Streamflow data:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Summer flows</td>
<td>0.07 – 0.52</td>
<td>Babcock and Cushing (1942)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Winter flows</td>
<td>0.37 – 1.05</td>
<td>Babcock and Cushing (1942)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average for all events</td>
<td>0.54</td>
<td>Babcock and Cushing (1942)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Seepage losses in pools ²</td>
<td>&gt;2.0</td>
<td>Babcock and Cushing (1942)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Salt River, Granite Reef Dam to 7th Ave.</td>
<td>Streamflow data</td>
<td>0.75 – 1.25</td>
<td>Briggs and Werho (1966)</td>
</tr>
<tr>
<td>San Pedro</td>
<td>Walnut Gulch</td>
<td>Streamflow data</td>
<td>1.1 – 4.5</td>
<td>Keppel (1960)</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td>Keppel and Renard (1962)</td>
</tr>
<tr>
<td></td>
<td>Walnut Gulch</td>
<td>Streamflow data</td>
<td>2.4</td>
<td>Peebles (1975)</td>
</tr>
<tr>
<td>San Simon</td>
<td>San Simon Creek</td>
<td>—</td>
<td>0.18</td>
<td>Peterson (1962)</td>
</tr>
</tbody>
</table>

¹ Transmission losses estimated from streamflow data.
² Measurement of loss rates from seepage in isolated pools.
Table 19C–3  Range of seepage rates in unlined canals 1/

<table>
<thead>
<tr>
<th>Effective hydraulic conductivity (in/h)</th>
<th>Description of materials 2/</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12–0.18</td>
<td>Clay-loam, described as impervious</td>
</tr>
<tr>
<td>0.25–0.38</td>
<td>Ordinary clay loam</td>
</tr>
<tr>
<td>0.38–0.50</td>
<td>Sandy loam or gravelly clay-loam with sand and clay</td>
</tr>
<tr>
<td>0.50–0.75</td>
<td>Sandy loam</td>
</tr>
<tr>
<td>0.75–0.88</td>
<td>Loose sandy soil</td>
</tr>
<tr>
<td>1.0–1.25</td>
<td>Gravelly sandy soils</td>
</tr>
<tr>
<td>1.5–3.0</td>
<td>Very gravelly soils</td>
</tr>
</tbody>
</table>

2/ Does not reflect the flashy, sediment-laden character of many ephemeral streams.