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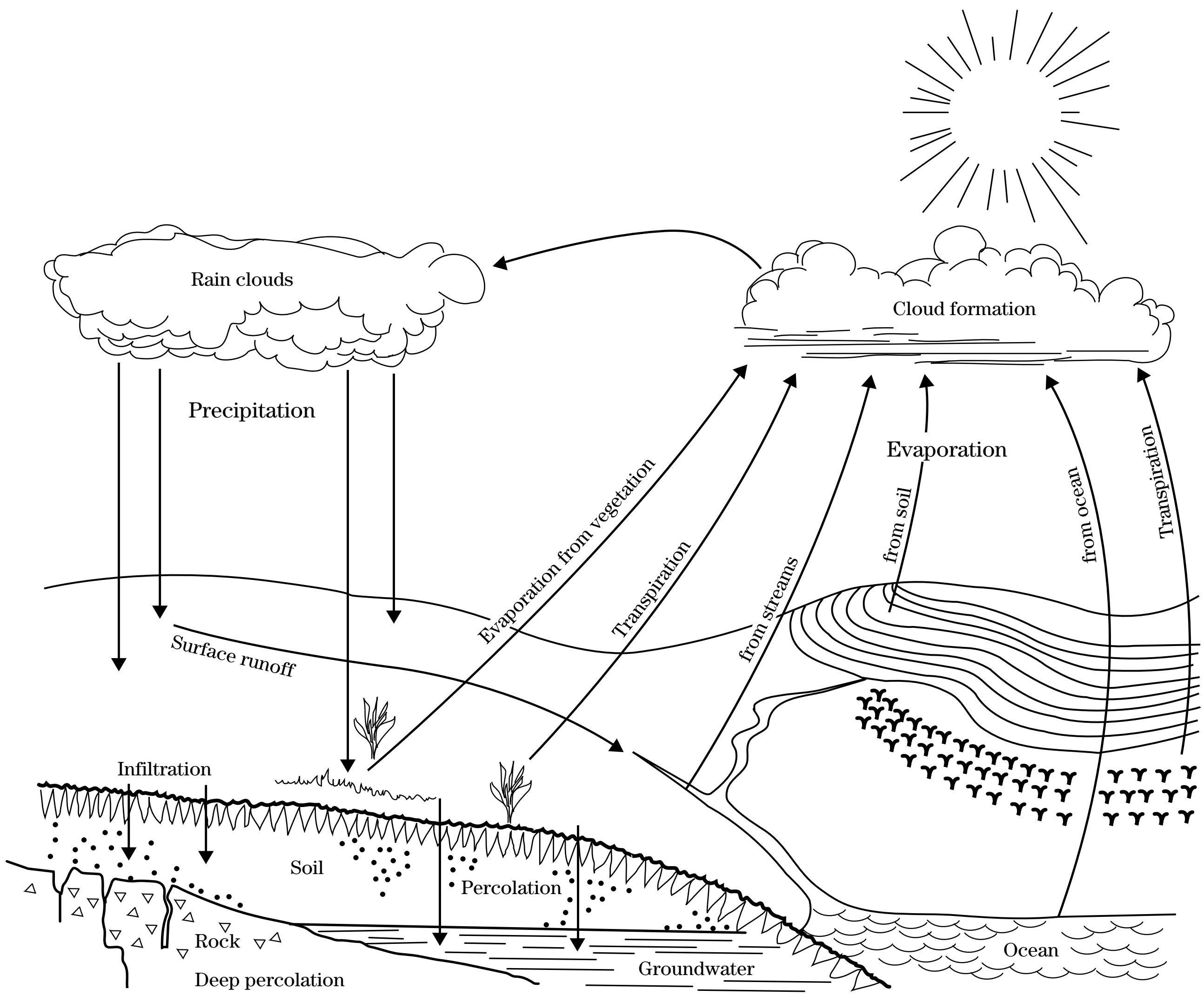
Chapter 18 Selected Statistical Methods



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Chapter 18 Selected Statistical Methods

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630.1800 Introduction

Chapter 18 is a guide for applying selected statistical methods to solve hydrologic problems. The chapter includes a review of basic statistical concepts, a description of selected statistical procedures, and references to procedures in other available documents. Examples illustrate how statistical procedures apply to typical problems in hydrology.

In project evaluation and design, the hydrologist and/or engineer must estimate the frequency of individual hydrologic events. This is necessary when making economic evaluations of flood protection projects; determining floodways; and designing irrigation systems, reservoirs, and channels. Frequency studies are based on past records and, where records are insufficient, on simulated data.

Meaningful relationships sometimes exist between hydrologic and other types of data. The ability to generalize about these relationships may allow data to be transferred from one location to another. Some procedures used to perform such transfers, called regionalization, are covered in this chapter.

The examples in this chapter contain many computer-generated tables. Some table values (especially logarithmic transformations) may not be as accurate as values calculated by other methods. Numerical accuracy is a function of the number of significant digits and the algorithms used in data processing, so some slight differences in numbers may be found if examples are checked by other means.

630.1801 Basic data requirements

(a) Basic concepts

To analyze hydrologic data statistically, the user must know basic definitions and understand the intent and limitations of statistical analysis. Because collection of all data or the entire population from a physical system generally is not feasible and recorded data from the system may be limited, observations must be based on a sample that is representative of the population.

Statistical methods are based on the assumption of randomness, which implies an event cannot be predicted with certainty. By definition, *probability* is an indicator for the likelihood of an event's occurrence and is measured on a scale from zero to one, with zero indicating no chance of occurrence and one indicating certainty of occurrence.

An event or value that does not occur with certainty is often called a random variable. The two types of random variables are discrete and continuous. A *discrete random variable* is one that can only take on values that are whole numbers. For example, the outcome of a toss of a die is a discrete random variable because it can only take on the integer values 1 to 6. The concept of risk as it is applied in frequency analysis is also based on a discrete probability distribution. A *continuous random variable* can take on values defined over a continuum; for example, peak discharge takes on values other than discrete integers.

A function that defines the probability that a random value will occur is called a *probability distribution function.* For example, the log-Pearson Type III distribution, often used in frequency analyses, is a probability distribution function. A *probability mass function* is used for discrete random variables, while a *density function* is used for continuous random variables. If values of a distribution function are added (discrete) or integrated (continuous), then a *cumulative distribution function* is formed. Usually, hydrologic data that are analyzed by frequency analysis are presented as a cumulative distribution function.

(b) Types of data

The application of statistical methods in hydrologic studies requires measurement of physical phenomena. The user should understand how the data are collected and processed before they are published. This knowledge helps the user assess the accuracy of the data. Some types of data used in hydrologic studies include rainfall, snowmelt, stage, streamflow, temperature, evaporation, and watershed characteristics.

Rainfall is generally measured as an accumulated depth over time. Measurements represent the amount caught by the gage opening and are valid only for the gage location. The amount collected may be affected by gage location and physical factors near the gage. Application over large areas requires a study of adjacent gages and determinations of a weighted rainfall amount. More complete descriptions of rainfall collection and evaluation procedures are in National Engineering Handbook (NEH) 630.04.

Snowfall is measured as depth or as water equivalent on the ground. As with rainfall, the measurement represents only the depth at the measurement point. The specific gravity of the snow times the depth of the snow determines the water equivalent of the snowpack, which is the depth of water that would result from melting the snow. To use snow information for such things as predicting water yield, the user should thoroughly know snowfall, its physical characteristics, and its measurement. NEH, Section 22, Snow Survey and Water Supply Forecasting (1972) further describes these subjects.

Stages are measurements of the elevation of the water surface as related to an established datum, either the channel bottom or mean sea level, called the National Geodetic Vertical Datum (NGVD). Peak stages are measured by nonrecording gages, crest-stage gages, or recording gages. Peak stages from nonrecording gages may be missed because continuous visual observations are not available. Crest-stage gages record only the maximum gage height and recording gages provide a continuous chart or record of stage.

Streamflow or discharge rates are extensions of the stage measurements that have been converted using rating curves. Discharge rates indicate the runoff from the drainage area above the gaging station and are expressed in cubic feet per second (ft^3/s). Volume of flow past a gage, expressed as a mean daily or hourly flow in cubic feet per second per day or cubic feet per second per hour (ft^3/s-d or ft^3/s-h), can be calculated if the record is continuous. Accuracy of streamflow data depends largely on physical features at the gaging site, frequency of observation, and the type and adequacy of the equipment used. Flows can be affected by upstream diversion and storage. U.S. Geological Survey (USGS) Water Supply Paper 888 (Corbett 1962) gives further details on streamflow data collection.

Daily temperature data are usually available, with readings published as maximum, minimum, and mean measurements for the day. Temperatures are recorded in degrees Fahrenheit or degrees Celsius. National Weather Service, Observing Handbook No. 2, Substation Observations (1972), describes techniques used to collect meteorological data.

Evaporation data are generally published as pan evaporation in inches per month. Pan evaporation is often adjusted to estimate gross lake evaporation. The National Weather Service has published pan evaporation values in "Evaporation Atlas for the Contiguous 48 United States" (Farnsworth, Thompson, and Peck 1982).

Watershed characteristics used in hydrologic studies include drainage area, channel slope, geology, type and condition of vegetation, and other features. Maps, field surveys, and studies are used to obtain this information. Often data on these physical factors are not published, but the USGS maintains a file on watershed characteristics for most streamgage sites. Many Federal and State agencies collect and publish hydrometeorological data (table 18–1). Many other organizations collect hydrologic data that are not published, but may be available upon request.

(c) Data errors

The possibility of instrumental and human error is inherent in data collection and publication for hydrologic studies. Instrumental errors are caused by the type of equipment used, its location, and conditions at the time measurements are taken. Instrumental errors can be accidental if they are not constant or do not create a trend, but they may also be systematic if they occur regularly and introduce a bias into the record. Human errors by the observer or by others who process or publish the information can also be accidental or systematic. Examples of human errors include improper operation or observation of equipment, misinterpretation of data, and errors in transcribing and publishing.

The user of the hydrologic data should be aware of the possibility of errors in observations and should recognize observations that are outside the expected range of values. Knowledge of the procedures used in collecting the data is helpful in recognizing and resolving any questionable observations, but the user should consult the collection agency when data seem to be in error.

(d) Types of series

Hydrologic data are generally presented in chronological order. If all the data for a certain increment of observation (for example, daily readings) are presented for the entire period of record, this is a *complete-duration series*. Many of these data do not have significance and can be excluded from hydrologic studies. The complete-duration series is only used for duration curves or mass curves. From the complete-duration series, two types of series are selected: the partial-duration series and the extreme-event series.

The *partial-duration series* includes all events in the complete-duration series with a magnitude above a selected base for high events or below a selected base for low events. Unfortunately, independence of events that occur in a short period is hard to establish because long-lasting watershed effects from one event can influence the magnitude of succeeding events. Also, in many areas the extreme events occur during a relatively short period during the year. Partial-duration frequency curves are developed either by graphically fitting the plotted sample data or by using empirical coefficients to convert the partial-duration series to another series.

The *extreme-event series* includes the largest (or smallest) values from the complete-duration series, with each value selected from an equal time interval in the period of record. If the time interval is taken as 1 year, then the series is an *annual series*; for example, a tabulation of the largest peak flows in each year through the period of record is an annual peak flow series at the location. Several high peak flows may occur within the same year, but the annual peak series includes only the largest peak flow per year. Table 18–2 illustrates a partial-duration and annual peak flow series.

Table 18–1 Sources of basic hydrologic data collected by Federal agencies

Agency	Data					
	Rainfall	Snow	Streamflow	Evaporation	Air temp.	Water stage
USDA Agricultural Research Service (ARS)	X	X	X	X	X	X
U.S. Army Corps of Engineers (USACE)	X	X	X	X		X
USDA Forest Service (FS)	X	X	X		X	X
U.S. Geological Survey, National Water Information Service (NWIS)		X	X	X	X	X
International Boundary and Water Commission	X		X	X	X	X
River Basin Commissions	X		X			X
DOI Bureau of Reclamation	X	X	X	X	X	X
USDA Natural Resources Conservation Service (NRCS)	X	X	X		X	X
Tennessee Valley Authority (TVA)	X		X	X	X	X
National Climatic Data Center, National Oceanic and Atmospheric Administration (NOAA)	X	X		X	X	X

Table 18–2 Flood peaks for East Fork Big Creek near Bethany, Missouri (06897000) [1/]

Year	Peaks above base (ft³/s)	Year	Peaks above base (ft³/s)	Year	Peaks above base (ft³/s)	Year	Peaks above base (ft³/s)
1940	1,780 *	1947	2,240	1959	3,800	1969	2,990
	1,120		8,120 *		3,000		3,110 *
			2,970		1,500		1,730
1941	2,770		3,700		2,660		2,910
	2,950 *		4,920		5,100 *		2,270
					3,660		2,060
1942	1,190	1948	1,260		2,280		
	1,400		2,310 *		1,890	1970	2,090
	925						3,070 *
	925	1949	2,000 *	1960	2,280		2,060
	1,330				4,650		
	1,330	1950	1,160		1,960	1971	2,000 *
	5,320		1,300 *		1,680		
	6,600 *				4,740 *	1972	3,190 *
		1951	1,090		2,040		
1943	958		2,920 *				
	1,680		1,090	1961	1,760		
	2,000		1,720		1,520		
	3,110 *		2,030		3,100		
	925		1,060		5,700 *		
	2,470		1,000		2,300		
	1,330						
	1,190	1952	1,440	1962	2,630		
	2,240		1,610		2,750		
	3,070		1,090		1,760		
			1,230		1,820		
1944	1,120		2,970 *		3,880 *		
	3,210 *		2,280				
	2,620			1963	2,100 *		
	2,170	1953	925 *				
				1964	1,880		
1945	3,490	1954	1,330 *		1,910 *		
	4,120 *						
	2,310			1965	1,730		
	2,350	1955	1,500		3,480 *		
			2,240 *				
1946	4,400		1,500	1966	2,430 *		
	1,520						
	1,720	1956	1,560	1967	1,640		
	6,770 *		2,500 *		3,350 *		
	1,960				1,640		
		1957	1,620 *				
				1968	3,150 *		
		1958	1,780 *				
			1,780				

1/ Partial-duration base is 925 ft³/s, the lowest annual flood for this series.
* Annual series values. Data from USGS Water Supply Papers.

Some data indicate seasonal variation, monthly variation, or causative variation. Major storms or floods may occur consistently during the same season of the year or may be caused by more than one factor; for example, by rainfall and snowmelt. Such data may require the development of a series based on a separation by causative factors or a particular timeframe.

(e) Data transformation

In many instances, complex data relationships require that variables be transformed to approximate linear relationships or other relationships with known shapes. Types of data transformation include:

- Linear transformation, which involves addition, subtraction, multiplication, or division by a constant
- Inverted transformation by use of the reciprocal of the data variables
- Logarithmic transformation by use of the logarithms of the data variables
- Exponential transformation, which includes raising the data variables to a power
- Any combination of the above

The appropriate transformation may be based on a physical system or may be entirely empirical. All data transformations have limitations. For example, the reciprocal of data greater than +1 yields values between zero and +1. Logarithms commonly used in hydrologic data can only be derived from positive data.

(f) Distribution parameters and moments

A probability distribution function, as previously defined, is represented by a mathematical formula that includes one or more of the following parameters:

- Location—provides reference values for the random variable
- Scale—characterizes the relative dispersion of the distribution
- Shape—describes the outline or form of a distribution

A parameter is *unbiased* if the average of estimates taken from repeated samples of the same size converges to the average population value. A parameter is *biased* if the average estimate does not converge to the average population value.

A probability density function can be characterized by its moments, which are also used in characterizing data samples. In hydrology, three moments of special interest are mean, variance, and skew.

The first moment about the origin is the *mean*, a location parameter that measures the central tendency of the data and is computed by:

$$\bar{X} = \frac{1}{N}\left(\sum_{i=1}^{N} X_i\right) \qquad (18\text{–}1)$$

where:

$\bar{X}$ = sample arithmetic mean having N observations
X_i = the i^{th} observation of the sample data
N = number of observations

The first moment about the mean is always zero. The remaining two moments of interest are taken about the mean instead of the origin.

The *variance*, a scale parameter and the second moment about the mean, measures the dispersion of the sample elements about the mean. The unbiased estimate of the variance (S^2) is given by:

$$S_X^2 = \left[\frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2\right] \qquad (18\text{–}2)$$

A biased estimate of the variance results when the divisor (N – 1) is replaced by N. An alternative form for computing the unbiased sample variance is given by:

$$S_X^2 = \frac{1}{N-1}\left[\sum_{i=1}^{N} X_i^2 - \frac{1}{N}\left(\sum_{i=1}^{N} X_i\right)^2\right] \qquad (18\text{–}3)$$

This equation is often used for computer applications because it does not require prior computation of the mean. However, because of the sensitivity of equation 18–3 to the number of significant digits carried through the computation, equation 18–2 is often preferred.

The *standard deviation of X* (S_X) is the square root of the variance and is used more frequently than the variance because its units are the same as those of the mean.

$$S_X = \left[\frac{1}{N-1} \sum_{i=1}^{N} \left(X_i - \bar{X} \right)^2 \right]^{0.5} \quad (18\text{–}4)$$

The *skew*, a shape parameter and the third moment about the mean, measures the symmetry of a distribution. The sample skew (G) can be computed by:

$$G = \frac{N}{(N-1)(N-2)S^3} \left[\sum_{i=1}^{N} \left(X_i - \bar{X} \right)^3 \right] \quad (18\text{–}5)$$

Although the range of the skew is theoretically unlimited, a mathematical limit based on sample size limits the possible skew (Kirby 1974). A skew of zero indicates a symmetrical distribution. Another equation for computing skew that does not require prior computation of the mean is:

$$G = \frac{N^2 \left(\sum_{i=1}^{N} X_i^3 \right) - 3N \left(\sum_{i=1}^{N} X_i \right) \left(\sum_{i=1}^{N} X_i^2 \right) + 2 \left(\sum_{i=1}^{N} X_i \right)^3}{N(N-1)(N-2)S^3} \quad (18\text{–}6)$$

This equation is extremely sensitive to the number of significant digits used during computation and may not give an accurate estimate of the sample skew.

630.1802 Frequency analysis

(a) Basic concepts

Frequency analysis is a statistical method commonly used to analyze a single random variable. Even when the population distribution is known, uncertainty is associated with the occurrence of the random variable. When the population is unknown, there are two sources of uncertainty: randomness of future events and accuracy of estimation of the relative frequency of occurrence. The cumulative density function is estimated by fitting a frequency distribution to the sample data. A *frequency distribution* is a generalized cumulative density function of known shape and range of values.

The probability scale of the frequency distribution differs from the probability scale of the cumulative density function by the relation (1–p) where:

$$p + q = 1 \quad (18\text{–}7)$$

The variables p and q represent the accumulation of the density function for all values less than and greater than, respectively, the value of the random variable. The accumulation is made from the right end of the probability density function curve when one considers high values, such as peak discharge. Exhibit 18–3 presents the accumulation of the Pearson III density function for both p and q for a range of skew values.

When minimum values (p) such as low flows are considered, the accumulation of the probability density function is from the left end of the curve. The resulting curve represents values less than the random variable.

(b) Plotting positions and probability paper

Statistical computations of frequency curves are independent of how the sample data are plotted. Therefore, the data should be plotted along with the calculated frequency curve to verify that the general trend of the data reasonably agrees with the frequency distribution curve. Various plotting formulas are used; many are of the general form:

$$PP = \frac{100(M-a)}{N-a-b+1} \qquad (18\text{–}8)$$

where:

PP = plotting position for a value in percent chance

M = ordered data (largest to smallest for maximum values and smallest to largest for minimum values)

N = size of the data sample

a and b = constants

Some commonly used constants for plotting position formulas are:

Name	a	b
Weibull	0	0
Hazen	–M+1	–N+M
California	0	1
Blom	3/8	3/8

The Weibull plotting position is used to plot the sample data in the chapter examples:

$$PP = \frac{100(M)}{N+1} \qquad (18\text{–}9)$$

Each probability distribution has its own probability paper for plotting. The probability scale is defined by transferring a linear scale of standard deviates (K values) into probabilities for that distribution. The frequency curve for a distribution will be a straight line on paper specifically designed for that distribution.

Probability paper for logarithmic normal and extreme value distributions is readily available. Distributions with a varying shape statistic (log-Pearson III and gamma) require paper with a different probability scale for each value of the shape statistic. For these distributions, a special plotting paper is not practical. The log-Pearson III and gamma distributions are generally plotted on logarithmic normal probability paper. The plotted frequency line may be curved, but this is more desirable than developing a new probability scale each time these distributions are plotted.

(c) Probability distribution functions

(1) Normal

The normal distribution, used to evaluate continuous random variables, is symmetrical and bell-shaped. The range of the random variable is –X to +X. Two parameters (location and scale) are required to fit the distribution. These parameters are approximated by the sample mean and standard deviation. The normal distribution is the basis for much of statistical theory, but generally does not fit hydrologic data.

The log-normal distribution (normal distribution with logarithmically transformed data) is often used in hydrology to fit high or low discharge data or in regionalization analysis. Its range is zero to +X. Example 18–1 illustrates the development of a log-normal distribution curve.

(2) Pearson III

Karl Pearson developed a system of 12 distributions that can approximate all forms of single-peak statistical distributions. The system includes three main distributions and nine transition distributions, all of which were developed from a single differential equation. The distributions are continuous, but can be fitted to various forms of discrete data sets (Chisman 1968).

The type III (negative exponential) is the distribution frequently used in hydrologic analysis. It is nonsymmetrical and is used with continuous random variables. The probability density function can take on many shapes. Depending on the shape parameter, the random variable range can be limited on the lower end, the upper end, or both. Three parameters are required to fit the Pearson type III distribution. The location and scale parameters (mean and standard deviation) are the same as those for the normal distribution. The shape (or third) parameter is approximated by the sample skew.

When a logarithmic transformation is used, a lower bound of zero exists for all shape parameters. The log-Pearson type III is used to fit high and low discharge values, snow, and volume duration data.

(3) Two-parameter gamma

The two-parameter gamma distribution is nonsymmetrical and is used with continuous random variables to fit high and low volume duration, stage, and discharge data. Its probability density function has a lower limit of zero and a defined upper limit of infinity (∞). Two parameters are required to fit the distribution: ß, a scale parameter, and γ, a shape parameter. A detailed description of how to fit the distribution with the two parameters and incomplete gamma function tables is in Technical Publication (TP)–148 (Sammons 1966). As a close approximation of this solution, a three-parameter Pearson type III fit can be made and exhibit 18–3 tables used. The mean and γ must be computed and converted to standard deviation and skewness parameters. Greenwood and Durand (1960) provide a method to calculate an approximation for γ that is a function of the relationship (R) between the arithmetic mean and geometric mean (G_m) of the sample data:

$$G_m = \left[X_1(X_2)(X_3)\ldots(X_N)\right]^{\frac{1}{N}} \qquad (18\text{–}10)$$

$$R = \ln\left[\frac{\bar{X}}{G_m}\right] \qquad (18\text{–}11)$$

where:
ln = natural logarithm

(a) If $0 \le R \le 0.5772$

$$\gamma = R^{-1}\left(0.5000876 + 0.1648852R - 0.0544274R^2\right) \qquad (18\text{–}12)$$

(b) If $0.5772 < R \le 17.0$

$$\gamma = \frac{8.898919 + 9.059950R + 0.9775373R^2}{R\left(17.79728 + 11.968477R + R^2\right)} \qquad (18\text{–}13)$$

(c) If R > 17.0 the shape approaches a log-normal distribution, and a log-normal solution may be used.

The standard deviation and skewness can now be computed from γ and the mean:

$$S = \frac{\bar{X}}{\sqrt{\gamma}} \qquad (18\text{–}14)$$

$$G = \frac{2}{\sqrt{\gamma}} \qquad (18\text{–}15)$$

(4) Extreme value
The extreme value distribution, another nonsymmetrical distribution used with continuous random variables, has three main types. Type I is unbounded, type II is bounded on the lower end, and type III is bounded on the upper end. The type I (Fisher-Tippett) is used by the National Weather Service in some precipitation analysis. Other Federal, State, local, and private organizations also have publications based on extreme value theory.

(5) Binomial
The binomial distribution, used with discrete random variables, is based on four assumptions:

- The random variable may have only one of two responses (for example, yes or no, successful or unsuccessful, flood or no flood).
- There will be n trials in the sample.
- Each trial will be independent.
- The probability of a response will be constant from one trial to the next.

The binomial distribution is used in assessing risk, which is described later in the chapter.

(d) Cumulative distribution curve

Selected percentage points on the cumulative distribution curve for normal, Pearson III, or gamma distributions can be computed with the sample mean, standard deviation, and skewness. Exhibit 18–2 contains standard deviate (K_n) values for the normal probability distribution. Exhibit 18–3 contains standard deviate (K_p) values for various values of skewness and probabilities for the Pearson III distribution. The equation used to compute points along the cumulative distribution curve is:

$$Q = \bar{X} + KS \qquad (18\text{–}16)$$

where:
Q = random variable value at a selected exceedance probability
$\bar{X}$ = sample mean
S = sample standard deviation
K = standard deviate, which is designated K_n or K_p (for normal or Pearson) depending on the exhibit used

If a logarithmic transformation has been applied to the data, then the equation becomes:

$$\log Q = \bar{X} + K_p S \qquad (18\text{–}17)$$

where:

$\bar{X}$ and S are based on the moments of the logarithmically transformed sample data.

With the mean, standard deviation, and skew computed, a combination of K_p values from exhibit 18–3 and either equation 18–16 or 18–17 is used to calculate specified points along the cumulative distribution curve.

(e) Data considerations in analysis

(1) Outliers

If the population model is correct, outliers are population elements that occur, but are highly unlikely to occur in a sample of a given size. Therefore, outliers can result from sampling variation or from using the incorrect probability model. After the most likely probability model is selected, outlier tests can be performed for evaluating extreme events.

Outliers can be detected by use of test criteria in exhibit 18–1. Critical standard deviates (K_n values) for the normal distribution can be taken from the exhibit. Critical K values for other distributions are computed from the probability levels listed in the exhibit. Critical K values are used in either equation 18–16 or 18–17, along with sample mean and standard deviation, to determine an allowable range of sample element values.

The detection process is iterative:

Step 1 Use sample statistics, $\bar{X}$ and S, and K, with equation 18–16 or 18–17 to detect a single outlier.

Step 2 Delete detected outliers from the sample.

Step 3 Recompute sample statistics without the outliers.

Step 4 Begin again at step 1.

Continue the process until no outliers are detected. High and low outliers can exist in a sample data set. Two extreme values of about the same magnitude are not likely to be detected by this outlier detection procedure. In these cases, delete one value and check to see if the remaining value is an outlier. If the remaining value is an outlier, then both values should be called outliers or neither value should be called an outlier.

The detection process depends on the distribution of the data. A positive skewness indicates the possibility of high outliers, and a negative skewness indicates the possibility of low outliers. Thus, samples with a positive skew should be tested first for high outliers, and samples with negative skew should be tested first for low outliers.

If one or more outliers are detected, another frequency distribution should be considered. If a frequency distribution is found that appears to have fewer outliers, repeat the outlier detection process. If no better model is found, treat the outliers in the following order of preference:

Step 1 Reduce their weight or impact on the frequency curve.

Step 2 Eliminate the outliers from the sample.

Step 3 Retain the outliers in the sample.

When historic data are available, high outlier weighting can be reduced using appendix 6 of Water Resources Council (WRC) Bulletin #17B (1982). If such data are not available, decide whether to retain or delete the high outliers. This decision involves judgment concerning the impact of the outliers on the frequency curve and its intended use. Low outliers can be given reduced weighting by treating them as missing data as outlined in appendix 5 of WRC Bulletin #17B.

Although WRC Bulletin #17B was developed for peak flow frequency analysis, many of the methods are applicable to other types of data.

Example 18–1 illustrates the development of a log-Pearson type III distribution curve. Example 18–2 shows the development of a two-parameter gamma frequency curve.

Example 18–1 Development of log-normal and log-Pearson III frequency curves

Given: Annual maximum peak discharge data for East Fork San Juan River near Pagosa Springs, Colorado, (Station 09340000) are analyzed. Table 18–3 shows the water year (column 1) and annual maximum peak values (column 2). Other columns in the table are referenced by number in parentheses in the following steps:

Solution: ***Step 1***—Plot the data. Before plotting the data, arrange them in descending order (column 6). Compute Weibull plotting positions, based on a sample size of 44, from equation 18–9 (column 7), and then plot the data on logarithmic normal probability paper (fig. 18–1).

Step 2—Examine the trend of plotted data. The plotted data follow a single trend that is nearly a straight line, so a log-normal distribution should provide an adequate fit. The log-Pearson type III distribution is also included because it is computational, like the log normal.

Step 3—Compute the required statistics. Use common logarithms to transform the data (column 3). Compute the sample mean by using the summation of sample data logarithms and equation 18–1:

$$\bar{X} = \frac{130.1245}{44} = 2.957376$$

Compute differences between each sample logarithm and the mean logarithm. Use the sum of the squares and cubes of the differences (columns 4 and 5) in computing the standard deviation and skew. Compute the standard deviation of logarithms by using the sum of squares of the differences and the square root of equation 18–2:

$$S = \left[\frac{1.659318}{(44-1)}\right]^{0.5} = 0.1964403$$

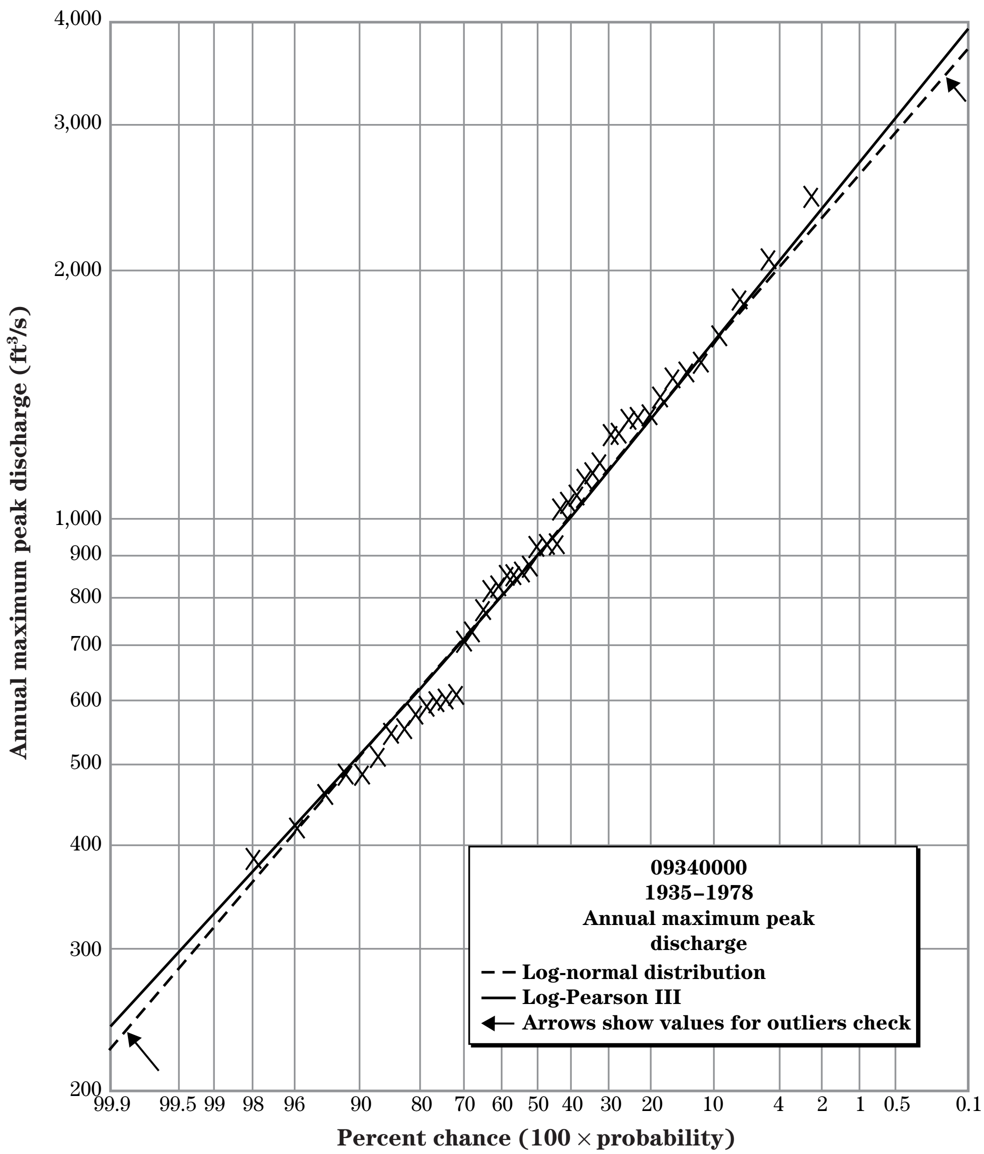
Example 18–1 Development of log-normal and log-Pearson III frequency curves—Continued

Table 18–3 Basic statistics data for example 18–1 (Station 09340000 E. Fork San Juan River near Pagosa Springs, CO; Drainage area = 86.9 mi^2, Elevation = 7,597.63 feet)

Water year (1)	Annual maximum peak (ft^3/s) (2)	X = log (peak) (3)	$(\bar{X} - X)^2$ (4)	$(\bar{X} - X)^3$ (5)	Ordered peak (ft^3/s) (6)	Weibull plot position 100 M/ (N+1) (7)
1935	1,480	3.170260	0.0453200	0.0096479	2,460	2.2
1936	931	2.968948	0.0001339	0.0000015	2,070	4.4
1937	1,120	3.049216	0.0084347	0.0007747	1,850	6.7
1938	1,670	3.222715	0.0704052	0.0186813	1,670	8.9
1939	580	2.763427	0.0376161	–0.0072956	1,550	11.1
1940	606	2.782472	0.0305914	–0.0053505	1,510	13.3
1941	2,070	3.315969	0.1285889	0.0461111	1,480	15.6
1942	1,330	3.123850	0.0277137	0.0046136	1,410	17.8
1943	830	2.919077	0.0014668	–0.0000562	1,340	20.0
1944	1,410	3.149218	0.0368034	0.0070604	1,330	22.2
1945	1,140	3.056904	0.0099059	0.0009859	1,320	24.4
1946	590	2.770850	0.0347917	–0.0064895	1,270	26.7
1947	724	2.859737	0.0095332	–0.0009308	1,270	28.9
1948	1,510	3.178975	0.0491064	0.0108819	1,170	31.1
1949	1,270	3.103803	0.0214409	0.0031395	1,140	33.3
1950	463	2.665580	0.0851447	–0.0248449	1,120	35.6
1951	709	2.850645	0.0113914	–0.0012158	1,070	37.8
1952	1,850	3.267170	0.0959725	0.0297318	1,050	40.0
1953	1,050	3.021188	0.0040720	0.0002598	1,030	42.2
1954	550	2.740361	0.0470952	–0.0102203	934	44.4
1955	557	2.745853	0.0447416	–0.0094638	931	46.7
1956	1,170	3.068185	0.0122787	0.0013606	923	48.9
1957	1,550	3.190331	0.0542680	0.0126420	880	51.1
1958	1,030	3.012836	0.0030758	0.0001706	865	53.3
1959	388	2.588830	0.1358257	–0.0500580	856	55.6
1960	865	2.937015	0.0004146	–0.0000084	856	57.8
1961	610	2.785329	0.0296001	–0.0050926	830	60.0
1962	880	2.944481	0.0001663	–0.0000021	820	62.2
1963	490	2.690195	0.0713854	–0.0190728	776	64.4
1964	820	2.913813	0.0018977	–0.0000827	724	66.7
1965	1,270	3.103803	0.0214409	0.0031395	709	68.9
1966	856	2.932472	0.0006202	–0.0000154	610	71.1
1967	1,070	3.029383	0.0051850	0.0003734	606	73.3
1968	934	2.970345	0.0001682	0.0000022	600	75.6
1969	856	2.932472	0.0006202	–0.0000154	590	77.8
1970	2,460	3.390934	0.1879728	0.0814972	580	80.0
1971	515	2.711805	0.0603047	–0.0148090	557	82.2
1972	422	2.625311	0.1102667	–0.0366157	550	84.4
1973	1,340	3.127104	0.0288077	0.0048895	515	86.7
1974	490	2.690195	0.0713854	–0.0190728	490	88.9
1975	1,320	3.120572	0.0266331	0.0043464	490	91.1
1976	923	2.965200	0.0000612	0.0000005	463	93.3
1977	600	2.778150	0.0321219	–0.0057571	422	95.6
1978	776	2.889860	0.0045583	–0.0003078	388	97.8
Summation		130.1245	1.659318	0.0235340		

Example 18–1 Development of log-normal and log-Pearson III frequency curves—Continued

Figure 18–1 Data and frequency curves for example 18–1



Example 18–1 Development of log-normal and log-Pearson III frequency curves—Continued

Compute the skew by using the sum of cubes of the differences (column 5) and equation 18–5:

$$G = \frac{44}{(44-1)(44-2)(0.1964403)^3} \times 0.023534 = 0.0756$$

For ease of use in the next step, round skew value to the nearest tenth (G = 0.1).

Step 4—Verify selection of distributions. Use exhibit 18–3 to obtain K_p values for required skew at sufficient exceedance probabilities to define the frequency curve. Use the mean, standard deviation, skew, and equation 18–17 to compute discharges at the selected exceedance probabilities. Exhibit 18–3 K_p values and discharge computations are shown in table 18–4. Plot the frequency curves on the same graph as the sample data (fig. 18–1). A comparison between the plotted frequency curve and the sample data verifies the selection of the distributions. Other distributions can be tested the same way.

Table 18–4 Frequency curve solutions for example 18–1

Exceed. prob. (q)	Exhibit 18–3 K_p value (G = 0.0)	Log Q= $\bar{X} + K_pS$	Log-normal discharges (ft^3/s)	Exhibit 18–3 K_p value (G = 0.1)	Log Q = $\bar{X} + K_pS$	Log-Pearson III discharges (ft^3/s)
0.999	–3.09023	2.35033	224	–2.94834	2.37820	239
0.998	–2.87816	2.39199	247	–2.75706	2.41578	260
0.995	–2.57583	2.45138	283	–2.48187	2.46984	295
0.990	–2.32635	2.50039	317	–2.25258	2.51488	327
0.980	–2.05375	2.55394	358	–1.99973	2.56455	367
0.960	–1.75069	2.61347	411	–1.71580	2.62032	417
0.900	–1.28155	2.70563	508	–1.27037	2.70782	510
0.800	–0.84162	2.79205	620	–0.84611	2.79117	618
0.700	–0.52440	2.85436	715	–0.53624	2.85204	711
0.600	–0.25335	2.90761	808	–0.26882	2.90457	803
0.500	0.00000	2.95738	907	-0.01662	2.95411	900
0.400	0.25335	3.00714	1,017	0.23763	3.00406	1,009
0.300	0.52440	3.06039	1,149	0.51207	3.05797	1,143
0.200	0.84162	3.12270	1,326	0.83639	3.12168	1,323
0.100	1.28155	3.20912	1,619	1.29178	3.21113	1,626
0.040	1.75069	3.30128	2,001	1.78462	3.30795	2,032
0.020	2.05375	3.36082	2,295	2.10697	3.37127	2,351
0.010	2.32635	3.41436	2,596	2.39961	3.42876	2,684
0.005	2.57583	3.46337	2,907	2.66965	3.48180	3,033
0.002	2.87816	3.52276	3,332	2.99978	3.54665	3,521
0.001	3.09023	3.56442	3,668	3.23322	3.59251	3,913

Example 18–1 Development of log-normal and log-Pearson III frequency curves—Continued

Step 5—Check the sample for outliers. K_n values, based on sample size, are obtained from exhibit 18–1. The K_n value for a sample of 44 is 2.945. Compute the log-normal high outlier criteria from the mean, the standard deviation, the outlier K_n value, and equation 18–17 (using K_n instead of K_p):

$$\begin{aligned}\log Q_{HI} &= 2.957376 + (2.945)(0.1964403)\\ &= 3.5359\\ Q_{HI} &= 3,435 \text{ ft}^3/\text{s}\end{aligned}$$

Use the negative of the outlier K_n value in equation 18–17 to compute the low outlier criteria:

$$\begin{aligned}\log Q_{LO} &= 2.957376 + (-2.945)(0.1964403)\\ &= 2.37886\\ Q_{LO} &= 239 \text{ ft}^3/\text{s}\end{aligned}$$

Because all of the sample data used in example 18–1 are between Q_{HI} and Q_{LO}, there are no outliers for the log-normal distribution.

High and low outlier criteria values for skewed distributions can be found by use of the high and low probability levels from exhibit 18–1. Read discharge values from the plotted log-Pearson III frequency curve at the probability levels listed for the sample size (in this case, 44). The high and low outlier criteria values are 3,700 and 250 cubic feet per second. Because all sample data are between these values, there are no outliers for the log-Pearson III distribution.

Example 18–2 Development of a two-parameter gamma frequency curve

Given: Table 18–5 contains 7-day mean low flow data for the Patapsco River at Hollifield, Maryland, (Station 01589000) including the water year (column 1) and 7-day mean low flow values (column 2). The remaining columns are referenced in the following steps.

Solution: ***Step 1***—Plot the data. Before plotting, arrange the data in ascending order (column 3). Weibull plotting positions are computed based on the sample size of 34 from equation 18–9 (column 4). Ordered data are plotted at the computed plotting positions on logarithmic-normal probability paper (fig. 18–2).

Step 2—Examine the trends of the plotted data. The data plot as a single trend with a slightly concave downward shape.

Step 3—Compute the required statistics. Compute the gamma shape parameter, γ, from the sample data (column 3), equations 18–1, 18–10, and 18–11, and either equation 18–12 or 18–13.

$$\bar{X} = \frac{1876}{34} = 55.17647$$

$$G_m = \left(3.308266 \times 10^{55}\right)^{\frac{1}{34}} = 42.94666$$

$$R = \ln\left[\frac{55.17647}{42.9466}\right] = 0.25058$$

Because R <0.5772 use equation 18–12 to compute γ.

$$\gamma = \left(\frac{1}{0.25058}\right)\left\{0.5000876 + (0.1648852)(0.25058) - (0.0544274)(0.25058)^2\right\}$$
$$\gamma = 2.14697$$

Using the mean and γ, compute the standard deviation and skew from equations 18–14 and 18–15:

$$S = \frac{55.17647}{\sqrt{2.14697}} = 37.65658$$

$$G = \frac{2}{\sqrt{2.14697}} = 1.36495$$

For ease of use in next step, round skew value to the nearest tenth (G = 1.4).

Step 4—Compute the frequency curve. Use exhibit 18–3 to obtain K_p values for the required skew at sufficient probability levels to define the frequency curve. Compute discharges at the selected probability levels (p) by equation 18–16. Exhibit 18–3 K_p values and computed discharges are shown in table 18–6. Then plot the frequency curve on the same graph as the sample data (fig. 18–2). Compare the plotted data and the frequency curve to verify the selection of the two-parameter gamma distribution.

Example 18–2 Development of a two-parameter gamma frequency curve—Continued

Table 18–5 Basic statistics data for example 18–2

Water year (1)	7-Day mean low flow (ft^3/s) (2)	Ordered data (ft^3/s) (3)	Weibull plot position 100 M/(N + 1) (4)
1946	107	11	2.9
1947	127	15	5.7
1948	79	16	8.6
1949	145	17	11.4
1950	110	19	14.3
1951	98	20	17.1
1952	99	22	20.0
1953	168	23	22.9
1954	60	23	25.7
1955	20	25	28.6
1956	23	25	31.4
1957	51	25	34.3
1958	17	27	37.1
1959	52	32	40.0
1960	25	40	42.9
1961	43	43	45.7
1962	27	44	48.6
1963	16	47	51.4
1964	11	48	54.3
1965	19	50	57.1
1966	22	51	60.0
1967	15	52	62.9
1968	47	59	65.7
1969	32	60	68.6
1970	25	69	71.4
1971	25	79	74.3
1972	59	80	77.1
1973	69	98	80.0
1974	50	99	82.9
1975	44	107	85.7
1976	80	110	88.6
1977	40	127	91.4
1978	23	145	94.3
1979	48	168	97.1
Sum		**1,876**	
Product		**3.308266×10^{55}**	

Table 18–6 Solution of frequency curve for example 18–2

Prob. (p)	Exhibit 18–3 K_p value (G = 1.4)	$Q = \bar{X} + K_pS$
0.999	5.09505	247.0
0.998	4.55304	227.0
0.995	3.82798	199.0
0.990	3.27134	178.0
0.980	2.70556	157.0
0.960	2.12768	135.0
0.900	1.33665	106.0
0.800	0.70512	82.0
0.700	0.31307	67.0
0.600	0.01824	56.0
0.500	– 0.22535	47.0
0.400	– 0.43949	39.0
0.300	– 0.63779	31.0
0.200	– 0.83223	24.0
0.100	– 1.04144	16.0
0.040	– 1.19842	10.0
0.020	– 1.26999	7.4
0.010	– 1.31815	5.5
0.005	– 1.35114	4.3
0.002	– 1.37981	3.2
0.001	– 1.39408	2.7

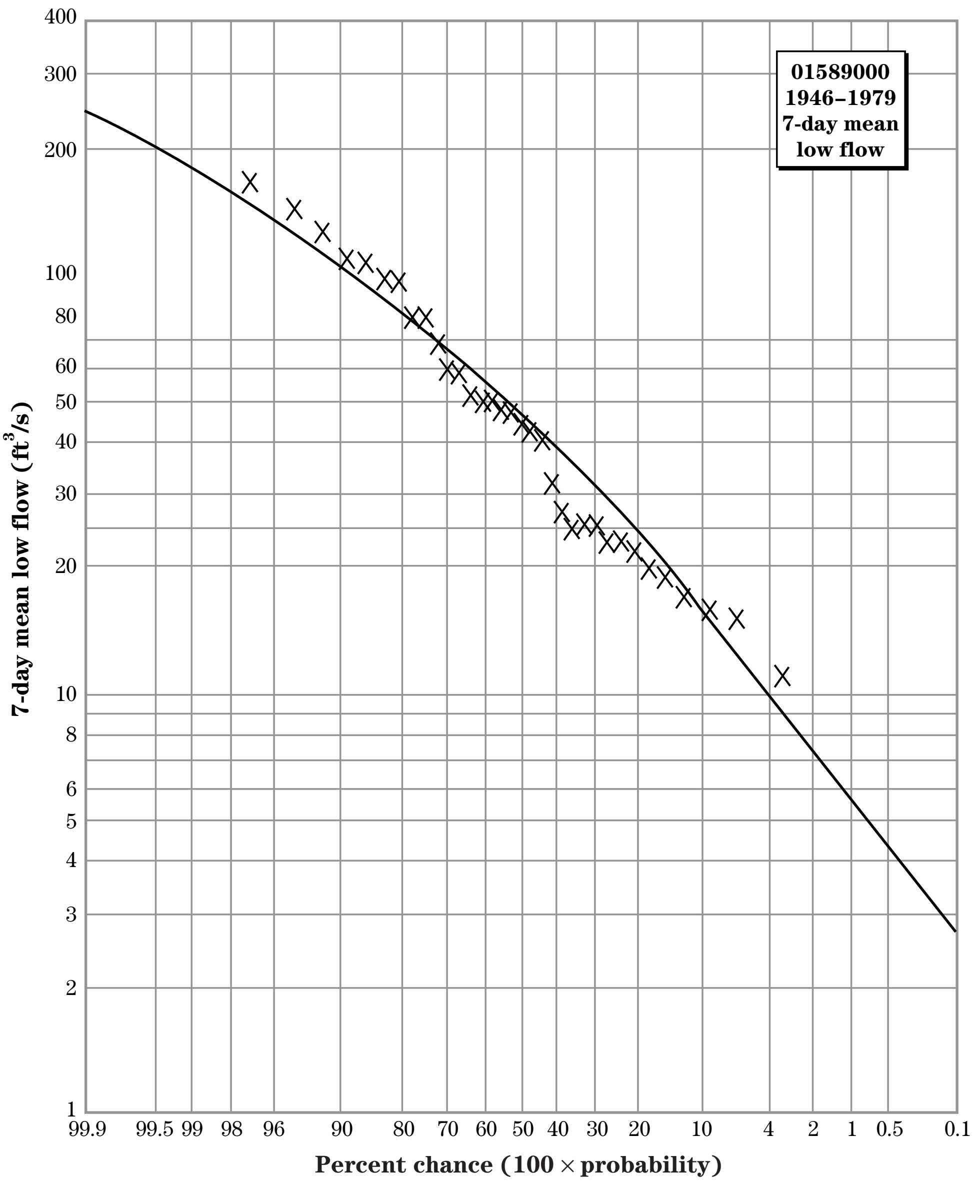
Example 18–2 Development of a two-parameter gamma frequency curve—Continued

Step 5**—**Check the sample for outliers. Obtain outlier probability levels from exhibit 18–1 for a sample size of 34. The probability levels are 0.9977863 and 0.0022137. From figure 18–2, read the discharge rates associated with these probability levels. The outlier criteria values are 220 and 3.3 cubic feet per second. Because all sample data are between these values, there are no outliers.

Step 6**—**Estimate discharges. Use the frequency curve to estimate discharges at desired probability levels.

Example 18–2 Development of a two-parameter gamma frequency curve—Continued

Figure 18–2 Data and frequency curve for example 18–2



(2) Mixed distributions

A mixed distribution occurs when at least two events in the population result from different causes. In flow frequency analysis, a sample of annual peak discharges at a given site can be drawn from a single distribution or mixture of distributions. A mixture occurs when the series of peak discharges are caused by various types of runoff-producing events, such as generalized rainfall, local thunderstorms, hurricanes, snowmelt, or any combination of these.

Previously discussed frequency analysis techniques may be valid for mixed distributions. If the mixture is caused by a single or small group of values, these values may appear as outliers. After these values are identified as outliers, the sample can then be analyzed. However, if the number of values departing from the trend of the data becomes significant, a second trend may be evident. Two or more trends may be evident when the data are plotted on probability paper.

Populations with multiple trends cause problems in analysis. The skewness of the entire sample is greater than the skewness of samples that are separated by cause. The larger skewness causes the computed frequency curve to differ from the sample data plot in the region common to both trends.

The two methods that can be used to develop a mixed distribution frequency curve are illustrated in example 18–3. The preferred method (method 1) involves separating the sample data by cause, analyzing the separated data, and combining the frequency curves. The detailed procedure is as follows:

Step 1 Determine the cause for each annual event. If a specific cause cannot be found for each event, method 1 cannot be used.

Step 2 Separate the data into individual series for each cause in step 1. Some events may be common to more than one series and, therefore, belong to more than one series. For example, snowmelt and generalized rainfall could form an event that would belong to both series.

Step 3 Collect the necessary data to form an annual series for each cause. Some series will not have an event for each year. An example of this is a hurricane series in an area where hurricanes occur about once every 10 years. If insufficient data for any series are a problem, then the method needs a truncated series with conditional probability adjustment. See appendix 5 of WRC Bulletin #17B.

Step 4 Compute the statistics and frequency curve for each annual series separately.

Step 5 Use the addition rule of probability to combine the computed frequency curves.

$$P\{A \cup B\} = P\{A\} + P\{B\} - \left[P\{A\} \times P\{B\}\right] \qquad (18\text{–}18)$$

where:

$P\{A \cup B\}$ = probability of an event of given magnitude occurring from either or both series

$P\{A\}$ and $P\{B\}$ = probabilities of an event of given magnitude occurring from each series

$P\{A\} \times P\{B\}$ = probability of an event from each series occurring in a single year

An alternative method (method 2) that requires only the sample data may be useful in estimating the frequency curve for $q \leq 0.5$. This method is less reliable than method 1 and requires that at least the upper half of the data be generally normal or log-normal if log-transformed data are used. A straight line is fitted to at least the upper half of the frequency range of the series. The standard deviation and mean are developed by use of the expected values of normal order statistics. The equations are:

$$S = \left[\frac{\left(\sum_{i=1}^{N} X_i^2\right) - \frac{\left(\sum_{i=1}^{N} X_i\right)^2}{N}}{\left(\sum_{i=1}^{N} K_i^2\right) - \frac{\left(\sum_{i=1}^{n} K_i\right)^2}{N}} \right]^{0.5} \qquad (18\text{–}19)$$

$$\bar{X} = \frac{1}{N}\left[\left(\sum_{i=1}^{N} X_i\right) - S\left(\sum_{i=1}^{N} K_i\right)\right] \qquad (18\text{–}20)$$

where:

N = number of elements in the truncated series

K_i = expected value of normal order statistics (K_n) for the i^{th} element of the complete sample

Expected values of normal order statistics are shown in exhibit 18–2 at the back of this chapter.

(3) Incomplete record and zero flow years

An *incomplete record* refers to a sample in which some data are missing either because they were too low or too high to record or because the measuring device was out of operation. In most instances, the agency collecting the data provides estimates for missing high flows. When the missing high values are estimated by someone other than the collecting agency, it should be documented and the data collection agency advised. Most agencies do not routinely provide estimates of low flow values. The procedure that accounts for missing low values is a conditional probability adjustment explained in appendix 5 of WRC Bulletin #17B.

Data sets containing zero values present a problem when one uses logarithmic transformations. The logarithm of zero is undefined and cannot be included. When a logarithmic transformation is desired, zeros should be treated as missing low data.

(4) Historic data

At many locations, information is available about major hydrologic occurrences either before or after the period of systematic data collection. Such information, called *historic data*, can be used to adjust the frequency curve. The historic data define an extended time period during which rare events, either recorded or historic, have occurred. Historic data may be obtained from other agencies, from newspapers, or by interviews. A procedure for incorporating historic data into the frequency analysis is in appendix 6 of WRC Bulletin #17B.

(5) Frequency analysis reliability

(The information in this section originally appeared in U.S. Corps of Engineers, Hydrologic Engineering Methods, Volume 3, Hydrologic Frequency Analysis (1975). This information concisely covers the main points of frequency reliability, including examples based on flood frequencies.)

The reliability of frequency estimates is influenced by:

- amount of information available
- variability of the events
- accuracy with which the data were measured

Amount of information available—Generally, errors of estimate are inversely proportional to the square root of the number of independent items in the frequency array. Therefore, errors of estimates based on 40 years of record would normally be half as large as errors of estimates based on 10 years of record, other conditions being the same.

Variability of events—The variability of events in a record is generally the most important factor affecting the reliability of frequency estimates. For example, the ratio of the largest to the smallest annual flood of record on the Mississippi River at Red River Landing, Louisiana, is about 2.7, whereas the ratio of the largest to the smallest annual flood of record on the Kings River at Piedra, California, is about 100, or 35 times as great. Statistical studies show that as a consequence of this factor, a flow corresponding to a given frequency can be estimated within 10 percent on the Mississippi River, but can be estimated only within 40 percent on the Kings River.

Accuracy with which the data were measured—The accuracy of data measurement normally has a relatively small influence on the reliability of a frequency estimate. This is true because such errors ordinarily are not systematic and tend to cancel and because the influence of chance events is great in comparison with that of measurement errors. For this reason, it is usually better to include an estimated magnitude for a major flood; for example, one that was not recorded because of gage failure, rather than to omit it from the frequency array even though its magnitude can only be estimated approximately. However, it is advisable always to use the most reliable sources of data and, in particular, to guard against systematic errors that result from using an unreliable rating curve.

The possible errors in estimating flood frequencies are very large principally because of the chance of having a nonrepresentative sample. Sometimes the occurrence of one or two abnormal floods can change the apparent exceedance frequency of a given magnitude from once in 1,000 years to once in 200 years. Nevertheless, the frequency curve technique is considerably better than any other tool available for some purposes and represents a substantial improvement over using an array restricted to observed flows only.

(6) Effects of watershed modification

The analysis of streamflow data is complicated by the fact that watershed conditions are rarely constant during the period of record. Fire, floods, changing land use, channel modification, reservoir construction, and land treatment all contribute to changes in the hydrologic responses of a watershed. If the changes are significant, then standard statistical procedures cannot be used to develop the frequency curve.

(f) Frequency analysis procedures

Obtain site information, historic data, and systematic data:

- Examine record period for changes in physical conditions. Use only data that are from periods of constant physical conditions (homogeneous).
- Estimate missing high data. The effort expended in estimating data depends on the use of the final frequency analysis.
- Obtain historic information.

Plot sample data:

- Use normal (logarithmic normal) probability paper.
- Observe general trend of plotted data.
- Select distribution:
  - — For single-trend data, select the distribution that best defines the population from which the sample is drawn.
  - — For multiple-trend data, use one of the mixed distribution techniques.

Compute frequency curve:

- The procedure in WRC Bulletin #17B should be used.
- Use station statistics including skew and distribution tables (such as exhibit 18–3). There are many computer programs available including USGS program PeakFQ.
- Plot curve on the paper with sample data.
- Compare general shape of curve with sample data. If the computed curve does not fit the data, check for outliers or for another distribution that may fit the population.

Detect outliers:

- Check for outliers according to the value of skewness, high first for positive skewness and low first for negative skewness.
- Delete outliers and recompute sample statistics.
- Continue the process until no outliers remain in sample.

Treat outliers and missing, low, and zero data.

- Check another frequency distribution model.
- For high outliers:
  - — If historical data are available, use appendix 6 of WRC Bulletin #17B.
  - — If historic data are not available, decide whether outliers should be retained in the sample.
- For low outliers and missing, low, and zero data, use appendix 5 of WRC Bulletin #17B.

Check reliability of results:

- Frequency curve estimates are based on prior experience and should be used with caution.
- Uncertainty of estimates increases as estimated values depart from the mean.

Example 18–3 Development of a mixed distribution frequency curve by separating the data by cause and by using at least the upper half of the data

Method 1 The Causative Factor Method

Given: Annual maximum peak discharge data for Carson River near Carson City, Nevada, (Station 10311000) are given in table 18–7. Column 1 contains the water year, and column 2 contains annual maximum peak discharge. The Weibull plotting position in column 4 of table 18–7 (100 M/(N+1)) is expressed in percent. The other columns will be referenced in the following steps:

Table 18–7 Annual maximum peak discharge data for example 18–3

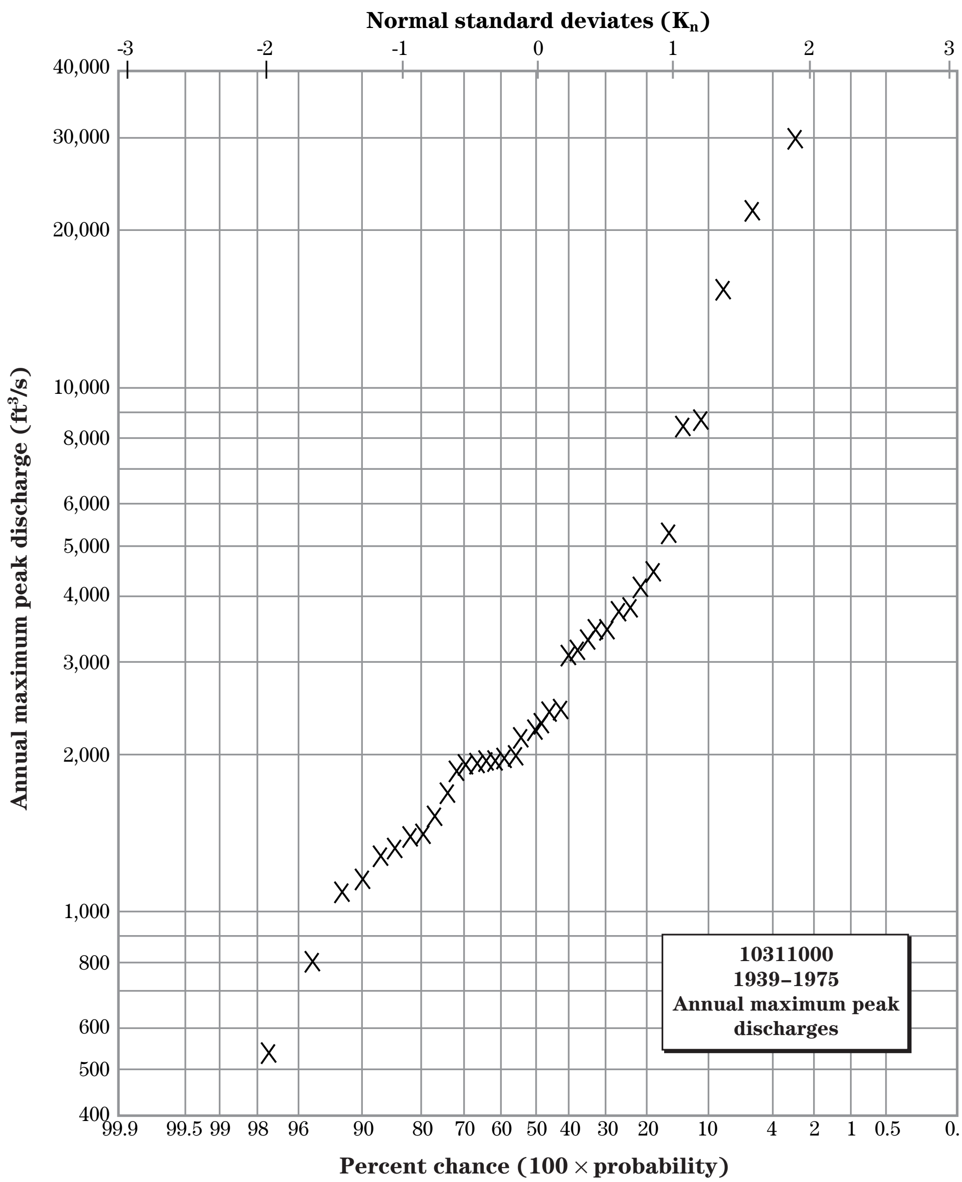
Water year (1)	Annual maximum peak discharge (ft^3/s) (2)	Ordered annual maximum peaks (ft^3/s) (3)	Weibull plotting position 100 M/(N + 1) (4)	Water year (1)	Annual maximum peak discharge (ft^3/s) (2)	Ordered annual maximum peaks (ft^3/s) (3)	Weibull plotting position 100 M/(N + 1) (4)
1939	541	30,000	2.6	1958	3,100	2,160	52.6
1940	2,300	21,900	5.3	1959	1,690	1,990	55.3
1941	2,430	15,500	7.9	1960	1,100	1,970	57.9
1942	5,300	8,740	10.5	1961	808	1,950	60.5
1943	8,500	8,500	13.2	1962	1,950	1,950	63.2
1944	1,530	5,300	15.8	1963	21,900	1,930	65.8
1945	3,860	4,430	18.4	1964	1,160	1,900	68.4
1946	1,930	4,190	21.1	1965	8,740	1,870	71.1
1947	1,950	3,860	23.7	1966	1,280	1,690	73.7
1948	1,870	3,750	26.3	1967	4,430	1,530	76.3
1949	2,420	3,480	28.9	1968	1,390	1,410	78.9
1950	2,160	3,480	31.6	1969	4,190	1,390	81.6
1951	15,500	3,330	34.2	1970	3,480	1,330	84.2
1952	3,750	3,180	36.8	1971	2,260	1,280	86.8
1953	1,990	3,100	39.5	1972	1,330	1,160	89.5
1954	1,970	2,430	42.1	1973	3,330	1,100	92.1
1955	1,410	2,420	44.7	1974	3,180	808	94.7
1956	30,000	2,300	47.4	1975	3,480	541	97.4
1957	1,900	2,260	50.0				

Example 18–3 Development of a mixed distribution frequency curve by separating the data by cause and by using at least the upper half of the data—Continued

Procedure: ***Step 1***—Plot the data. Before plotting, order the data in table 18–7 from high to low (column 3). Compute plotting positions using sample size of 37 and equation 18–9 (column 4). Then plot ordered data at the computed plotting positions on logarithmic-normal probability paper (fig. 18–3).

Step 2—Examine the plotted data. The data plot in an S-shape with a major trend break at 20 percent chance.

Figure 18–3 Annual maximum peak discharge data for example 18–3



Example 18–3 Development of a mixed distribution frequency curve by separating the data by cause and by using at least the upper half of the data—Continued

Step 3—Determine what caused the maximum peak discharge. Based on streamgage and weather records, two causative factors were rainfall and snowmelt. Annual maximum peak discharge series for each cause are tabulated in table 18–8.

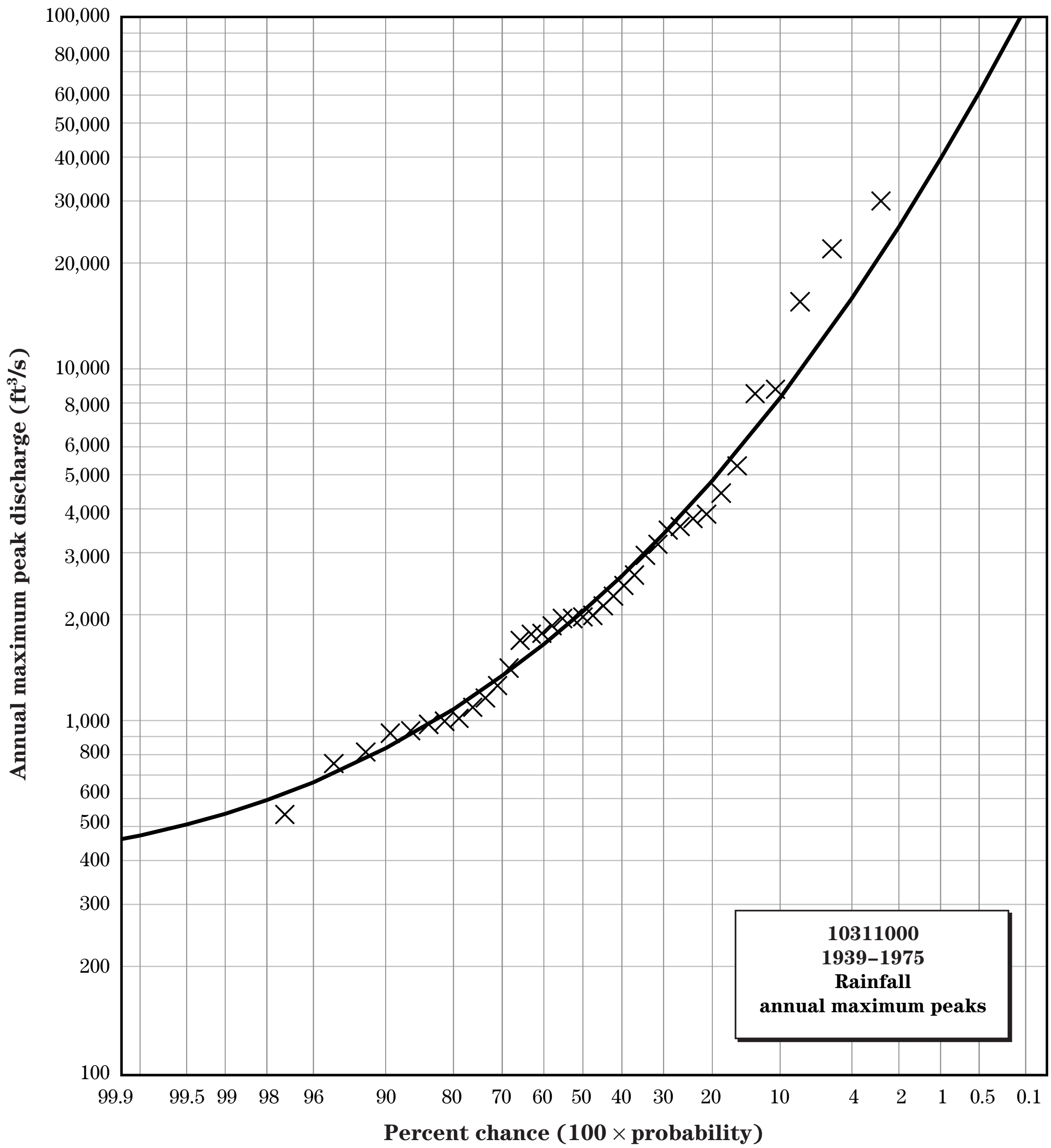
Step 4—Plot each annual maximum series. As in step 1, arrange the data in descending order (rainfall, column 4; snowmelt, column 5) and compute plotting positions (column 6). Rainfall data are plotted in figure 18–4, and snowmelt data are plotted in figure 18–5.

Table 18–8 Annual maximum rainfall/snowmelt peak discharge for example 18–3

Water year (1)	Annual maximum rainfall peak discharge (ft^3/s) (2)	Annual maximum snowmelt peak discharge (ft^3/s) (3)	Ordered rainfall maximum peak discharge (ft^3/s) (4)	Ordered snowmelt maximum peak discharge (ft^3/s) (5)	Weibull plot position 100 M/(N + 1) (6)
1939	541	355	30,000	4,290	2.6
1940	1,770	2,300	21,900	4,190	5.3
1941	1,015	2,434	15,500	3,480	7.9
1942	5,298	2,536	8,740	3,330	10.5
1943	8,500	2,340	8,500	3,220	13.2
1944	995	1,530	5,298	3,100	15.8
1945	3,860	1,420	4,430	2,980	18.4
1946	1,257	1,930	3,860	2,759	21.1
1947	1,950	1,680	3,750	2,536	23.7
1948	755	1,870	3,560	2,460	26.3
1949	2,420	1,680	3,480	2,434	28.9
1950	1,760	2,158	3,172	2,417	31.6
1951	15,500	1,750	2,946	2,340	34.2
1952	3,750	2,980	2,590	2,300	36.8
1953	1,990	972	2,420	2,158	39.5
1954	1,970	1,640	2,260	2,010	42.1
1955	1,410	1,360	2,120	1,930	44.7
1956	30,000	3,220	1,990	1,900	47.4
1957	1,860	1,900	1,970	1,900	50.0
1958	2,120	3,100	1,950	1,870	52.6
1959	1,690	698	1,950	1,750	55.3
1960	1,090	895	1,860	1,680	57.9
1961	814	620	1,770	1,680	60.5
1962	1,950	1,900	1,760	1,640	63.2
1963	21,900	2,417	1,690	1,530	65.8
1964	1,160	800	1,410	1,420	68.4
1965	8,740	2,460	1,257	1,360	71.1
1966	920	1,280	1,160	1,360	73.7
1967	4,430	4,290	1,090	1,309	76.3
1968	936	1,360	1,015	1,280	78.9
1969	3,560	4,190	995	972	81.6
1970	3,480	2,010	975	895	84.2
1971	2,260	837	936	837	86.8
1972	975	1,309	920	800	89.5
1973	2,946	3,330	814	698	92.1
1974	3,172	2,759	755	620	94.7
1975	2,590	3,480	541	355	97.4

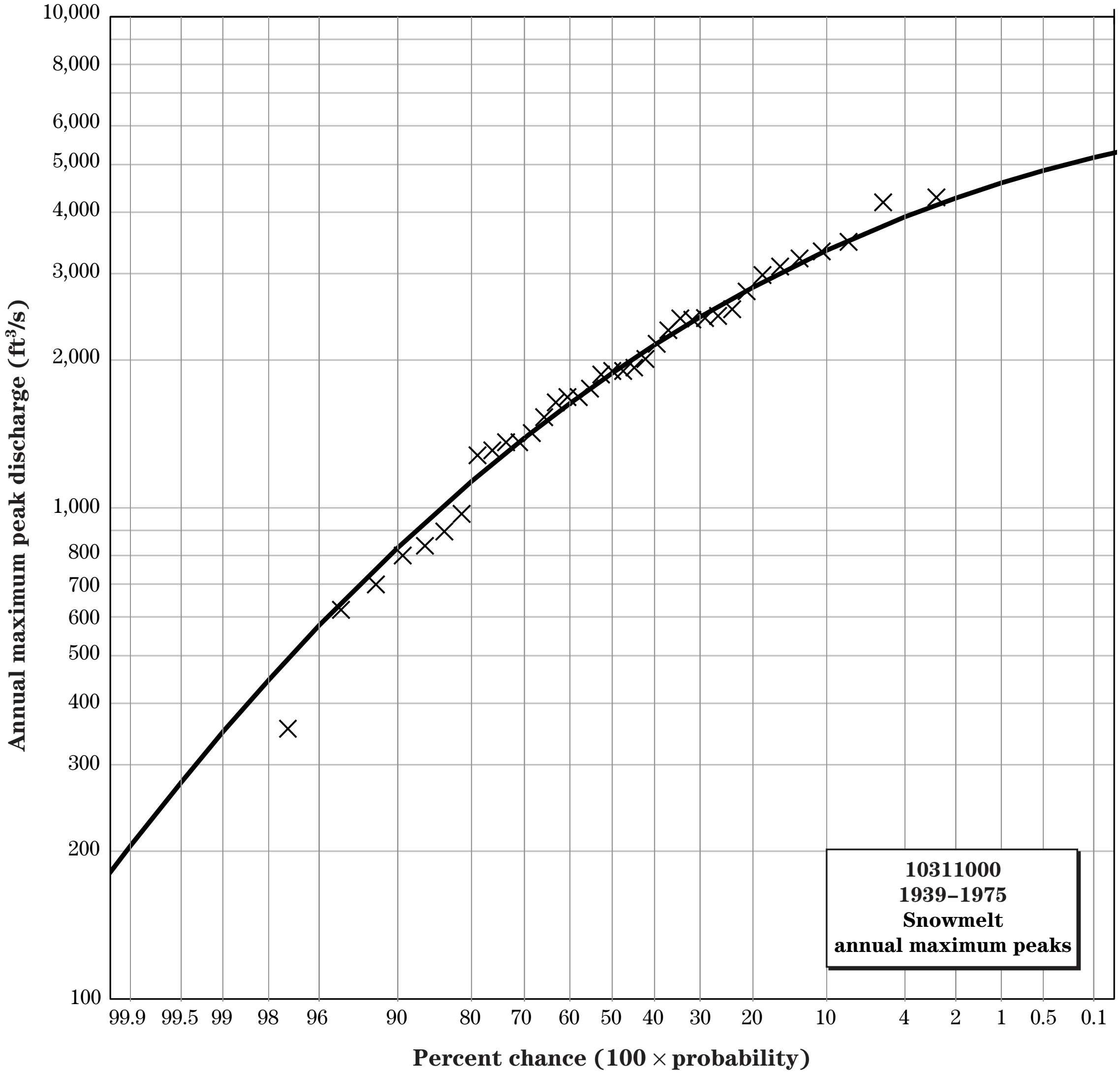
Example 18–3 Development of a mixed distribution frequency curve by separating the data by cause and by using at least the upper half of the data—Continued

Figure 18–4 Data and frequency curve for rainfall annual maximum peaks in example 18–3



Example 18–3 Development of a mixed distribution frequency curve by separating the data by cause and by using at least the upper half of the data—Continued

Figure 18–5 Data and frequency curve for snowmelt annual maximum peaks in example 18–3



Example 18–3 Development of a mixed distribution frequency curve by separating the data by cause and by using at least the upper half of the data—Continued

Step 5—Compute the required statistics. Using the procedure in step 3 of example 18–1, compute the sample mean, standard deviation, and skewness for each series. The results of these computations follow:

Series	$\bar{X}$	S	G	Use G
Rainfall	3.37611	0.40385	1.03	1.0
Snowmelt	3.24241	0.24176	–0.77	–0.8

Step 6—Compute the log-Pearson III frequency curve for each series. The frequency curve solution for each series, as computed in step 4 of example 18–1, is listed in table 18–9. Log-Pearson frequency curves are plotted for the rainfall and the snowmelt series in figures 18–4 and 18–5, respectively.

Table 18–9 Frequency curve solutions for example 18–3

Exceed prob. (q)	Rainfall: Exhibit 18–3 K_p value (G = 1.0)	Rainfall: Log Q = $\bar{X} + K_pS$	Rainfall: Log-Pearson III discharges (ft^3/s)	Snowmelt: Exhibit 18–3 K_p value (G = – 0.8)	Snowmelt: Log Q = $\bar{X} + K_pS$	Snowmelt: Log-Pearson III discharges (ft^3/s)
0.999	– 1.78572	2.65495	452	– 4.24439	2.21629	165
.998	– 1.74062	2.67316	471	– 3.84981	2.31168	205
.995	– 1.66390	2.70414	506	– 3.31243	2.44160	276
.99	– 1.53838	2.73464	543	– 2.89101	2.54348	350
.98	– 1.49188	2.77361	594	– 2.45298	2.64938	446
.96	– 1.36584	2.82452	668	– 1.99311	2.76056	576
.90	– 1.12762	2.92072	833	– 1.33640	2.91932	830
.80	– 0.85161	3.03219	1,077	– 0.77986	3.05387	1,132
.70	– 0.61815	3.12816	1,343	– 0.41309	3.14254	1,388
.60	– 0.39434	3.21686	1,648	– 0.12199	3.21292	1,633
.50	– 0.16397	3.30989	2,041	0.13199	3.27432	1,881
.40	0.08763	3.41150	2,579	0.36889	3.33159	2,146
.30	0.38111	3.53002	3,389	0.60412	3.38846	2,446
.20	0.75752	3.68203	4,809	0.85607	3.44937	2,814
.10	1.34039	3.91743	8,268	1.16574	3.52424	3,344
.04	2.04269	4.20105	15,887	1.44813	3.59251	3,913
.02	2.54206	4.40272	25,277	1.60604	3.63069	4,273
.01	3.02256	4.59677	39,516	1.73271	3.66131	4,585
.005	3.48874	4.78504	60,959	1.83660	3.68643	4,858
.002	4.08802	5.02706	106,428	1.94806	3.71337	5,169
.001	4.53112	5.20600	160,695	2.01739	3.73013	5,372

Example 18–3 Development of a mixed distribution frequency curve by separating the data by cause and by using at least the upper half of the data—Continued

Step 7—Check each sample for outliers. Read high and low outlier criterion values from the frequency curve plots (figures 18–4 and 18–5) at the probability levels given in exhibit 18–1 for the sample size of 37. The high and low probability levels from exhibit 18–1 are 0.9980116 (99.8 percent) and 0.0019884 (0.2 percent). Outlier criterion values read from the plots are:

Series (ft^3/s)	High outlier criterion (ft^3/s)	Low outlier criterion (ft^3/s)
Rainfall	106,000	470
Snowmelt	5,200	200

All of the rainfall and snowmelt data are between the outlier criterion values, so there are no outliers.

Step 8—Combine the rainfall and snowmelt series frequency curves. For selected discharge values, read the rainfall and snowmelt frequency curve probability levels from figures 18–4 and 18–5. Using equation 18–18, combine the probabilities for the two series. Table 18–10 contains the individual and combined probabilities of selected discharges. The snowmelt frequency curve approaches an upper bound of 5,400 cubic feet per second; therefore, only the rainfall curve is used above this value.

Step 9—Plot the combined series frequency curve. Figure 18–6 shows the combined and annual frequency curves plotted on the same sheet as the annual series. The combined series frequency curve will not necessarily fit the annual series, as additional data were used to develop it, but the curve does represent the combined effect of the two causes.

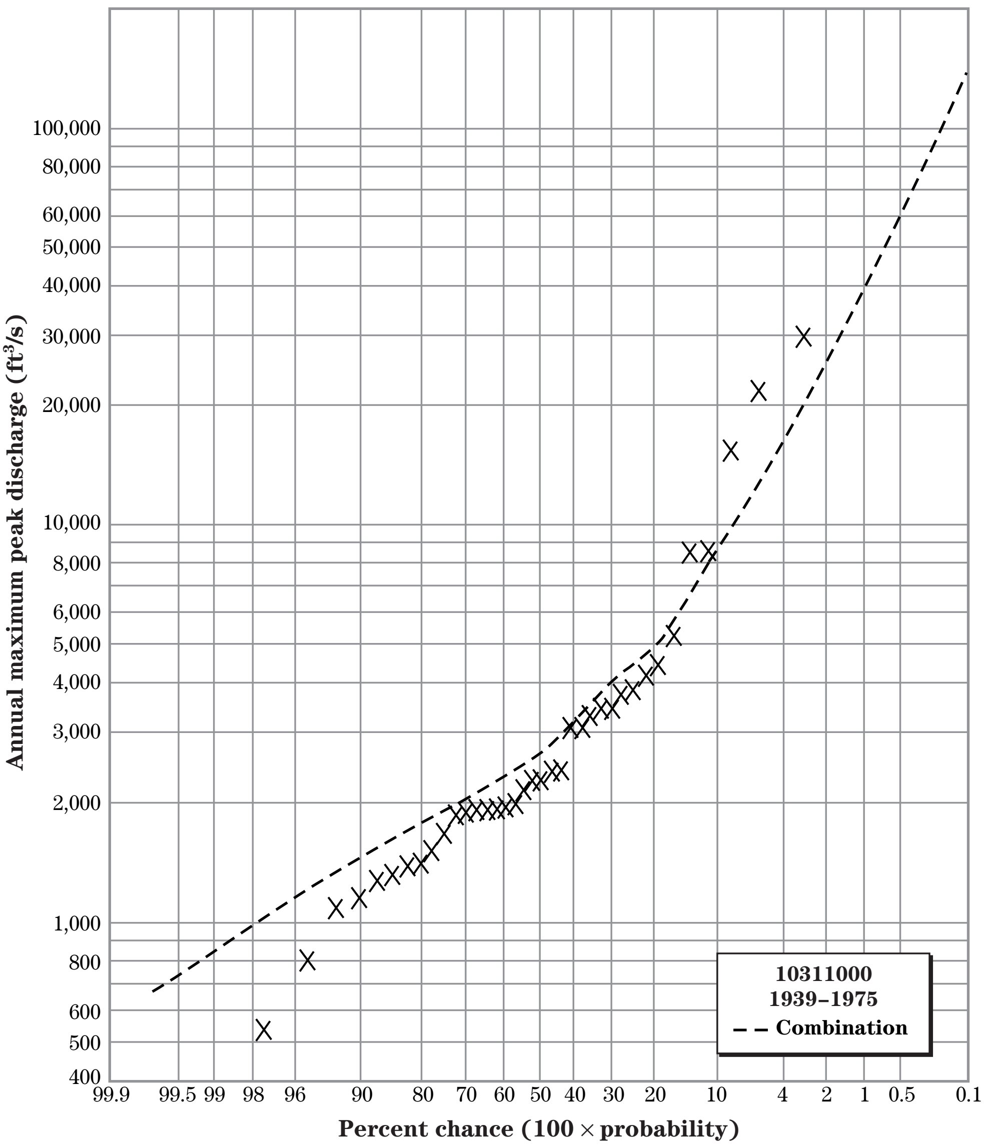
Table 18–10 Combination of frequency curves for example 18–3

Peak discharge (ft^3/s) (1)	$P_R = P$ (rain) (2)	$P_S = P$ (snow) (3)	$P = P_R + P_S - P_R P_S$ (4)
600	0.98	0.955	0.999
830	.90	.90	.990
1,640	.60	.60	.840
2,450	.42	.30	.594
3,360	.30	.10	.370
4,840	.20	.005	.204
8,268	.10	– 1/	.100
15,887	.04	–	.040
39,516	.01	–	.010
160,695	.001	–	.001

1/ Probability is too small to be considered.

Example 18–3 Development of a mixed distribution frequency curve by separating the data by cause and by using at least the upper half of the data—Continued

Figure 18–6 Combination of annual maximum rainfall and annual maximum snowmelt frequency curves for example 18–3



Note 1: Combination curve is plot of columns 1 and 4 of table 18–10.
Note 2: X's represent plot of columns 1 and 4 of table 18–7.

Example 18–3 Development of a mixed distribution frequency curve by separating the data by cause and by using at least the upper half of the data—Continued

Method 2—Truncated Series

An alternative method of mixed distribution analysis is to fit a log-normal distribution to only part of the data. At least the upper half of the data must be included and must be basically log-normal (i.e., approximate a straight line when plotted on logarithmic-normal paper). Steps 1 and 2 of method 1 help to determine that the data are mixed and that the major trend break occurs at 20 percent. While the upper half of the data include data from both major trends, a log-normal fit is used as an illustration of the procedure.

Procedure: ***Steps 1 and 2***—See method 1.

Step 3—Select K_n values. Select the normal K_n values for a sample size of 37 from exhibit 18–2. A tabulation of these values along with the ordered annual maximum peaks and their logarithms is in table 18–11.

Step 4—Plot the ordered annual maximum peaks at the normal K_n values tabulated in table 18–11. These are plotted in figure 18–7. For plotting the data, use the normal K_n-value scale at the top of the figure.

Step 5—Compute the statistics based on the upper half of data. Use equations 18–19 and 18–20 to compute the standard deviation and mean from the sums given in table 18–11.

$$S = \left[\frac{260.757 - \frac{(70.11699)^2}{19}}{17.25002 - \frac{(14.44423)^2}{19}} \right]^{0.5} = 0.56475$$

$$\bar{X} = \frac{\left[(70.11699) - (0.56475)(14.44423)\right]}{19} = 3.26103$$

Step 6—Compute the log-normal frequency curve for the data. Use the same procedure as explained in step 4 of example 18–1. As a log-normal curve is to be fit, it will be a straight line on logarithmic-normal paper, and solution of only two points is required.

Exceedance probability level	**K_n normal**	**Log Q = $\bar{X} + K_n S$**	**Q**
0.50	0.0	3.26103	1,824
0.01	2.32635	4.57484	37,570

Step 7—Plot the computed frequency curve. The curve is plotted on the same page as the sample data, figure 18–7.

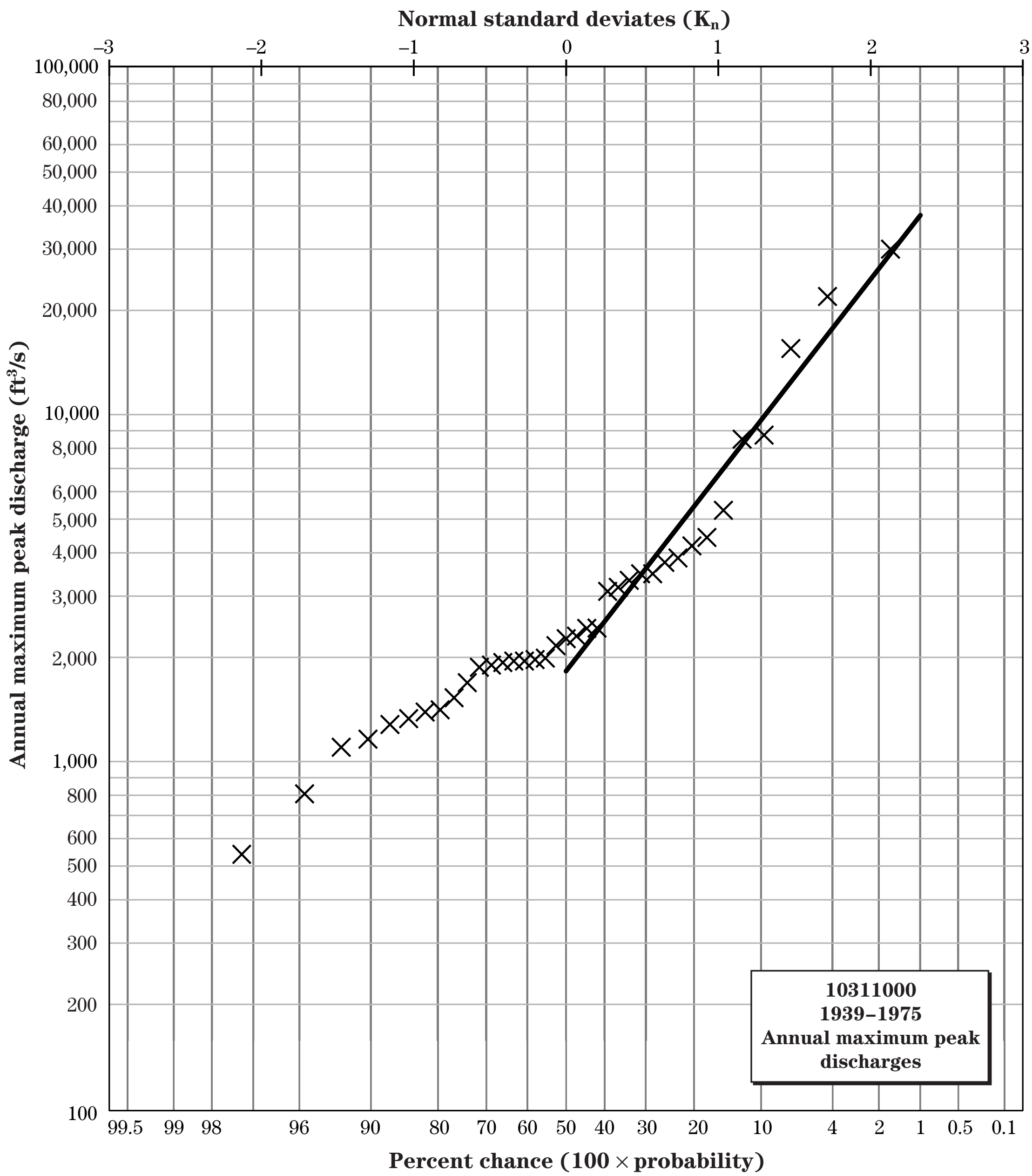
Example 18–3 Development of a mixed distribution frequency curve by separating the data by cause and by using at least the upper half of the data—Continued

Table 18–11 Data and normal K_n values for example 18–3

Ordered annual maximum peaks (ft^3/s) (1)	Logarithm of ordered annual maximum peaks (2)	Expected normal K_n value (3)	Expected normal K_n value (4)
30,000	4.47712	2.12928	
21,900	4.34044	1.71659	
15,500	4.19033	1.47676	
8,740	3.94151	1.30016	
8,500	3.92942	1.15677	
5,300	3.72428	1.03390	
4,430	3.64640	0.92496	
4,190	3.62221	0.82605	
3,860	3.58659	0.73465	
3,750	3.57403	0.64902	
3,480	3.54158	0.56793	
3,480	3.54158	0.49042	
3,330	3.52244	0.41576	
3,180	3.50243	0.34336	
3,100	3.49136	0.27272	
2,430	3.38561	0.20342	
2,420	3.38382	0.13509	
2,300	3.36173	0.06739	
2,260	3.35411	0.00000	
2,160			–0.06739
1,990			–0.13509
1,970			–0.20342
1,950			–0.27272
1,950			–0.34336
1,930			–0.41576
1,900			–0.49042
1,870			–0.56793
1,690			–0.64902
1,530			–0.73465
1,410			–0.82605
1,390			–0.92496
1,330			–1.03390
1,280			–1.15677
1,160			–1.30016
1,100			–1.47676
808			–1.71659
541			–2.12928
Sum (values)	70.11699	14.44423	
Sum (values^2)	260.75700	17.25002	

Example 18–3 Development of a mixed distribution frequency curve by separating the data by cause and by using at least the upper half of the data—Continued

Figure 18–7 Data and top half frequency curve for example 18–3



630.1803 Flow duration

A flow duration curve indicates the percentage of time a streamflow was greater than or less than a specific discharge during a period of record. A flow duration curve does not show the chronological sequence of flows. Because daily flows are nonrandom and nonhomogeneous, a flow duration curve cannot be considered a frequency or probability curve. Duration curves are normally constructed from mean daily flows.

Although a flow duration curve indicates only the distribution of mean daily flows that have been recorded, it can be used as an estimate of the flow duration distribution expected. Flow duration curves help determine availability of streamflow for beneficial uses.

USGS Water Supply Paper 1542–A (Searcy 1959) gives procedures for preparing and using flow duration curves. Many flow duration curves are available in USGS publications. Unpublished curves may be available at USGS district offices.

630.1804 Correlation and regression

(a) Correlation analysis

Correlation is an index that measures the linear variation between variables. While several correlation coefficients exist, the most frequently used is the Pearson product-moment correlation coefficient (r):

$$r = \frac{\sum_{i=1}^{N}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)}{\left[\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2 \sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2\right]^{0.5}} \qquad (18\text{–}21)$$

where:

X_i and Y_i = values of the i[th] observation of the two variables X and Y, respectively

$\bar{X}$ and $\bar{Y}$ = means of the two samples

N = number of common elements in the samples.

Equation 18–21 is used to measure the relationship between two variables. As an example, one may be interested in examining whether there is a significant linear relationship between the T-year peak discharge (Y) and the fraction of the drainage area in impervious land cover (X). To examine this relationship, values for X and Y must be obtained from N watersheds with widely different values of the X variable, and then a quantitative index of the relation is determined using equation 18–21.

Values of r range between +1 and –1. A correlation of +1 indicates a perfect direct relationship between variables X and Y, while a correlation of –1 indicates a perfect inverse relationship. Zero correlation indicates no linear relationship between the variables. Correlation values between 0 and ±1 indicate the degree of relationship between the variables. Figure 18–8 illustrates various linear correlation values between two variables.

Because correlation coefficient values can be misleading at times, the sample data should be plotted and examined. Some situations that may cause low correlation values are:

- No relation exists between variables—random variation.
- A relation exists, but is nonlinear, such as a parabolic or circular relation.
- Data values can depart significantly from the linear trend of the remaining data. The extreme values not only can change the correlation coefficient, but also can change the sign of the correlation coefficient.

High correlation can be attributed to:

- Significant relation between variables.
- Small sample size—For example, two points defining a straight line will result in a correlation coefficient of r = 1 or –1. Other small samples are influenced by this effect and may also have high correlation values.
- Data clustering—Two data clusters, each with low correlation, can exhibit high correlation values. Each cluster acting as a unit value may act as a small sample size.

The correlation between two variables will change if either of the variables is transformed nonlinearly. A new correlation coefficient should be developed for the transformed variables and will apply only to the variables in their transformed state.

(b) Regression

Regression is a method of developing a relation between a *criterion* variable (Y) and one or more *predictor* variables (X), with the objective of predicting the criterion variable for given values of the predictor variables.

Correlation analysis is quite different from regression analysis, although they are frequently used together. Regression is a predictive technique that distinguishes between the predictor and criterion variables. A regression equation that is developed to predict Y should not be transformed to predict the X variable for a given value of Y. Regression is based on an assumption that no error exists in the independent variable; errors occur only in the dependent variable. Thus, regression is directional. Correlation is not directional in that the correlation between Y and X is the same as that between X and Y. Also, correlation is different from regression in that correlation is only a standardized index of the degree of a linear relation.

Wang and Huber (1967) list additional assumptions that form the basis for regression as:

- The predictor variables are statistically independent.
- The variance of the criterion variable does not change with changes in magnitude of the predictor variables.
- The observed values of the criterion variable are uncorrelated events.
- The population of the criterion variable is normally distributed about the regression line for any fixed level of the predictor variables under consideration.

Generally, hydrologic data do not meet all of the assumptions of regression analysis, but regression is still used because it provides an easy method for analyzing many factors simultaneously. The error caused by failure to meet all of the assumptions is generally minor.

Forms of regression analysis include linear bivariate, linear multiple, and curvilinear. The *linear bivariate regression* relates a criterion variable (Y) and a single predictor variable (X) by using:

$$Y = a + bX \qquad (18\text{–}22)$$

where:
a and b = intercept and slope regression coefficients, respectively

Linear multiple regression relates a criterion variable (Y) and p predictor variables (X_j where j = 1, 2, …, p):

$$Y = b_0 + b_1X_1 + b_2X_2 + \ldots b_pX_p \qquad (18\text{–}23)$$

where:
b_j(j = 0, 1,. . ., p) = partial regression coefficients

The *curvilinear regression* technique is used when powers of the predictor variable(s) are included in the equation. For a single variable, the following regression equation can be used:

$$Y = b_0 + b_1X + b_2X^2 + \ldots b_qX^q \qquad (18\text{–}24)$$

where:
q = order of the polynomial

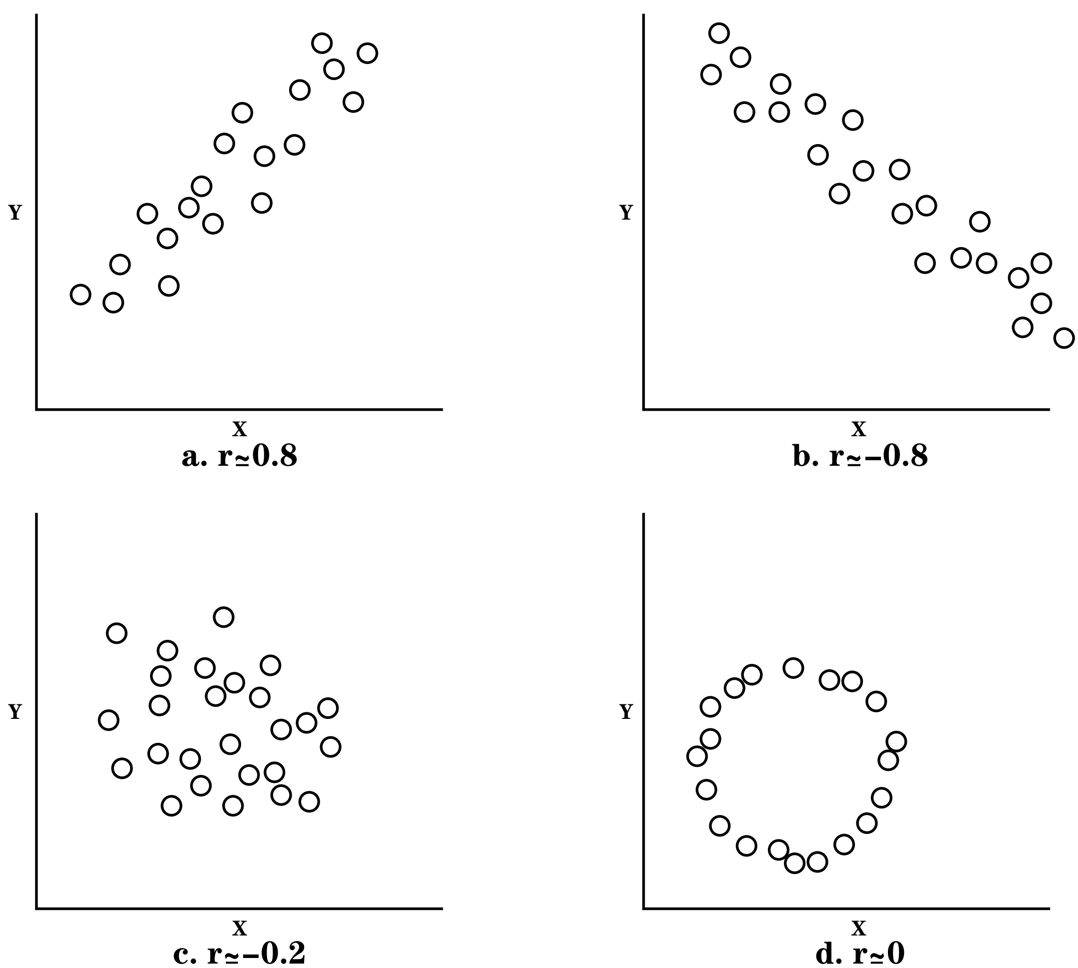
This equation can be expanded to include other predictor variables.

More than one regression equation can be derived to fit data, so some technique must be selected to evaluate the "best fit" line. The *method of least squares* is generally used because it minimizes the sum of the square of the differences between the sample criterion values and the estimated criterion values.

A cause-and-effect relation is implied between the predictor and the criterion variables. If there is no physical relation between a predictor and the criterion, do not use the predictor. Always carefully examine the sign of the coefficients for rationality. Do not use any equation outside the range of the sample data that were used to derive the coefficients.

A detailed procedure of how to develop regression equations is not given in this chapter. Regression analysis is usually performed by use of programmed

Figure 18–8 Linear correlation values



procedures on a calculator or computer. The following section highlights the basic concepts and terminology of regression analysis.

(c) Evaluating regression equations

After the regression coefficients are developed, it is necessary to examine the quality of a regression equation. The following means of evaluating the quality are described:

- analysis of the residuals
- standard error of estimate
- coefficient of determination
- analysis of the rationality of the sign and magnitude of the regression coefficients
- analysis of the relative importance of the predictor variables, as measured by the standardized partial regression coefficients

A *residual* is the difference between the value predicted with the regression equation and the criterion variable. A residual measures the amount of criterion variation left unexplained by the regression equation. The least squares concept assumes that the residual should exhibit the following properties:

- mean value equals zero
- independent of criterion and predictor variables
- variance is constant
- have a normal distribution

The mean of zero is easily verified by simply summing the residuals; a nonzero mean may result if not enough digits are used in the partial regression coefficients. Their independence and constant variance can be checked by plotting the residuals against the criterion and each predictor. Such plots should not exhibit any noticeable trends. Figure 18–9 illustrates some general trends that might occur when residuals are plotted. Nonconstant variance generally indicates an incorrect model form.

In theory, the residuals are normally distributed. The distribution can often be identified by use of a frequency analysis. However, if the sample is small, conclusive statements are difficult to make about the distribution of the residuals. Frequently, the model can be improved if a cause for a residual or trend in residuals is found.

Just as the individual residuals are of interest, the moments of the residuals are also worth examining. While the mean of the residuals is zero, the standard deviation of the residuals is called the *standard error of estimate*, which is denoted by S_e and is computed by:

$$S_e = \left[\frac{\sum_{i=1}^{N} \left(\hat{Y}_i - Y_i \right)^2}{d_f} \right]^{0.5} \qquad (18\text{–}25)$$

where:

$\hat{Y}_i$ = predicted value

Y_i = observed value of the i^{th} observation on the criterion variable

d_f = degrees of freedom

The *degrees of freedom* equal the number of independent pieces of information required to form the estimate. For a regression equation, this equals the number of observations in the data sample N minus the number of unknowns estimated from the data. A regression equation with p predictor variables and an intercept coefficient would have N–p–1 degrees of freedom.

Compare S_e with the standard deviation of the criterion variable (S_y) as a measure of the quality of a regression equation. Both S_e and S_y have the same units as the criterion variable. If the regression equation does not provide a good fit to the observed values of the criterion variable, then S_e should approach S_y, with allowance being made for the differences in degrees of freedom (S_e has N–p–1 while S_y has N–1). However, if the regression provides a good fit, S_e will approach zero. Thus, S_e can be compared with the two extremes, zero and S_y, to assess the quality of the regression.

The part of the total variation in the criterion variable that is explained by the regression equation should be considered. This part is called the *coefficient of determination* and can be computed by:

$$r^2 = \frac{\sum_{i=1}^{N}\left(\hat{Y}_i - \bar{Y}\right)^2}{\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2} \qquad (18\text{–}26)$$

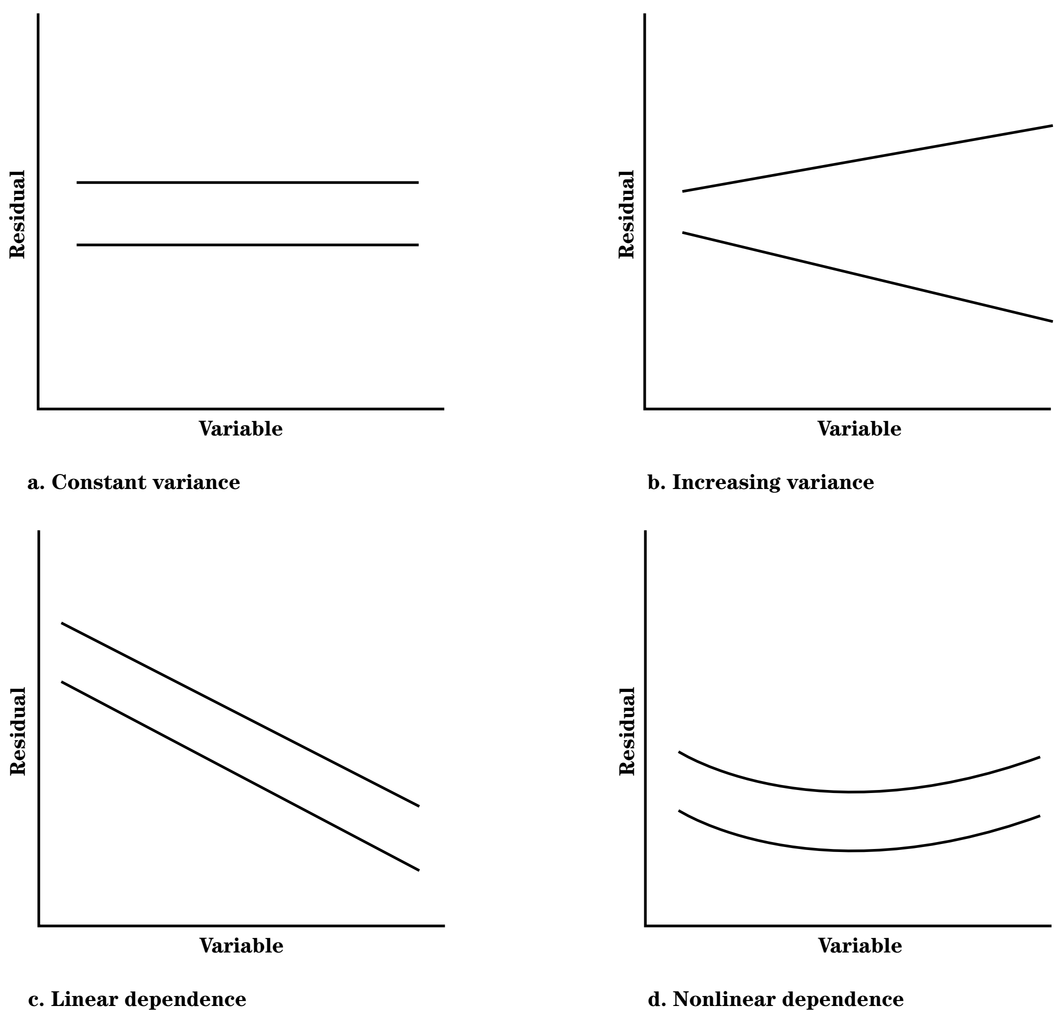
The value of r^2 ranges from zero to one, with a value of zero indicating no relationship between the criterion and predictor variables and a value of one indicating a perfect fit of the sample data to the regression line. The value of r^2 is a decimal percentage of the variation in Y explained by the regression equation.

An inverse relation between r^2 and S_e is:

$$S_e = S\sqrt{1 - r^2} \qquad (18\text{–}27)$$

While this relation may be acceptable for large samples, it should not be used for small samples because S_e is based on N–p–1 degrees of freedom, while S is based on N–1 degrees of freedom and r^2 is based on N degrees of freedom. Therefore, equations 18–25 and 18–26 should be used to compute S_e and r^2.

Figure 18–9 Sample plots of residuals



A regression equation describes the relation that exists between the variables, with a partial regression coefficient reflecting the effect of the corresponding predictor variable on the criterion variable. As such, the magnitude and sign of each coefficient should be checked for rationality. While the rationality of the magnitude of a coefficient is sometimes difficult to assess, the rationality of the sign of the coefficient is generally easy to assess. Irrationality of either sign or magnitude often results from significant correlations between predictor variables. Thus, the use of highly correlated predictor variables should be avoided. The potential accuracy of estimates is rarely increased significantly by including a predictor variable that is highly correlated with one or more other predictor variables in the equation.

Regression equations can be developed for any number of predictor variables, but selecting the proper number is important. Having too few predictor variables may reduce the accuracy of the criterion estimate. Having too many makes the equation more complex than necessary and wastes time and money in collecting and processing unneeded data that do not significantly improve accuracy.

Step-type regressions can be used to evaluate the importance or significance of individual predictor variables in a regression equation. A step consists of adding or deleting a predictor variable from the regression equation and measuring the increase or decrease in the ability of the equation to predict the criterion variable.

The significance of predictor variables and the total equation are evaluated by using F-tests. Two F-tests are used. The partial F-test (F_p) checks the significance of predictor variables that are added or deleted from a regression equation. The total F-test (F_t) checks the significance of the entire regression equation. The partial F-test is computed by:

$$F_p = \frac{\left(r_p^2 - r_{p-1}^2\right)}{\frac{\left(1 - r_p^2\right)}{(N - p - 1)}} \qquad (18–28)$$

where:

r_p and r_{p-1} = coefficients of determination for the p and p–1 predictor models

The equation is significant if the computed F is greater than the value found in an F distribution table. The degrees of freedom needed for use of the F table are $1(d_{f1})$ and $N–p–1$ (d_{f2}). F distribution tables for 0.05 and 0.01 levels of significance are in most standard statistics texts. The 0.05 probability table is generally used.

F_t is computed by:

$$F_t = \frac{\frac{r_p^2}{p}}{\frac{\left(1 - r_p^2\right)}{(N - p - 1)}} \qquad (18–29)$$

where:

p = number of predictors in the equation

r_p^2 = coefficient of determination for the p predictor equation

The degrees of freedom required to use the tables are $p(d_{f1})$ and $N–p–1(d_{f2})$.

Step backward regression starts with all predictors in the regression equation. The least important predictor is deleted and the F_p computed. If the predictor is not significant, the next least important of the remaining predictors is deleted and the process repeated. When a significant predictor is found, the previous equation that includes that predictor should be used.

Step forward regression starts with the most important predictor as the only variable in the equation. The most important of the remaining predictors is added and the F_p computed. If this predictor is significant, the next most important of the remaining predictors is added and the process repeated. When a nonsignificant predictor is found, the previous equation that does not include that predictor should be used.

Stepwise regression combines features of both step backward and step forward regression. Stepwise is basically a step forward regression with a step backward partial F test of all predictors in the equation after each step. When predictors are added to an equation, two or more may combine their prediction ability to make previously included predictors insignificant. As these “older” predictors are no longer needed in the equation, they are deleted.

(d) Outline of procedures

(1) Correlation

The procedure for correlation is:

Step 1 Determine that a cause-and-effect relation exists for all variable pairs to be tested.

Step 2 Plot every combination of one variable versus another to examine data trends.

Step 3 Make adjustments, such as transformation of data, if required. This step is optional.

Step 4 Compute linear correlation coefficients between each pair of variables.

(2) Regression

The procedure for regression is:

Step 1 Compile a list of predictor variables that are related to the criterion variable by some physical relation and for which data are available.

Step 2 Plot each predictor variable versus the criterion variable.

Step 3 Determine the form of the desired equation; i.e., linear or curvilinear.

Step 4 Compute the correlation matrix; i.e., the correlation coefficient between each pair of variables.

Step 5 Compute the regression coefficients for the predictor variable(s) that have high correlation coefficients with the criterion variable and low correlation coefficients with any other included predictor variables.

Step 6 Compute standard error of estimate, S_e; standard deviation of the criterion variable, S_y; and the coefficient of determination, r^2.

Step 7 Evaluate the regression equation by the following methods:

- Standard error of estimate has the bounds $0 \leq S_e \leq S_y$; as $S_e \longrightarrow 0$ more of the variance is explained by the regression.
- Coefficient of determination has the bounds $0 \leq r^2 \leq 1$; as $r^2 \longrightarrow 1$ the better the "fit" is of the regression line to the data.
- Partial and total F-tests are used to evaluate each predictor and total equation significance.
- The sign of each regression coefficient should be compared to the correlation coefficient for the appropriate predictor criterion. The signs should be the same.
- Examine the residuals to identify deficiencies in the regression equation and check the assumptions of the model.

Step 8 If regression equation accuracy is not acceptable, reformulate the regression equation or transform some of the variables. A satisfactory solution is not always possible from data available.

Example 18–4 illustrates the development of a multiple regression equation.

Example 18–4 Development of a multiple regression equation

Given: Peak flow data for watershed W-11, Hastings, Nebraska, are used. Table 18–12 contains basic data for peak flow and three other variables.

Solution: ***Step 1***—Plot one variable versus another to establish that a linear or nonlinear data trend exists. Figure 18–10 is a plot of peak flow (Y) versus maximum average 1-day flow (X_i). Similar plots are done for all combinations of variable pairs. The plot indicates a linear trend exists between peak flow and maximum average 1-day flow.

Step 2—Determine the linear correlation coefficients between each pair of variables. Table 18–12 contains the product of differences required for the computation. Use equation 18–21 to compute the linear correlation. The array of the computed linear correlations follows:

	Linear Correlation Matrix q = Y	**Q = X_1**	**Q_m = X_2**	**P_m = X_3**
Y	1.0000	0.9230	0.7973	0.5748
X_1		1.0000	0.9148	0.7442
X_2			1.0000	0.8611
X_3				1.0000

Step 3—Develop a multiple regression equation based on maximum 1-day flow (X_1) and maximum monthly rainfall (X_3). Maximum monthly runoff (X_2) is not included as a predictor because it is highly correlated (0.9148) with maximum average 1-day flow (X_1). Predictor variables should be correlated with the criterion, but not highly correlated with the other predictors. Two highly correlated predictors will explain basically the same part of the criterion variation. The predictor with the highest criterion correlation is retained. High correlation between predictor variables may cause irrational regression coefficients. The following regression coefficients were developed from a locally available multiple linear regression computer program (Dixon 1975):

$b_0 =$ 0.0569
$b_1 =$ 0.1867
$b_2 =$ –0.0140

The regression equation is:

$$Y = 0.0569 + 0.1867X_1 - 0.0140X_3$$

In the equation, peak flow varies directly with the maximum average 1-day flow and inversely with maximum monthly rain. The inverse relation between Y and X_3 is not rational and should be included only if the increased significance is meaningful.

Step 4—Analyze the residuals. Compute the standard deviation of the criterion variable (square root of equation 18–2), standard error of estimate (equation 18–25), and coefficient of determination (equation 18–26). Table 18–13 contains the data needed for this step.

$$d_f = 29 - 2 - 1 = 26$$

Example 18–4 Development of a multiple regression equation—Continued

$$S_y = \left[\frac{\sum\left(Y_i - \bar{Y}\right)^2}{N-1}\right]^{0.5} = \left(\frac{0.4343}{28}\right)^{0.5} = 0.1245$$

$$S_e = \left[\frac{\sum\left(\hat{Y}_i - Y_i\right)^2}{d_f}\right]^{0.5} = \left(\frac{0.0508}{26}\right)^{0.5} = 0.044$$

$$r^2 = \frac{\sum\left(\hat{Y}_i - \bar{Y}_i\right)^2}{\sum\left(Y_i - \bar{Y}\right)^2} = \left(\frac{0.3822}{0.4343}\right) = 0.880$$

The regression equation is a good predictor of the peak flow. The equation explains 88 percent of the variation in Y, and the standard error of estimate is much smaller than the standard deviation of the criterion variable, S_y.

Maximum monthly rainfall is not really needed in the equation, but is included to illustrate a multiple predictor model. The correlation coefficient between peak flow and maximum 1-day flow, from the correlation matrix, indicates that the maximum 1-day flow will explain 85 percent of the variation in peak flow; i.e., $r^2 = (0.9230)^2 = 0.85$.

The sum of residuals from table 18–13 is –0.0020. The number of significant digits was not sufficient to produce truly accurate regression coefficients. More significant digits would improve the accuracy of the coefficients.

Step 5—Plot the residuals as shown in figure 18–11. Similar plots can be made for the predictor variables and residuals. The greatest amount of underprediction (negative residual) occurs near a peak flow of 0.3 ft^3/s. Two data points (1952 and 1954) in the region account for 46 percent of the sum of residuals squared. The greatest amount of overprediction (positive residuals) occurs at the maximum peak flow value. Large residual values (positive or negative) may be a problem when the regression equation is used in the upper range of peak flow values.

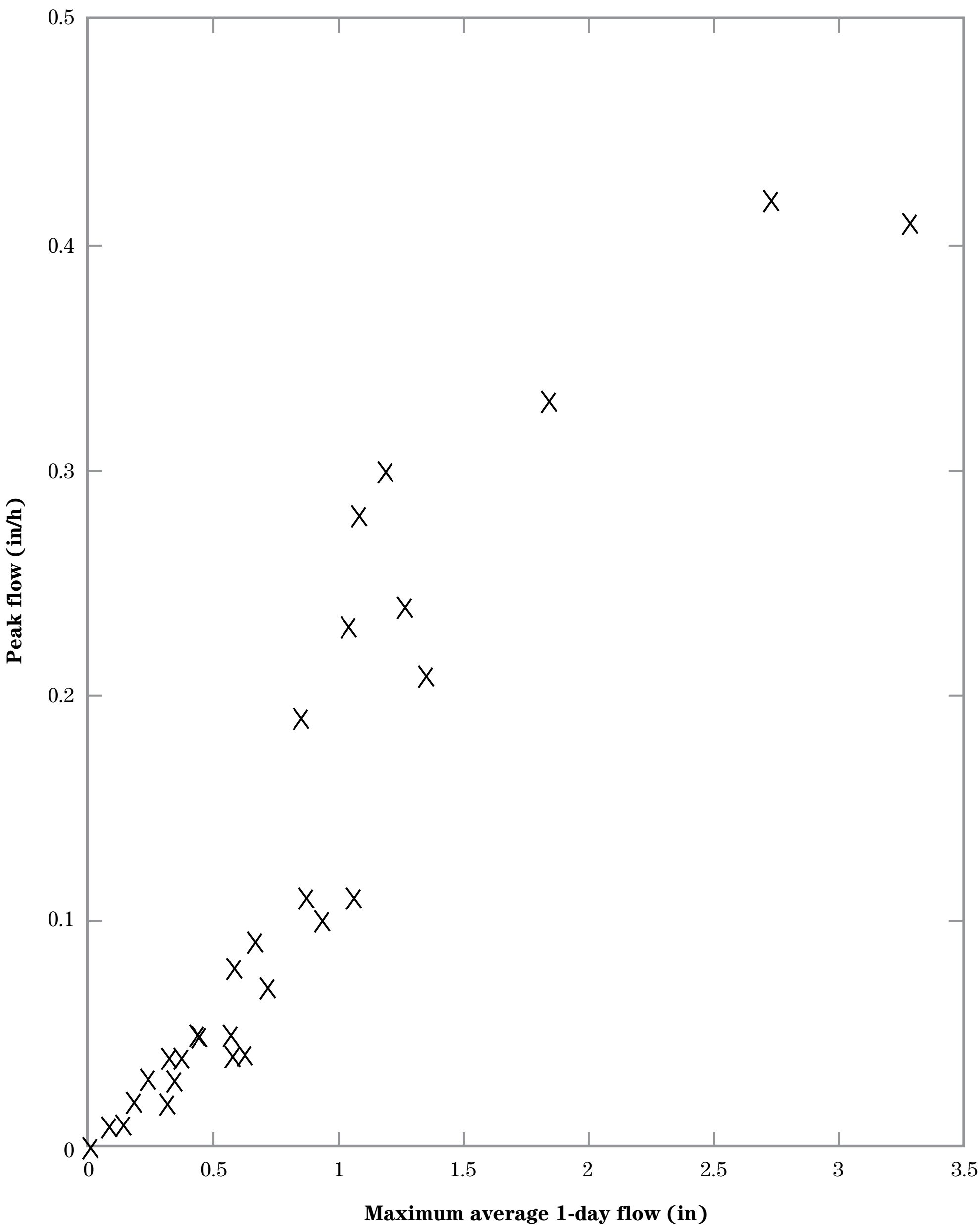
Example 18–4 Development of a multiple regression equation—Continued

Table 18–12 Basic correlation data for example 18–4 (linear correlation coefficient computation)

Water year	Y = peak flow (in/h)	X_1 = max. avg. 1-day flow (in)	X_2 = max. monthly runoff (in)	X_3 = max. monthly rainfall (in)	$(X - \bar{X})$ for Y	$(X - \bar{X})$ for X_1	$(X - \bar{X})$ for X_2	$(X - \bar{X})$ for X_3	Product of differences for Y, X_1	Y, X_2	Y, X_3	X_1, X_2	X_1, X_3	X_2, X_3
1939	0.01	0.08	0.12	3.57	–0.1141	–0.7393	–1.1852	–2.5583	0.0844	0.1352	0.2919	0.8762	1.8913	3.0321
1940	0.00	0.00	0.02	2.00	–0.1241	–0.8193	–1.2852	–4.1283	0.1017	0.1595	0.5123	1.0530	3.3823	5.3057
1941	0.04	0.56	1.41	8.31	–0.0841	–0.2593	0.1048	2.1817	0.0218	–0.0088	–0.1835	–0.0272	–0.5657	0.2286
1942	0.05	0.55	2.31	8.39	–0.0741	–0.2693	1.0048	2.2617	0.0200	–0.0745	–0.1676	–0.2706	–0.6091	2.2726
1943	0.08	0.57	1.58	5.95	–0.0441	–0.2493	0.2748	–0.1783	0.0110	–0.0121	0.0079	–0.0685	0.0445	–0.0490
1944	0.11	1.05	1.74	8.14	–0.0141	0.2307	0.4348	2.0117	–0.0033	–0.0061	–0.0284	0.1003	0.4641	0.8747
1945	0.09	0.66	0.67	3.82	–0.0341	–0.1593	–0.6352	–2.3083	0.0054	0.0217	0.0787	0.1012	0.3677	1.4662
1946	0.02	0.31	0.83	5.34	–0.1041	–0.5093	–0.4752	–0.7883	0.0530	0.0495	0.0821	0.2420	0.4015	0.3746
1947	0.04	0.31	0.75	5.46	–0.0841	–0.5093	–0.5552	–0.6683	0.0428	0.0467	0.0562	0.2828	0.3404	0.3710
1948	0.02	0.17	0.33	4.38	–0.1041	–0.6493	–0.9752	–1.7483	0.0676	0.1015	0.1820	0.6332	1.1352	1.7049
1949	0.11	0.86	1.60	7.21	–0.0141	0.0407	0.2948	1.0817	–0.0006	–0.0042	–0.0153	0.0120	0.0440	0.3189
1950	0.21	1.33	1.37	5.69	0.0859	0.5107	0.0648	–0.4383	0.0439	0.0056	–0.0376	0.0331	–0.2238	–0.0284
1951	0.33	1.83	3.04	10.27	0.2059	1.0107	1.7348	4.1417	0.2081	0.3572	0.8528	1.7534	4.1860	7.1850
1952	0.30	1.17	1.59	5.76	0.1759	0.3507	0.2848	–0.3683	0.0617	0.0501	–0.0648	0.0999	–0.1292	–0.1049
1953	0.19	0.84	0.85	3.28	0.0659	0.0207	–0.4552	–2.8483	0.0014	–0.0300	–0.1877	–0.0094	–0.0590	1.2965
1954	0.28	1.07	1.55	6.35	0.1559	0.2507	0.2448	0.2217	0.0391	0.0382	0.0346	0.0614	0.0556	0.0543
1955	0.05	0.43	0.90	5.18	–0.0741	–0.3893	–0.4052	–0.9483	0.0288	0.0300	0.0703	0.1577	0.3692	0.3843
1956	0.03	0.23	0.39	3.61	–0.0941	–0.5893	–0.9152	–2.5183	0.0555	0.0861	0.2370	0.5393	1.4840	2.3047
1957	0.41	3.27	5.22	11.77	0.2859	2.4507	3.9148	5.6417	0.7007	1.1192	1.6130	9.5940	13.8261	22.0861
1958	0.03	0.33	0.38	4.80	–0.0941	–0.4893	–0.9252	–1.3283	0.0460	0.0871	0.1250	0.4527	0.6499	1.2289
1959	0.24	1.25	1.26	6.49	0.1159	0.4307	–0.0452	0.3617	0.0499	–0.0052	0.0419	–0.0195	0.1558	–0.0163
1960	0.23	1.03	1.73	5.70	0.1059	0.2107	0.4248	–0.4283	0.0223	0.0450	–0.0454	0.0895	–0.0902	–0.1819
1961	0.10	0.92	0.86	7.09	–0.0241	0.1007	–0.4452	0.9617	–0.0024	0.0107	–0.0232	–0.0448	0.0968	–0.4281
1962	0.07	0.70	0.81	5.10	–0.0541	–0.1193	–0.4952	–1.0283	0.0065	0.0268	0.0556	0.0591	0.1227	0.5092
1963	0.04	0.61	1.08	8.93	–0.0841	–0.2093	–0.2252	2.8017	0.0176	0.0189	–0.2356	0.0471	–0.5864	–0.6309
1964	0.05	0.42	0.93	5.76	–0.0741	–0.3993	–0.3752	–0.3683	0.0296	0.0278	0.0273	0.1498	0.1471	0.1382
1965	0.42	2.72	3.33	9.38	0.2959	1.9007	2.0248	3.2517	0.5624	0.5991	0.9622	3.8485	6.1805	6.5840
1966	0.01	0.13	0.24	3.86	–0.1141	–0.6893	–1.0652	–2.2683	0.0786	0.1215	0.2588	0.7342	1.5635	2.4162
1967	0.04	0.36	0.96	6.13	–0.0841	–0.4593	–0.3452	0.0017	0.0386	0.0290	–0.0001	0.1586	–0.0008	–0.0006
Sum	3.60	23.76	37.85	177.72					2.3921	3.0255	4.5004	20.639	34.644	58.6966
Mean	0.1241	0.8193	1.3052	6.1283										
			Squared sum		0.4359	15.4095	33.0335	140.6424						

Example 18–4 Development of a multiple regression equation—Continued

Figure 18–10 Variable plot for example 18–4



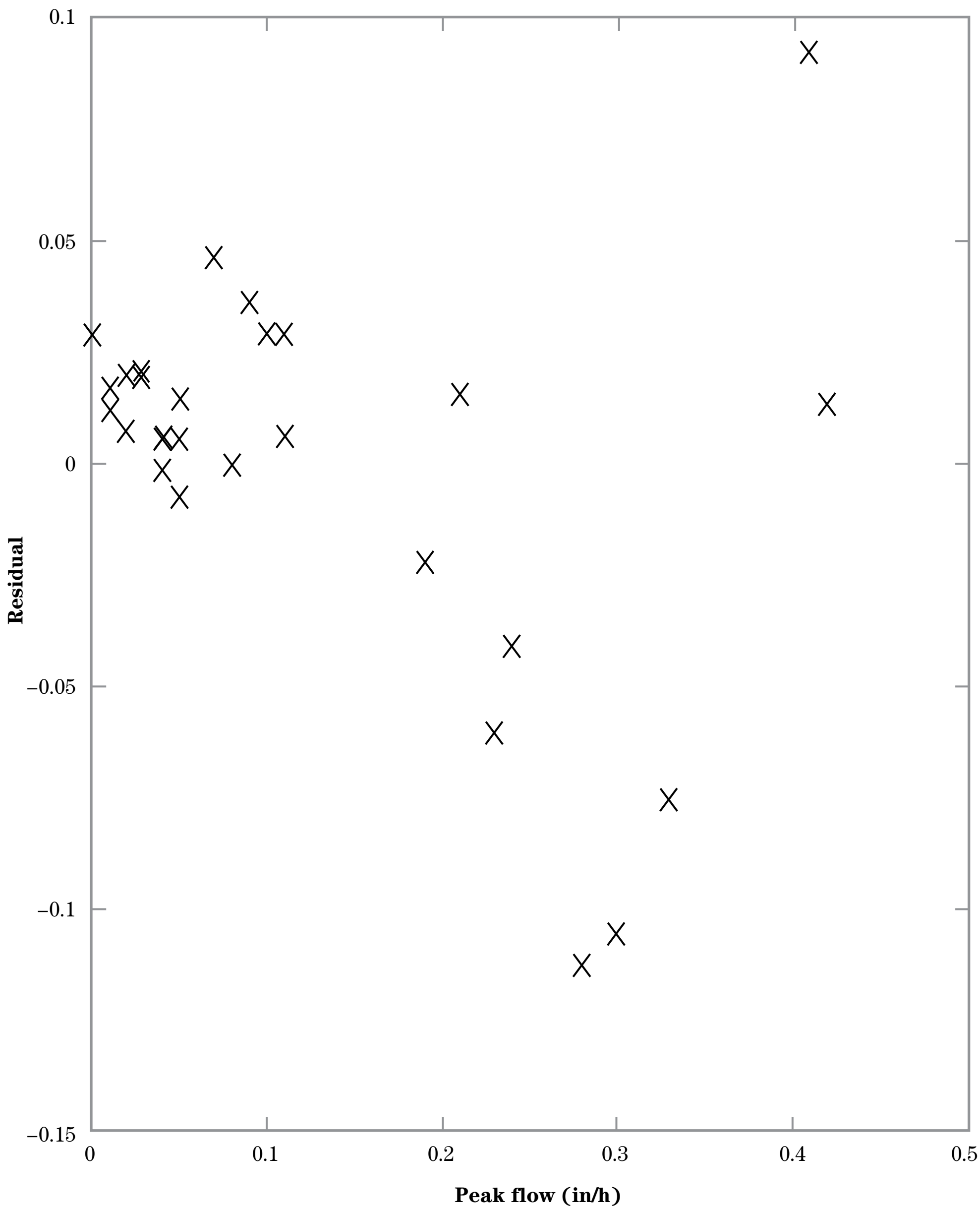
Example 18–4 Development of a multiple regression equation—Continued

Table 18–13 Residual data for example 18–4 (analysis of residuals for $\hat{Y} = 0.0569 + 0.1867X_1 - 0.0140X_3$)

Water year	Y = peak flow (in/h)	X_1= max. avg. 1-day flow (in)	X_3= max. monthly rainfall (in)	$\hat{Y}$	$(\hat{Y} - Y)$	$(\hat{Y} - Y)^2$	$(\hat{Y} - \bar{Y})^2$	$(Y - \bar{Y})^2$
1939	0.01	0.08	3.57	0.0219	0.0119	0.0001	0.0104	0.0130
1940	0.00	0.00	2.00	0.0289	0.0289	0.0008	0.0090	0.0154
1941	0.04	0.56	8.31	0.0451	0.0051	0.0000	0.0062	0.0070
1942	0.05	0.55	8.39	0.0421	–0.0079	0.0000	0.0067	0.0054
1943	0.08	0.57	5.95	0.0800	–0.0000	0.0000	0.0019	0.0019
1944	0.11	1.05	8.14	0.1390	0.0290	0.0008	0.0002	0.0001
1945	0.09	0.66	3.82	0.1266	0.0366	0.0013	0.0000	0.0011
1946	0.02	0.31	5.34	0.0400	0.0200	0.0003	0.0070	0.0108
1947	0.04	0.31	5.46	0.0383	–0.0017	0.0000	0.0073	0.0070
1948	0.02	0.17	4.38	0.0273	0.0073	0.0000	0.0093	0.0108
1949	0.11	0.86	7.21	0.1165	0.0065	0.0000	0.0000	0.0001
1950	0.21	1.33	5.69	0.2256	0.0156	0.0002	0.0103	0.0073
1951	0.33	1.83	10.27	0.2548	-0.0752	0.0056	0.0170	0.0423
1952	0.30	1.17	5.76	0.1947	–0.1053	0.0110	0.0049	0.0309
1953	0.19	0.84	3.28	0.1678	–0.0222	0.0004	0.0019	0.0043
1954	0.28	1.07	6.35	0.1678	–0.1122	0.0125	0.0019	0.0243
1955	0.05	0.43	5.18	0.0647	0.0147	0.0002	0.0035	0.0054
1956	0.03	0.23	3.61	0.0493	0.0193	0.0003	0.0055	0.0088
1957	0.41	3.27	11.77	0.5026	0.0926	0.0085	0.1432	0.0817
1958	0.03	0.33	4.80	0.0513	0.0213	0.0004	0.0052	0.0088
1959	0.24	1.25	6.49	0.1994	–0.0406	0.0016	0.0056	0.0134
1960	0.23	1.03	5.70	0.1694	–0.0606	0.0036	0.0020	0.0112
1961	0.10	0.92	7.09	0.1294	0.0294	0.0008	0.0000	0.0005
1962	0.07	0.70	5.10	0.1162	0.0462	0.0021	0.0000	0.0029
1963	0.04	0.61	8.93	0.0458	0.0058	0.0000	0.0061	0.0070
1964	0.05	0.42	5.76	0.0547	0.0047	0.0000	0.0048	0.0054
1965	0.42	2.72	9.38	0.4334	0.0134	0.0001	0.0956	0.0875
1966	0.01	0.13	3.86	0.0271	0.0171	0.0002	0.0094	0.0130
1967	0.04	0.36	6.13	0.0383	–0.0017	0.0000	0.0073	0.0070
Sum					–0.0020	0.0508	0.3822	0.4343

Example 18–4 Development of a multiple regression equation—Continued

Figure 18–11 Residual plot for example 18–4



630.1805 Analysis based on regionalization

(a) Purpose

Many watersheds analyzed by NRCS are in locations for which few data are available, so techniques have been developed to transfer or regionalize available data to other locations.

One purpose of regionalization is to synthesize a frequency curve at an ungaged location or at a location where data are inadequate for developing a frequency curve by using the methods in NEH 630.1802, Frequency analysis. The most common forms of regionalization use watershed and hydrometeorological characteristics as predictor variables. Data may be regionalized by either direct or indirect estimation.

(b) Direct estimation

The most commonly used technique is to relate selected values at various exceedance frequencies to the physical characteristics of the watershed. For example, the 10-year, 7-day mean flow may be related to drainage area and percentage of forest cover. The predictor variables can include both physical and hydrometeorological characteristics.

Previous studies have included the following as predictors: drainage area, mean watershed slope, mean basin elevation, length and slope of the main watercourse, the weighted runoff curve number, percentage of watershed in lakes or various cover types, and geological characteristics.

Meteorological characteristics include: mean annual precipitation, mean annual snowfall, mean annual temperature, mean monthly temperature, mean monthly precipitation, and the 24-hour duration precipitation for various frequencies. Latitude, longitude, and watershed orientation have been included as location parameters. This list of various predictor variables is not complete, but has been included to give some concept of the characteristics that can be used.

The USGS uses stepwise multiple regression to develop predictive equations for selected flow values. The results are published in open file reports that generally include predictive equations for major river basins, physiographic regions, or States. Meteorological and physical characteristics listed in the reports can be used to develop applicable predictive equations for NRCS hydrologic studies.

Example 18–5 illustrates the development of a direct probability estimate using stepwise regression.

Example 18–5 Development of a direct probability estimate by use of stepwise regression

A sample power form prediction equation is:

$$\hat{Y} = b_o X_1^{b_1} X_2^{b_2} X_3^{b_3} \ldots X_n^{b_n}$$

where:

$\hat{Y}$ = estimated criterion variable
$X_1, X_2, X_3 \ldots X_n$ = predictor variables
$b_0, b_1, b_2 \ldots b_n$ = regression coefficients

Given: The regression coefficients are developed from a multiple linear regression of the logarithms of the data. When the variables are transformed back to original units, the regression coefficients become powers.

Table 18–14 includes 9 variables for 18 north coastal California watersheds used to develop a power equation for estimating the 1 percent maximum 7-day mean runoff ($V_{0.01}$). A locally available stepwise regression computer program (Dixon 1975) is used in the analysis.

The correlation matrix of the logarithms of the data is in table 18–15. The highest correlations of logarithms between runoff volume and the other variables are between channel length (–0.62) and drainage area (–0.53). These two variables are highly correlated (0.96) themselves, so only one would be expected to be used in the final equation. Rainfall intensity (0.48) and annual precipitation (0.45) are the variables with the next highest correlations to $V_{0.01}$. One or both of these variables may appear in the final regression equation.

The results of the stepwise regression analysis are in tables 18–16 and 18–17. Table 18–16 has the regression coefficients for each step of the regression, and table 18–17 shows the regression equation data for each step. Equation 5 in table 18–17 was selected as the best because the regression coefficients are rational and including additional variables does not significantly decrease the standard error of estimate.

All equations are significant based on the total F-test at the 1 percent level. The least significant variable is slope (S) based on a 1 percent level F with 4 and 13 degrees of freedom. From a standard F table, for these degrees of freedom, $F_{0.01} = 3.18$. The partial F value required to enter the slope variable is 5.3. Equation 5 in table 18–17 explains 83.6 percent of the variation (r^2) in the logarithm of $V_{0.01}$, and addition of all remaining variables only raises this to 87.3 percent.

Procedure: Examine the residuals to evaluate the quality of the selected regression equation. Table 18–18 has the predicted and observed $V_{0.01}$ logarithms as well as the residuals and their sum. A plot of the residuals with the predicted values in figure 18–12 shows no correlation between $V_{0.01}$ logarithms and the residuals. The residual variation is also constant over the range of the $V_{0.01}$ logarithms.

The final power equation is:

$$V_{0.01} = 4.7337\, L^{(-0.4650)}\, P^{(0.6735)}\, F^{(0.1432)}\, S^{(-0.1608)}$$

Example 18–5 Development of a direct probability estimate by use of stepwise regression—Continued

For data from station 11372000 (table 18–14), the estimated $V_{0.01}$ is:

$$V_{0.01} = (4.7337)(48.7)^{(-0.4650)}(56)^{(0.6735)}(99)^{(0.1432)}(63)^{(-0.1608)}$$
$$V_{0.01} = 11.60 \text{ watershed inches}$$

Similar procedures can be used to develop regression equations for 0.50, 0.20, 0.10, 0.04, and 0.02 exceedance probabilities. Because each equation may not contain the same predictor variables, inconsistencies may develop from one exceedance probability to another. A method of eliminating inconstancies is to smooth estimated values over the range of exceedance probabilities. Figure 18–13 illustrates the smoothing for station 11372000.

Table 18–14 Basic data for example 18–5

Station number	Drainage area (A) mi²	Mean annual precipitation (P)	2-year, 24-hour rainfall intensity (I)	Evaporation (E)	Channel slope (S) ft/mi	Channel length (L) mi	Altitude (Al) 1,000 ft	Percent forest (F) % + 1	Runoff volume ($V_{0.01}$) inches
		----	----inches----	----					
11372000	228.00	56	3.5	48	63	48.7	2.1	99	11.1966
11374400	249.00	41	2.8	48	58	43.5	1.6	53	7.6804
11379500	92.90	36	2.8	51	170	19.6	2.0	92	10.3144
11380500	126.00	28	2.7	51	93	42.7	1.8	84	6.6278
11382000	194.00	35	2.8	49	126	36.5	2.7	98	11.5990
11448500	6.36	41	4.5	46	374	4.2	2.1	95	18.9540
11448900	11.90	37	4.0	45	125	5.3	1.9	85	20.8693
11451500	197.00	39	3.0	52	40	34.0	1.7	96	10.1729
11451720	100.00	30	3.8	51	17	38.0	1.3	90	8.8838
11453500	113.00	52	3.5	49	55	21.6	1.4	89	18.8469
11453600	78.30	35	4.0	49	30	18.0	0.8	60	17.7086
11456000	81.40	48	3.3	49	46	19.4	0.5	79	16.2089
11456500	52.10	35	3.3	49	140	14.3	1.0	87	11.1178
11457000	17.40	35	3.3	49	72	10.8	1.2	29	13.1009
11458200	9.79	30	2.4	45	258	8.9	1.1	98	14.6669
11458500	58.40	35	3.0	46	82	17.3	0.3	72	15.9474
11459000	30.90	28	3.0	43	95	10.3	0.4	1	7.3099
11460000	18.10	42	3.0	42	125	7.5	0.5	50	19.0027

Example 18–5 Development of a direct probability estimate by use of stepwise regression—Continued

Table 18–15 Correlation matrix of logarithms for example 18–5

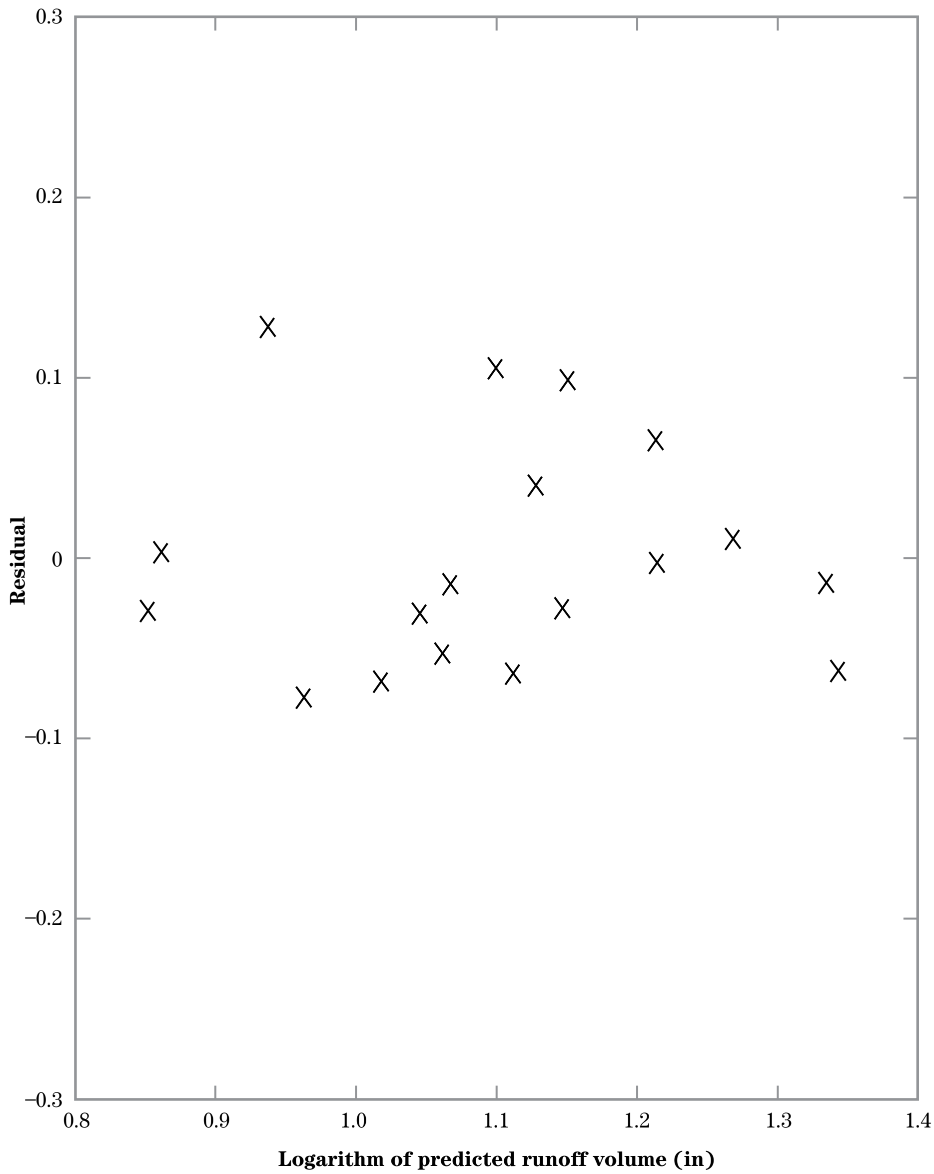
Variable	Drainage area (A) mi^2	Mean annual precipitation (P)	2-year, 24-hour rainfall intensity (I)	Evaporation (E)	Channel slope (S) ft/mi	Channel length (L) mi	Altitude (Al) 1,000 ft	Percent forest (F) % + 1	Runoff volume ($V_{0.01}$) inches
		----------inches---------							
Area	1.00								
Precipitation	0.23	1.00							
Intensity	–0.25	0.32	1.00						
Evaporation	0.63	0.01	–0.03	1.00					
Slope	–0.60	–0.10	–0.19	–0.44	1.00				
Length	0.96	0.11	–0.32	0.68	–0.61	1.00			
Altitude	0.22	0.14	0.11	0.50	0.16	0.27	1.00		
Forest	0.19	0.36	0.11	0.49	0.01	0.22	0.49	1.00	
Runoff volume	–0.53	0.45	0.48	–0.37	0.22	–0.62	–0.17	0.34	1.00

Table 18–16 Stepwise regression coefficients for example 18–5

Equation number	Constant	L	P	F	S	Al	A	E	I
1	1.0997								
2	1.4745	–0.3010							
3	–0.0022	–0.3281	0.9615						
4	0.1739	–0.3605	0.7380	0.1210					
5	0.6752	–0.4650	0.6735	0.1432	–0.1608				
6	0.5178	–0.4257	0.6731	0.1675	–0.1231	–0.1046			
7	0.6604	–0.5722	0.5803	0.1756	–0.1242	–0.1012	0.0985		
8	2.6010	–0.5796	0.4824	0.1980	–0.1509	–0.0681	0.1233	–1.0785	
9	2.6392	–0.5971	0.4949	0.1983	–0.1623	–0.0608	0.1257	–1.0705	–0.0637

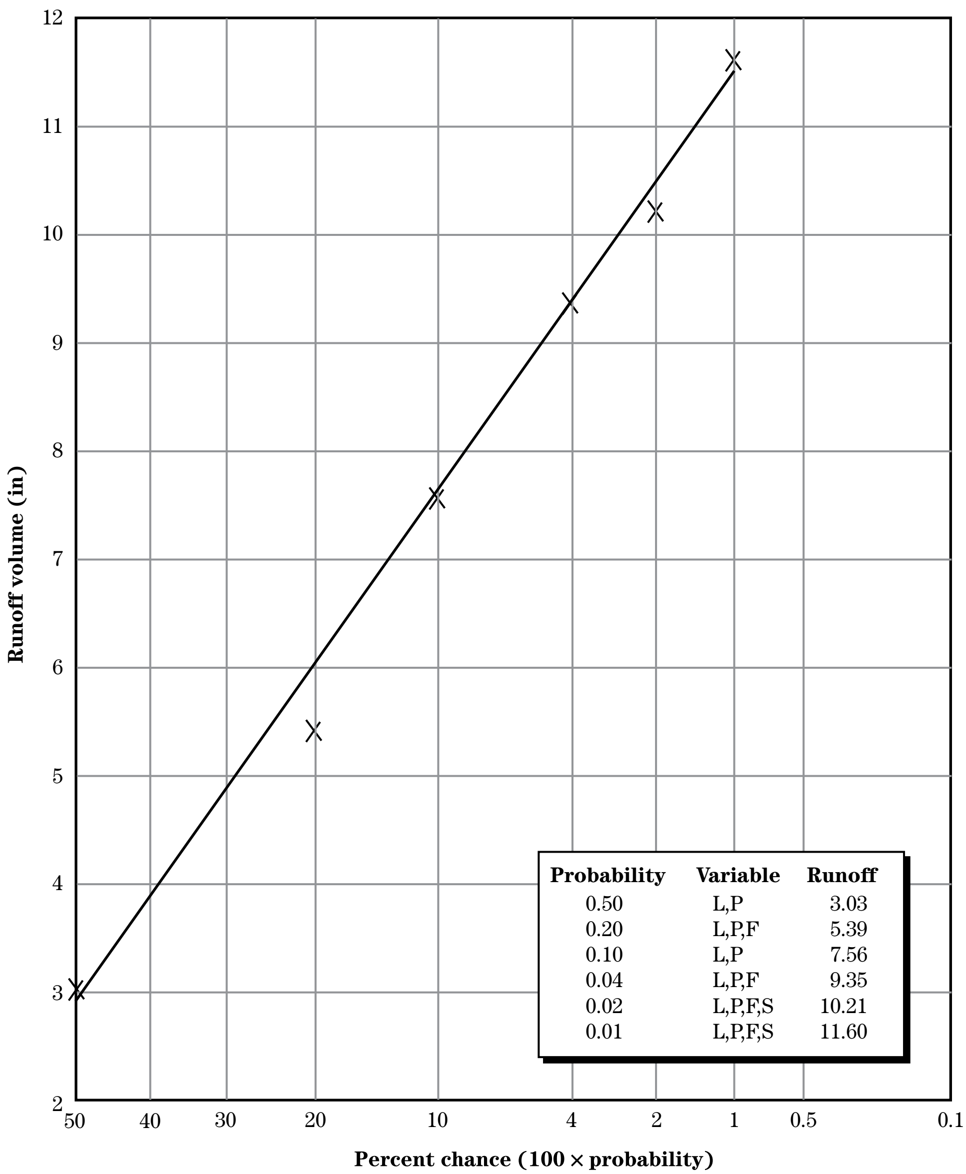
Example 18–5 Development of a direct probability estimate by use of stepwise regression—Continued

Figure 18–12 Residual plot for example 18–5



Example 18–5 Development of a direct probability estimate by use of stepwise regression—Continued

Figure 18–13 Estimate smoothing for example 18–5



Probability	Variable	Runoff
0.50	L,P	3.03
0.20	L,P,F	5.39
0.10	L,P	7.56
0.04	L,P,F	9.35
0.02	L,P,F,S	10.21
0.01	L,P,F,S	11.60

Example 18–5 Development of a direct probability estimate by use of stepwise regression—Continued

Table 18–17 Regression equation evaluation data for example 18–5

Equation no.	Predictor variables	r^2	Δr^2	S_e	SS/df regression	SS/df residuals	F_t ratio	F_p ratio
1	---			0.1566*				
2	L	0.390	0.390	0.1260	0.1627/1	0.2542/16	10.2	10.2
3	L,P	0.661	0.271	0.0971	0.2754/2	0.1415/15	14.6	11.9
4	L,P,F	0.769	0.108	0.0830	0.3204/3	0.0964/14	15.5	6.5
5	L,P,F,S	0.836	0.067	0.0725	0.3485/4	0.0684/13	16.6	5.3
6	L,P,F,S,Al	0.858	0.022	0.0703	0.3575/5	0.0593/12	14.5	1.8
7	L,P,F,S,Al,A	0.864	0.006	0.0718	0.3601/6	0.0567/11	11.6	0.5
8	L,P,F,S,Al,A,E	0.873	0.009	0.0728	0.3639/7	0.0530/10	9.8	0.7
9	L,P,F,S,Al,A,E,I	0.873	0.000	0.0766	0.3640/8	0.0529/9	7.7	0.2

r^2 Coefficient of determination
Δr^2 Change in r^2
S_e Standard error of estimate
SS/df Sum of squares to degrees of freedom ratio for regression or residuals
F_t Total F-test value
F_p Partial F-test value
*S_y of criterion variable, $V_{0.01}$

Table 18–18 Residuals for example 18–5

Station no.	Predicted runoff volume (logs)	Observed runoff volume (logs)	Residual	Station no.	Predicted runoff volume (logs)	Observed runoff volume (logs)	Residual
11372000	1.0646	1.0491	–0.0155	11453500	1.2099	1.2752	0.0653
11374400	0.9631	0.8854	–0.0777	11453600	1.1487	1.2482	0.0995
11379500	1.0453	1.0137	–0.0316	11456000	1.2133	1.2098	–0.0035
11380500	0.8510	0.8214	–0.0296	11456500	1.1108	1.0460	–0.0648
11382000	0.9363	1.0644	0.1281	11457000	1.1455	1.1173	–0.0282
11448500	1.3413	1.2777	–0.0636	11458200	1.1261	1.1663	0.0402
11448900	1.3339	1.3195	–0.0144	11458500	1.0979	1.2027	0.1048
11451500	1.0611	1.0074	–0.0537	11459000	0.8610	0.8639	0.0029
11451720	1.0177	0.9486	–0.0691	11460000	1.2679	1.2788	0.0109
						Sum	0.0000

(c) Indirect estimation

The second technique for regionalization of watershed data is to use regression equations to relate the statistical characteristics of selected values to various basin characteristics. The probability level estimates are then derived from the frequency curve, based on the predicted statistical characteristics. Example 18–6 illustrates this technique.

Example 18–6 Development of indirect probability estimates

Given: The mean and standard deviations of the 1-day and 15-day high flow frequency curves were related to basin characteristics for 25 sites in the north coastal region of California, using the units cubic feet per second per day or days (ft^3/s-d), that is, the volume of water represented by a flow of 1 cubic foot per second for a period of 1 day or 15 days. Figures 18–14 through 18–17 show the relationships of the 25 stations used. The relationships of drainage area, mean annual precipitation, 1-day and 15-day high flow means and standard deviations were developed by regression. The predictor variables were selected because of availability of data. Tests were performed on each regression equation to verify that the mean of residuals is zero, the residuals are independent of each variable, the variance is constant, and that S_e is smaller than S_y the standard deviation of the criterion.

Activity: Develop 1- and 15-day high flow frequency curves for a 50-square-mile drainage area in the north coastal region of California with a mean annual precipitation of 60 inches.

$$\bar{X}_1 = 3{,}100 \text{ ft}^3\text{/s-d}$$ from figure 18–14

$$S_1 = 1{,}600 \text{ ft}^3\text{/s-d}$$ from figure 18–15

$$\gamma_1 = \frac{(\bar{X})^2}{S^2}$$ solution of equation 18–14 for γ

$$\gamma_1 = \frac{(3{,}100)^2}{(1{,}600)^2} = 3.75$$

$$G_1 = \frac{2}{\sqrt{3.75}} = 1.03$$ from equation 18–15

use 1.0

$$\bar{X}_{15} = 900 \text{ ft}^3\text{/s-d}$$ from figure 18–16

$$S_{15} = 360 \text{ ft}^3\text{/s-d}$$ from figure 18–17

$$\gamma_{15} = \frac{(900)^2}{(360)^2} = 6.25$$

$$G_{15} = \frac{2}{\sqrt{6.25}} = 0.8$$

use 0.8

Example 18–6 Development of indirect probability estimates—Continued

Using equation 18–16 as shown in table 18–19, determine the 1-day and 15-day high flow values for selected exceedance frequencies using the Pearson III probability distribution.

where:

$\bar{X}_1 = 3{,}100 \text{ ft}^3/\text{s-d}$ $\bar{X}_{15} = 900 \text{ ft}^3/\text{s-d}$

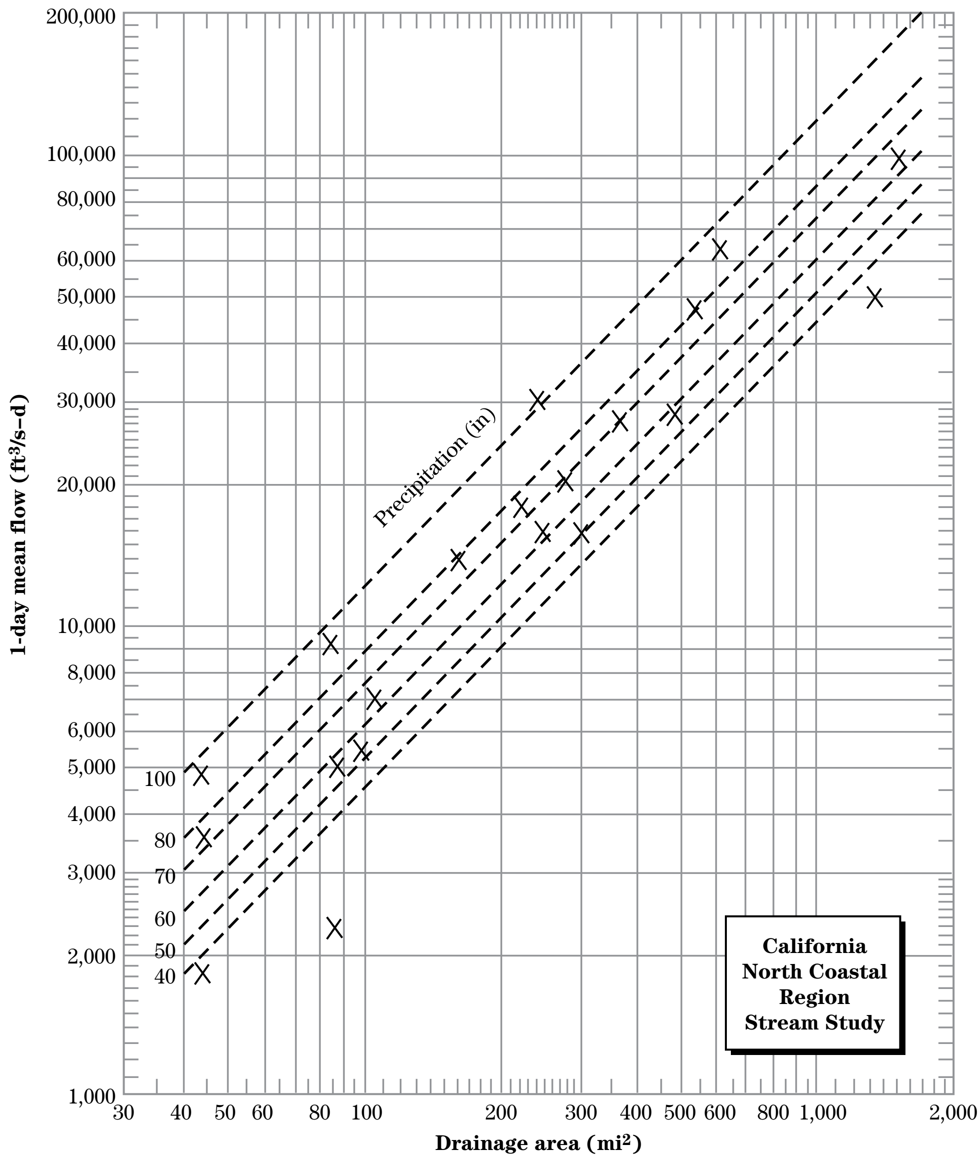
$S_1 = 1{,}600 \text{ ft}^3/\text{s-d}$ $S_{15} = 360 \text{ ft}^3/\text{s-d}$

Table 18–19 Frequency curve solutions for example 18–6

Exceed prob. (q)	Exhibit 18–3 K_p value (G = 1.0)	$V_1 = \bar{X}_1 + K_pS_1$ (ft^3/s-d)	Exhibit 18–3 K_p value (G = 0.8)	$V_{15} = \bar{X}_{15} + K_pS_{15}$ (ft^3/s-d)
99	–1.58838	559	–1.73271	276
95	–1.31684	993	–1.38855	400
80	–0.85161	1,737	–0.85607	592
50	–0.16397	2,838	–0.13199	852
20	0.75752	4,312	0.77986	1,181
10	1.34039	5,245	1.33640	1,381
4	2.04269	6,368	1.99311	1,618
2	2.54206	7,167	2.45298	1,783
1	3.02256	7,936	2.89101	1,941

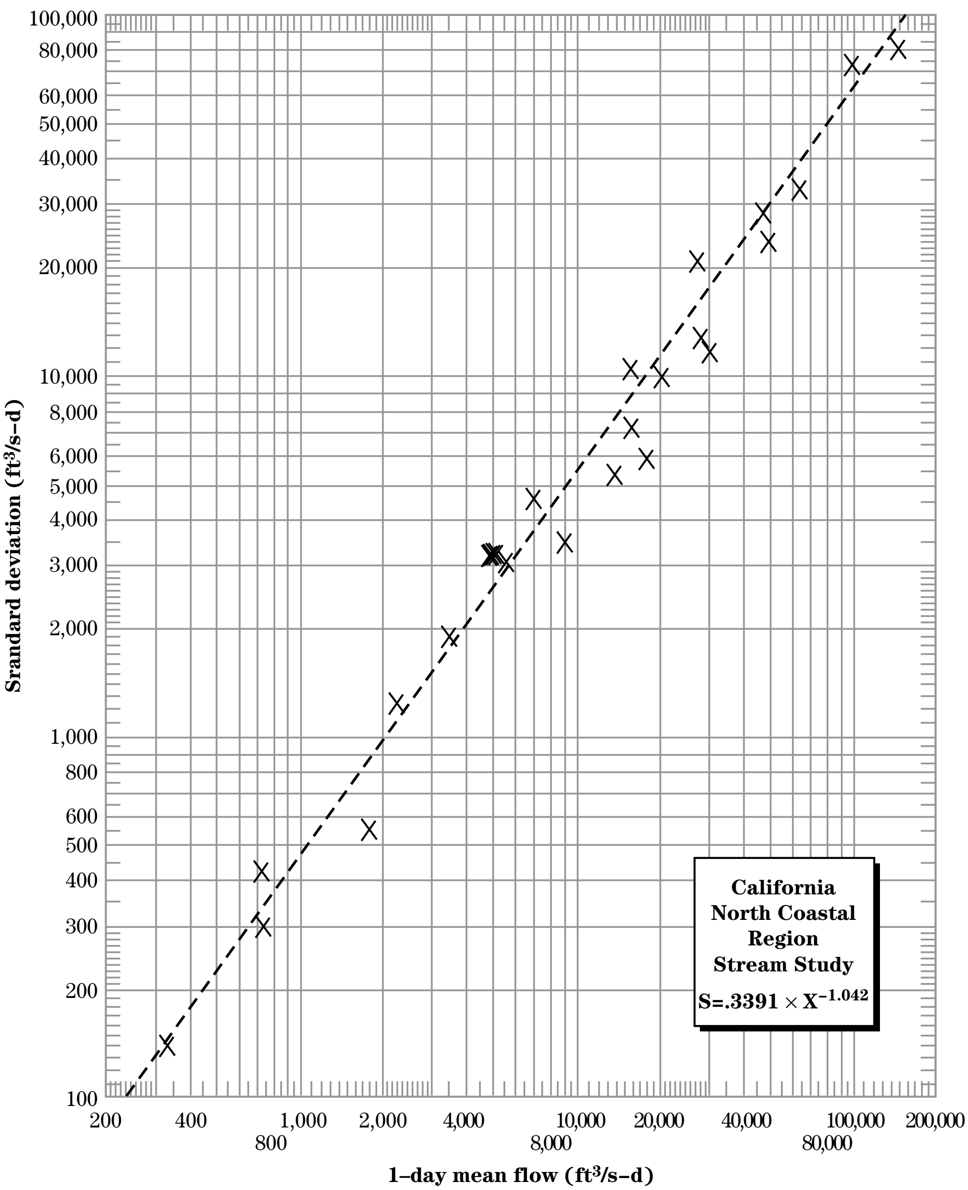
Example 18–6 Development of indirect probability estimates—Continued

Figure 18–14 Drainage area and mean annual precipitation for 1-day mean flow for example 18–6



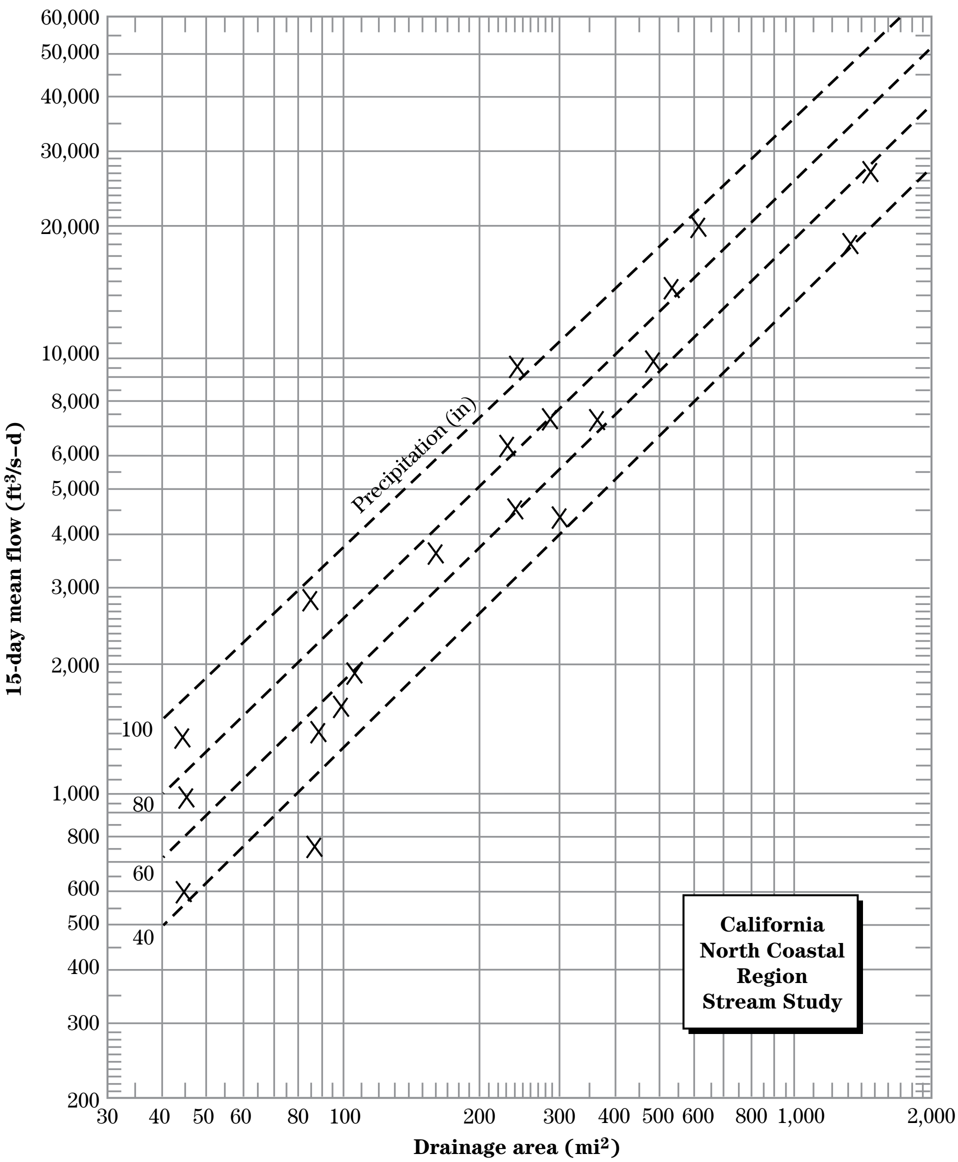
Example 18–6 Development of indirect probability estimates—Continued

Figure 18–15 One-day mean flow and standard deviation for example 18–6



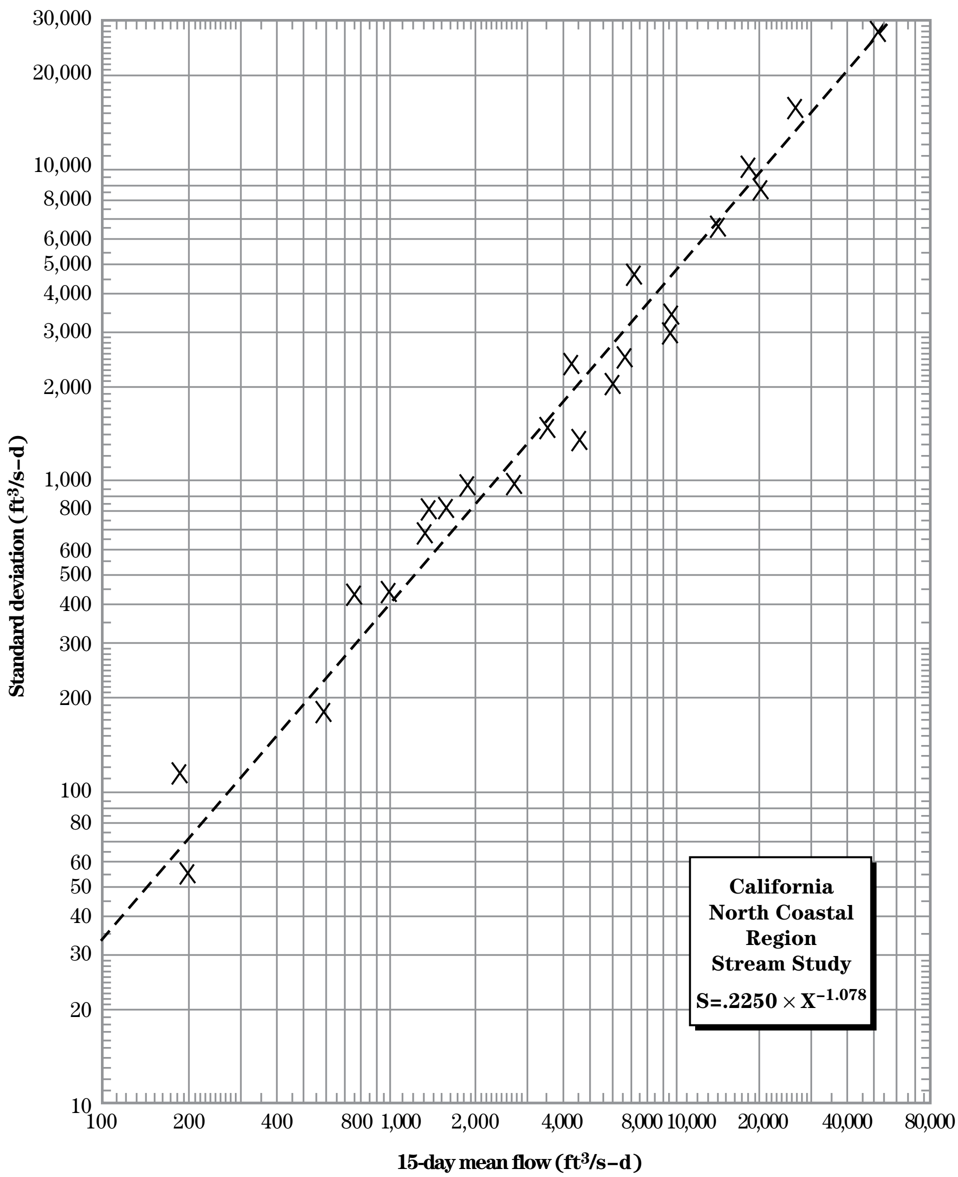
Example 18–6 Development of indirect probability estimates—Continued

Figure 18–16 Drainage area and mean annual precipitation for 15-day mean flow for example 18–6



Example 18–6 Development of indirect probability estimates—Continued

Figure 18–17 Fifteen-day mean flow and standard deviation for example 18–6



(d) Discussion

The basic uses of regionalization are to transfer data from gaged watersheds either to ungaged watersheds or to locations within gaged watersheds, and to calibrate water resource models. In using regionalization, however, certain basic limitations must be understood.

The prediction equation generally should be used only within the range of the predictor variables used to develop the equation. The prediction equation represents the "average" condition for the data. If the ungaged watershed varies significantly from the average condition, then the variation must be explained by one or more of the variables in the prediction equation. If the variation is not explained, the equation should not be used.

When the prediction equation is used to calibrate a watershed model, values estimated by the regression equation should deviate from the values computed by the model. The magnitude of this deviation is a function of how much the ungaged watershed differs from the average condition. For example, if most of the watersheds used to develop the prediction equation are flat and long and the ungaged watershed is steep and short, the peak flow computed with the watershed model could differ significantly from that estimated by the prediction equation. The prediction equation should not be used when the watershed characteristics are outside the range of those used to develop the equation.

The coefficients of the prediction equation must be rational. For example, peak flow is inversely proportional to the length of the main watercourse, if all other variables are constant. This means that when a logarithmic transformation is used, the power of the length variable should be negative. If a predictor variable has an irrational relationship in the equation, the correlation coefficients of all the predictor variables should be examined before the equation is used. A high correlation coefficient between two predictor variables means that one of the variables can be used to explain how the criterion variable varies with both predictor variables. The accuracy of the prediction equation is not improved by adding the second predictor variable; the equation merely becomes more complicated.

630.1806 Risk

Flood frequency analysis identifies the population from a sample of data. The population cannot be identified exactly when only a sample is available, and this represents an important element of uncertainty. A second source of uncertainty is that even if the population was known exactly, there is a finite chance that an event of a certain size will be exceeded.

The measurement of such uncertainty is called *risk*. Typical questions include:

- A channel is designed with a capacity of a 0.02 exceedance probability. Is it unreasonable to expect its capacity will be exceeded once or more in the next 10 years?
- What is the risk that an emergency spillway designed to pass a 2 percent chance flow will experience this flow twice or more in the next 10 years?
- Throughout the United States, the NRCS has built many flood-control structures. What percent will experience a 1 percent chance flood in the next 5 years? The next 10 years?

These problems can be solved by means of the binomial distribution. Basic assumptions in the use of the binomial distribution are given in the general discussion on distributions. These assumptions are generally valid for assessing risk in hydrology. The binomial expression for risk is:

$$R_I = \frac{N!}{I!(N-I)!} q^I (1-q)^{(N-I)} \qquad (18\text{–}30)$$

where:

R_I = estimated risk of obtaining in N time periods exactly I number of events with an exceedance probability q.

Examples 18–7 through 18–10 show the methods used to measure risk.

Example 18–7 Risk of future nonoccurrence

Problem: What is the probability that a 10 percent chance flood (q = 0.10) will not be exceeded in the next 5 years?

Solution: From equation 18–30, for N = 5, q = 0.10, and I = 0:

$$R_0 = \frac{(5)!}{0!(5)!} 0.10^0 (1-.10)^{(5-0)}$$

The probability of nonoccurrence is 0.59 or 59 percent; the probability of occurrence is $1 - R_0$ or 0.41.

Example 18–8 Risk of multiple occurrence

Problem: What is the probability that a 2 percent chance peak flow (q = 0.02) will be exceeded twice or more in the next 10 years?

Solution: For nonexceedance of the 2 percent chance event:

$$\begin{aligned} &N = 10,\ q = 0.02,\ I = 0 \\ R_0 &= \frac{(10)!}{0!(10)!}(0.02)^0 (1-0.02)^{10} \\ &= 0.817 \end{aligned}$$

For only one exceedance of the 2 percent chance event:

$$\begin{aligned} &N = 10,\ q = 0.02,\ I = 1 \\ R_1 &= \frac{(10)!}{1!(9)!}(0.02)^1 (1-0.02)^{9} \\ &= 0.167 \end{aligned}$$

For two or more exceedances of the 2 percent chance event:

$$\begin{aligned} R_{(2\text{ or more})} &= 1-(R_0 + R_1) \\ R_{(2\text{ or more})} &= 1-(0.817+0.167) \\ &= 0.016 \end{aligned}$$

In other words, there is a 1.6 percent chance of experiencing two or more peaks equal to or greater than the 2 percent chance peak flow within any 10-year period. If flood events are not related, probably no more than 16 locations in a thousand will, on the average, experience two or more floods equal to or greater than the 2 percent chance flood within the next 10 years.

Example 18–9 Risk of a selected exceedance probability

Given: 20-year record on a small creek.

Problem: What is the probability that the greatest flood of record is not a 5 percent chance event (q = 0.05)?

Solution: For nonoccurrence of the 5 percent chance event:

$$N = 20,\ q = 0.05,\ I = 0$$

$$\begin{aligned} R_0 &= \frac{20!}{0!20!}(0.05)^0(1-0.05)^{20} \\ &= 0.358 \end{aligned}$$

Therefore, there is a 36 percent chance of the 5 percent chance event not occurring and, conversely, a 64 percent chance that one or more will occur.

Example 18–10 Exceedance probability of a selected risk

Problem: What exceedance probability has a 50 percent chance of occurrence in a 20-year period?

Solution: For 50 percent occurrence in 20 years:

$$N = 20,\ q = ?,\ I = 0,\ R_0 = 0.5$$

$$\begin{aligned} 0.5 &= \frac{20!}{0!20!}(q)^0(1-q)^{(20-0)} \\ 0.5 &= (1-q)^{(20)} \\ 1-q &= (0.5)^{\frac{1}{20}} = 0.966 \\ q &= 0.034 \end{aligned}$$

Or, there is a 50 percent chance that a 3 percent chance event will occur within the 20-year period.

630.1807 Metric conversion factors

The English system of units is used in this report. To convert to the International System of units (metric), use the following factors:

To convert	To metric units	Multiply by English units
acres (acre)	hectares (ha)	0.405
square miles (mi^2)	square kilometers (km^2)	2.59
cubic feet per second (ft^3/s) 1/	cubic meters per second (m^3/s)	0.0283
cubic feet per second per day $ft^3/s–d$	cubic meters per second per day $m^3/s–d$	2,450
inches (in)	millimeters (mm)	25.4

1/ In converting stream discharge values, which are recorded in English units with only three significant digits, do not imply a greater precision than is present.

630.1808 References

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Exhibit 18–1 Five Percent Two-sided Critical Values for Outlier Detection

N	K_n	Low prob.	High prob.
10	2.294	0.9891048	0.0108952
11	2.343	0.9904353	0.0095647
12	2.387	0.9915068	0.0084932
13	2.426	0.9923669	0.0076331
14	2.461	0.9930725	0.0069275
15	2.493	0.9936665	0.0063335
16	2.523	0.9941821	0.0058179
17	2.551	0.9946293	0.0053707
18	2.577	0.9950169	0.0049831
19	2.600	0.9953388	0.0046612
20	2.623	0.9956420	0.0043580
21	2.644	0.9959034	0.0040966
22	2.664	0.9961391	0.0038609
23	2.683	0.9963517	0.0036483
24	2.701	0.9965434	0.0034566
25	2.717	0.9967061	0.0032939
26	2.734	0.9968715	0.0031285
27	2.751	0.9970293	0.0029707
28	2.768	0.9971799	0.0028201
29	2.781	0.9972904	0.0027096
30	2.794	0.9973969	0.0026031
31	2.808	0.9975075	0.0024925
32	2.819	0.9975913	0.0024087
33	2.833	0.9976943	0.0023057
34	2.846	0.9977863	0.0022137
35	2.858	0.9978684	0.0021316
36	2.869	0.9979411	0.0020589
37	2.880	0.9980116	0.0019884
38	2.890	0.9980738	0.0019262
39	2.900	0.9981341	0.0018659
40	2.910	0.9981928	0.0018072
41	2.919	0.9982442	0.0017558
42	2.925	0.9982777	0.0017223
43	2.937	0.9983429	0.0016571
44	2.945	0.9983852	0.0016148
45	2.954	0.9984316	0.0015684
46	2.960	0.9984618	0.0015382
47	2.970	0.9985110	0.0014890
48	2.978	0.9985493	0.0014507
49	2.985	0.9985821	0.0014179
50	2.993	0.9986187	0.0013813
51	3.000	0.9986501	0.0013499
52	3.007	0.9986808	0.0013192
53	3.013	0.9987066	0.0012934
54	3.020	0.9987361	0.0012639
55	3.025	0.9987568	0.0012432
56	3.032	0.9987853	0.0012147
57	3.040	0.9988171	0.0011829
58	3.046	0.9988404	0.0011596
59	3.051	0.9988596	0.0011404
60	3.058	0.9988859	0.0011141
61	3.063	0.9989043	0.0010957
62	3.070	0.9989297	0.0010703
63	3.075	0.9989474	0.0010526
64	3.082	0.9989719	0.0010281
65	3.086	0.9989856	0.0010144
66	3.090	0.9989992	0.0010008
67	3.096	0.9990192	0.0009808
68	3.101	0.9990356	0.0009644
69	3.105	0.9990486	0.0009514
70	3.110	0.9990645	0.0009355
71	3.115	0.9990802	0.0009198
72	3.121	0.9990988	0.0009012
73	3.125	0.9991109	0.0008891
74	3.130	0.9991260	0.0008740
75	3.134	0.9991378	0.0008622
76	3.138	0.9991494	0.0008506
77	3.142	0.9991609	0.0008391
78	3.148	0.9991780	0.0008220
79	3.152	0.9991892	0.0008108
80	3.157	0.9992030	0.0007970
81	3.161	0.9992138	0.0007862
82	3.164	0.9992219	0.0007781
83	3.168	0.9992325	0.0007675
84	3.172	0.9992430	0.0007570
85	3.176	0.9992533	0.0007467
86	3.180	0.9992636	0.0007364
87	3.184	0.9992737	0.0007263
88	3.188	0.9992837	0.0007163
89	3.191	0.9992911	0.0007089
90	3.194	0.9992984	0.0007016
91	3.198	0.9993080	0.0006920
92	3.202	0.9993176	0.0006824
93	3.205	0.9993247	0.0006753
94	3.208	0.9993317	0.0006683
95	3.211	0.9993386	0.0006614
96	3.214	0.9993455	0.0006545
97	3.217	0.9993523	0.0006477
98	3.220	0.9993590	0.0006410
99	3.224	0.9993679	0.0006321
100	3.228	0.9993767	0.0006233

Note: K_n values are positive for high outliers and negative for low outliers.

Exhibit 18–2 Expected Values of Normal Order Statistics, K_n

N	10	11	12	13	14	15	16
1	1.53875	1.58644	1.62923	1.66799	1.70338	1.73591	1.76599
2	1.00136	1.06192	1.11573	1.16408	1.20790	1.24794	1.28474
3	0.65606	0.72884	0.79284	0.84983	0.90113	0.94769	0.99027
4	0.37576	0.46198	0.53684	0.60285	0.66176	0.71488	0.76317
5	0.12267	0.22489	0.31225	0.38833	0.45557	0.51570	0.57001
6		0.00000	0.10259	0.19052	0.26730	0.33530	0.39622
7				0.00000	0.08816	0.16530	0.23375
8						0.00000	0.07729

N	17	18	19	20	21	22	23
1	1.79394	1.82003	1.84448	1.86748	1.88917	1.90969	1.92916
2	1.31878	1.35041	1.37994	1.40760	1.43362	1.45816	1.48137
3	1.02946	1.06573	1.09945	1.13095	1.16047	1.18824	1.21445
4	0.80738	0.84812	0.88586	0.92098	0.95380	0.98459	1.01356
5	0.61946	0.66479	0.70661	0.74538	0.78150	0.81527	0.84697
6	0.45133	0.50158	0.54771	0.59030	0.62982	0.66667	0.70115
7	0.29519	0.35084	0.40164	0.44833	0.49148	0.53157	0.56896
8	0.14599	0.20774	0.26374	0.31493	0.36203	0.40559	0.44609
9	0.00000	0.06880	0.13072	0.18696	0.23841	0.28579	0.32965
10			0.00000	0.06200	0.11836	0.16997	0.21755
11					0.00000	0.05642	0.10813
12							0.00000

N	24	25	26	27	28	29	30
1	1.94767	1.96531	1.98216	1.99827	2.01371	2.02852	2.04276
2	1.50338	1.52430	1.54423	1.56326	1.58145	1.59888	1.61560
3	1.23924	1.26275	1.28511	1.30641	1.32674	1.34619	1.36481
4	1.04091	1.06679	1.09135	1.11471	1.13697	1.15822	1.17855
5	0.87682	0.90501	0.93171	0.95705	0.98115	1.00414	1.02609
6	0.73354	0.76405	0.79289	0.82021	0.84615	0.87084	0.89439
7	0.60399	0.63690	0.66794	0.69727	0.72508	0.75150	0.77666
8	0.48391	0.51935	0.55267	0.58411	0.61385	0.64205	0.66885
9	0.37047	0.40860	0.44436	0.47801	0.50977	0.53982	0.56834
10	0.26163	0.30268	0.34105	0.37706	0.41096	0.44298	0.47329
11	0.15583	0.20006	0.24128	0.27983	0.31603	0.35013	0.38235
12	0.05176	0.09953	0.14387	0.18520	0.22389	0.26023	0.29449
13		0.00000	0.04781	0.09220	0.13361	0.17240	0.20885
14				0.00000	0.04442	0.08588	0.12473
15						0.00000	0.04148

Exhibit 18–2 Expected values of normal order statistics, K_n—Continued

N	31	32	33	34	35	36	37
1	2.05646	2.06967	2.08241	2.09471	2.10661	2.11812	2.12928
2	1.63166	1.64712	1.66200	1.67636	1.69023	1.70362	1.71659
3	1.38268	1.39985	1.41637	1.43228	1.44762	1.46244	1.47676
4	1.19803	1.21672	1.23468	1.25196	1.26860	1.28466	1.30016
5	1.04709	1.06721	1.08652	1.10509	1.12295	1.14016	1.15677
6	0.91688	0.93841	0.95905	0.97886	0.99790	1.01624	1.03390
7	0.80066	0.82359	0.84555	0.86660	0.88681	0.90625	0.92496
8	0.69438	0.71875	0.74204	0.76435	0.78574	0.80629	0.82605
9	0.59545	0.62129	0.64596	0.66954	0.69214	0.71382	0.73465
10	0.50206	0.52943	0.55552	0.58043	0.60427	0.62710	0.64902
11	0.41287	0.44185	0.46942	0.49572	0.52084	0.54488	0.56793
12	0.32686	0.35755	0.38669	0.41444	0.44091	0.46620	0.49042
13	0.24322	0.27573	0.30654	0.33582	0.36371	0.39032	0.41576
14	0.16126	0.19572	0.22832	0.25924	0.28863	0.31663	0.34336
15	0.08037	0.11695	0.15147	0.18415	0.21515	0.24463	0.27272
16	0.00000	0.03890	0.07552	0.11009	0.14282	0.17388	0.20342
17			0.00000	0.03663	0.07123	0.10399	0.13509
18					0.00000	0.03461	0.06739
19							0.00000

N	38	39	40	41	42	43	44
1	2.14009	2.15059	2.16078	2.17068	2.18032	2.18969	2.19882
2	1.72914	1.74131	1.75312	1.76458	1.77571	1.78654	1.79707
3	1.49061	1.50402	1.51702	1.52964	1.54188	1.55377	1.56533
4	1.31514	1.32964	1.34368	1.35728	1.37048	1.38329	1.39574
5	1.17280	1.18830	1.20330	1.21782	1.23190	1.24556	1.25881
6	1.05095	1.06741	1.08332	1.09872	1.11364	1.12810	1.14213
7	0.94300	0.96041	0.97722	0.99348	1.00922	1.02446	1.03924
8	0.84508	0.86343	0.88114	0.89825	0.91480	0.93082	0.94634
9	0.75468	0.77398	0.79259	0.81056	0.82792	0.84472	0.86097
10	0.67009	0.69035	0.70988	0.72871	0.74690	0.76448	0.78148
11	0.59005	0.61131	0.63177	0.65149	0.67052	0.68889	0.70666
12	0.51363	0.53592	0.55736	0.57799	0.59788	0.61707	0.63561
13	0.44012	0.46348	0.48591	0.50749	0.52827	0.54830	0.56763
14	0.36892	0.39340	0.41688	0.43944	0.46114	0.48204	0.50220
15	0.29954	0.32520	0.34978	0.37337	0.39604	0.41784	0.43885
16	0.23159	0.25849	0.28423	0.30890	0.33257	0.35533	0.37723
17	0.16469	0.19292	0.21988	0.24569	0.27043	0.29418	0.31701
18	0.09853	0.12817	0.15644	0.18345	0.20931	0.23411	0.25792
19	0.03280	0.06395	0.09362	0.12192	0.14897	0.17488	0.19972
20		0.00000	0.03117	0.06085	0.08917	0.11625	0.14219
21				0.00000	0.02969	0.05803	0.08513
22						0.00000	0.02835

Exhibit 18–2 Expected values of normal order statistics, K_n—Continued

N	45	46	47	48	49	50	51
1	2.20772	2.21639	2.22486	2.23312	2.24119	2.24907	2.25678
2	1.80733	1.81732	1.82706	1.83655	1.84582	1.85487	1.86371
3	1.57658	1.58754	1.59820	1.60860	1.61874	1.62863	1.63829
4	1.40784	1.41962	1.43108	1.44224	1.45312	1.46374	1.47409
5	1.27170	1.28422	1.29641	1.30827	1.31983	1.33109	1.34207
6	1.15576	1.16899	1.18186	1.19439	1.20658	1.21846	1.23003
7	1.05358	1.06751	1.08104	1.09420	1.10701	1.11948	1.13162
8	0.96139	0.97599	0.99018	1.00396	1.01737	1.03042	1.04312
9	0.87673	0.89201	0.90684	0.92125	0.93525	0.94887	0.96213
10	0.79795	0.81391	0.82939	0.84442	0.85902	0.87321	0.88701
11	0.72385	0.74049	0.75663	0.77228	0.78748	0.80225	0.81661
12	0.65353	0.67088	0.68768	0.70397	0.71978	0.73513	0.75004
13	0.58631	0.60438	0.62186	0.63881	0.65523	0.67117	0.68666
14	0.52166	0.54046	0.55865	0.57625	0.59331	0.60986	0.62592
15	0.45912	0.47868	0.49759	0.51588	0.53360	0.55077	0.56742
16	0.39833	0.41868	0.43834	0.45734	0.47573	0.49354	0.51080
17	0.33898	0.36016	0.38060	0.40034	0.41942	0.43789	0.45578
18	0.28081	0.30285	0.32410	0.34460	0.36441	0.38357	0.40211
19	0.22358	0.24652	0.26862	0.28992	0.31049	0.33036	0.34957
20	0.16707	0.19097	0.21396	0.23610	0.25746	0.27807	0.29799
21	0.11109	0.13600	0.15993	0.18296	0.20514	0.22653	0.24719
22	0.05546	0.08144	0.10637	0.13033	0.15338	0.17559	0.19702
23	0.00000	0.02712	0.05311	0.07805	0.10203	0.12511	0.14735
24			0.00000	0.02599	0.05095	0.07494	0.09803
25					0.00000	0.02496	0.04896
26							0.00000

N	52	53	54	55	56	57	58
1	2.26432	2.27169	2.27891	2.28598	2.29291	2.29970	2.30635
2	1.87235	1.88080	1.88906	1.89715	1.90506	1.91282	1.92041
3	1.64773	1.65695	1.66596	1.67478	1.68340	1.69185	1.70012
4	1.48420	1.49407	1.50372	1.51315	1.52237	1.53140	1.54024
5	1.35279	1.36326	1.37348	1.38346	1.39323	1.40278	1.41212
6	1.24132	1.25234	1.26310	1.27361	1.28387	1.29391	1.30373
7	1.14347	1.15502	1.16629	1.17729	1.18804	1.19855	1.20882
8	1.05550	1.06757	1.07934	1.09083	1.10205	1.11300	1.12371
9	0.97504	0.98762	0.99988	1.01185	1.02352	1.03493	1.04607
10	0.90045	0.91354	0.92629	0.93873	0.95086	0.96271	0.97427

Exhibit 18–2 Expected values of normal order statistics, K_n—Continued

N	52	53	54	55	56	57	58
11	0.83058	0.84417	0.85742	0.87033	0.88292	0.89520	0.90719
12	0.76455	0.77866	0.79240	0.80578	0.81883	0.83155	0.84397
13	0.70170	0.71633	0.73057	0.74444	0.75794	0.77111	0.78396
14	0.64152	0.65668	0.67143	0.68578	0.69976	0.71337	0.72665
15	0.58358	0.59928	0.61455	0.62940	0.64385	0.65793	0.67164
16	0.52755	0.54380	0.55960	0.57495	0.58989	0.60444	0.61860
17	0.47312	0.48995	0.50629	0.52217	0.53761	0.55263	0.56725
18	0.42007	0.43749	0.45439	0.47080	0.48675	0.50226	0.51736
19	0.36818	0.38621	0.40369	0.42065	0.43713	0.45314	0.46872
20	0.31726	0.33592	0.35400	0.37154	0.38856	0.40510	0.42117
21	0.26716	0.28648	0.30518	0.32331	0.34090	0.35797	0.37456
22	0.21772	0.23772	0.25708	0.27583	0.29400	0.31163	0.32875
23	0.16880	0.18953	0.20957	0.22896	0.24774	0.26595	0.28362
24	0.12029	0.14177	0.16252	0.18259	0.20201	0.22082	0.23906
25	0.07206	0.09434	0.11584	0.13661	0.15669	0.17614	0.19498
26	0.02400	0.04712	0.06940	0.09091	0.11170	0.13180	0.15127
27		0.00000	0.02312	0.04541	0.06693	0.08773	0.10785
28				0.00000	0.02229	0.04382	0.06463
29						0.00000	0.02153

N	59	60	61	62	63	64	65
1	2.31288	2.31928	2.32556	2.33173	2.33778	2.34373	2.34958
2	1.92786	1.93516	1.94232	1.94934	1.95624	1.96301	1.96965
3	1.70822	1.71616	1.72394	1.73158	1.73906	1.74641	1.75363
4	1.54889	1.55736	1.56567	1.57381	1.58180	1.58963	1.59732
5	1.42127	1.43023	1.43900	1.44760	1.45603	1.46430	1.47241
6	1.31334	1.32274	1.33195	1.34097	1.34982	1.35848	1.36698
7	1.21886	1.22869	1.23832	1.24774	1.25698	1.26603	1.27490
8	1.13419	1.14443	1.15445	1.16427	1.17388	1.18329	1.19252
9	1.05695	1.06760	1.07802	1.08821	1.09819	1.10797	1.11754
10	0.98557	0.99662	1.00742	1.01799	1.02833	1.03846	1.04838
11	0.91890	0.93034	0.94153	0.95247	0.96317	0.97365	0.98391
12	0.85609	0.86793	0.87950	0.89081	0.90187	0.91270	0.92329
13	0.79649	0.80873	0.82068	0.83237	0.84379	0.85496	0.86590
14	0.73960	0.75224	0.76459	0.77665	0.78843	0.79996	0.81123
15	0.68502	0.69807	0.71081	0.72324	0.73540	0.74727	0.75889

Exhibit 18–2 Expected values of normal order statistics, K_n—Continued

N	59	60	61	62	63	64	65
16	0.63241	0.64587	0.65901	0.67183	0.68436	0.69659	0.70856
17	0.58150	0.59538	0.60893	0.62214	0.63504	0.64764	0.65996
18	0.53205	0.54637	0.56033	0.57395	0.58723	0.60020	0.61288
19	0.48388	0.49864	0.51303	0.52705	0.54073	0.55408	0.56712
20	0.43681	0.45202	0.46685	0.48129	0.49537	0.50911	0.52252
21	0.39068	0.40637	0.42164	0.43652	0.45101	0.46515	0.47894
22	0.34538	0.36155	0.37729	0.39260	0.40752	0.42207	0.43625
23	0.30078	0.31745	0.33366	0.34944	0.36480	0.37976	0.39435
24	0.25677	0.27396	0.29066	0.30691	0.32272	0.33812	0.35312
25	0.21325	0.23098	0.24820	0.26494	0.28122	0.29706	0.31249
26	0.17013	0.18842	0.20618	0.22343	0.24019	0.25650	0.27237
27	0.12733	0.14621	0.16452	0.18230	0.19957	0.21636	0.23269
28	0.08476	0.10425	0.12315	0.14148	0.15927	0.17656	0.19337
29	0.04234	0.06248	0.08198	0.10089	0.11923	0.13704	0.15435
30	0.00000	0.02081	0.04096	0.06047	0.07938	0.09774	0.11556
31			0.00000	0.02014	0.03966	0.05858	0.07694
32					0.00000	0.01952	0.03844
33							0.00000

N	66	67	68	69	70	71	72
1	2.35532	2.36097	2.36652	2.37199	2.37736	2.38265	2.38785
2	1.97618	1.98260	1.98891	1.99510	2.00120	2.00720	2.01310
3	1.76071	1.76767	1.77451	1.78122	1.78783	1.79432	1.80071
4	1.60487	1.61228	1.61955	1.62670	1.63373	1.64063	1.64742
5	1.48036	1.48817	1.49584	1.50338	1.51078	1.51805	1.52520
6	1.37532	1.38351	1.39154	1.39942	1.40717	1.41478	1.42226
7	1.28360	1.29213	1.30051	1.30873	1.31680	1.32473	1.33252
8	1.20157	1.21044	1.21915	1.22769	1.23608	1.24431	1.25240
9	1.12693	1.13613	1.14516	1.15401	1.16270	1.17123	1.17961
10	1.05810	1.06762	1.07696	1.08612	1.09511	1.10393	1.11259
11	0.99395	1.00380	1.01345	1.02291	1.03220	1.04130	1.05024
12	0.93367	0.94383	0.95379	0.96355	0.97313	0.98252	0.99173
13	0.87660	0.88708	0.89735	0.90741	0.91728	0.92695	0.93644
14	0.82226	0.83306	0.84364	0.85400	0.86416	0.87412	0.88388
15	0.77025	0.78138	0.79226	0.80293	0.81338	0.82362	0.83366

Exhibit 18–2 Expected values of normal order statistics, K_n—Continued

N	66	67	68	69	70	71	72
16	0.72025	0.73170	0.74290	0.75387	0.76462	0.77514	0.78546
17	0.67200	0.68377	0.69529	0.70657	0.71761	0.72843	0.73903
18	0.62526	0.63737	0.64921	0.66080	0.67214	0.68325	0.69413
19	0.57985	0.59230	0.60447	0.61638	0.62803	0.63943	0.65060
20	0.53561	0.54841	0.56091	0.57314	0.58510	0.59681	0.60827
21	0.49240	0.50555	0.51839	0.53095	0.54323	0.55525	0.56701
22	0.45009	0.46360	0.47680	0.48969	0.50230	0.51463	0.52669
23	0.40857	0.42245	0.43601	0.44925	0.46219	0.47484	0.48721
24	0.36775	0.38201	0.39594	0.40953	0.42281	0.43579	0.44848
25	0.32753	0.34219	0.35649	0.37045	0.38408	0.39739	0.41041
26	0.28784	0.30290	0.31759	0.33192	0.34591	0.35958	0.37292
27	0.24859	0.26408	0.27917	0.29389	0.30825	0.32227	0.33596
28	0.20973	0.22565	0.24116	0.25627	0.27102	0.28540	0.29945
29	0.17118	0.18755	0.20349	0.21902	0.23416	0.24893	0.26333
30	0.13288	0.14972	0.16611	0.18207	0.19762	0.21277	0.22756
31	0.09478	0.11211	0.12896	0.14536	0.16134	0.17690	0.19208
32	0.05681	0.07465	0.09199	0.10885	0.12527	0.14125	0.15683
33	0.01893	0.03730	0.05514	0.07249	0.08936	0.10579	0.12178
34		0.00000	0.01837	0.03622	0.05357	0.07045	0.08688
35				0.00000	0.01785	0.03520	0.05209
36						0.00000	0.01736

N	73	74	75	76	77	78	79
1	2.39298	2.39802	2.40299	2.40789	2.41271	2.41747	2.42215
2	2.01890	2.02462	2.03024	2.03578	2.04124	2.04662	2.05191
3	1.80699	1.81317	1.81926	1.82525	1.83115	1.83696	1.84268
4	1.65410	1.66067	1.66714	1.67350	1.67976	1.68592	1.69200
5	1.53223	1.53914	1.54594	1.55263	1.55921	1.56569	1.57207
6	1.42961	1.43684	1.44395	1.45094	1.45782	1.46459	1.47125
7	1.34017	1.34770	1.35510	1.36237	1.36953	1.37657	1.38350
8	1.26034	1.26815	1.27583	1.28338	1.29080	1.29810	1.30529
9	1.18784	1.19592	1.20387	1.21168	1.21936	1.22691	1.23434
10	1.12110	1.12945	1.13766	1.14572	1.15365	1.16145	1.16912
11	1.05902	1.06764	1.07610	1.08442	1.09260	1.10063	1.10854
12	1.00078	1.00966	1.01838	1.02695	1.03537	1.04364	1.05178
13	0.94576	0.95490	0.96387	0.97269	0.98135	0.98986	0.99822
14	0.89346	0.90286	0.91209	0.92115	0.93005	0.93880	0.94739
15	0.84351	0.85317	0.86265	0.87196	0.88110	0.89008	0.89890

Exhibit 18–2 Expected values of normal order statistics, K_n—Continued

N	73	74	75	76	77	78	79
16	0.79558	0.80550	0.81524	0.82480	0.83418	0.84339	0.85244
17	0.74942	0.75960	0.76960	0.77940	0.78903	0.79848	0.80776
18	0.70480	0.71526	0.72551	0.73557	0.74544	0.75512	0.76463
19	0.66155	0.67227	0.68279	0.69310	0.70322	0.71314	0.72289
20	0.61950	0.63050	0.64128	0.65185	0.66222	0.67239	0.68237
21	0.57852	0.58980	0.60085	0.61168	0.62230	0.63272	0.64294
22	0.53850	0.55006	0.56138	0.57248	0.58336	0.59403	0.60449
23	0.49932	0.51117	0.52277	0.53414	0.54528	0.55621	0.56692
24	0.46089	0.47304	0.48493	0.49657	0.50798	0.51917	0.53013
25	0.42313	0.43558	0.44777	0.45970	0.47138	0.48283	0.49404
26	0.38597	0.39873	0.41122	0.42343	0.43540	0.44711	0.45859
27	0.34934	0.36242	0.37521	0.38772	0.39997	0.41196	0.42371
28	0.31317	0.32657	0.33968	0.35250	0.36504	0.37731	0.38934
29	0.27740	0.29114	0.30457	0.31770	0.33055	0.34311	0.35542
30	0.24199	0.25608	0.26984	0.28329	0.29645	0.30931	0.32190
31	0.20688	0.22133	0.23543	0.24922	0.26269	0.27586	0.28875
32	0.17202	0.18684	0.20130	0.21543	0.22923	0.24272	0.25591
33	0.13737	0.15257	0.16740	0.18188	0.19602	0.20983	0.22334
34	0.10289	0.11848	0.13370	0.14854	0.16303	0.17718	0.19101
35	0.06852	0.08453	0.10014	0.11536	0.13021	0.14471	0.15888
36	0.03424	0.05068	0.06670	0.08231	0.09754	0.11240	0.12691
37	0.00000	0.01689	0.03333	0.04935	0.06497	0.08020	0.09507
38			0.00000	0.01644	0.03247	0.04809	0.06333
39					0.00000	0.01602	0.03165
40							0.00000

N	80	81	82	83	84	85	86
1	2.42677	2.43133	2.43582	2.44026	2.44463	2.44894	2.45320
2	2.05714	2.06228	2.06735	2.07236	2.07729	2.08216	2.08696
3	1.84832	1.85387	1.85935	1.86475	1.87007	1.87532	1.88049
4	1.69798	1.70387	1.70968	1.71540	1.72104	1.72660	1.73209
5	1.57836	1.58455	1.59065	1.59665	1.60258	1.60841	1.61417
6	1.47781	1.48428	1.49064	1.49691	1.50309	1.50918	1.51518
7	1.39032	1.39704	1.40366	1.41017	1.41659	1.42292	1.42915
8	1.31236	1.31932	1.32617	1.33292	1.33957	1.34611	1.35257
9	1.24165	1.24884	1.25593	1.26290	1.26977	1.27653	1.28320
10	1.17666	1.18409	1.19139	1.19859	1.20567	1.21264	1.21951

Exhibit 18–2 Expected values of normal order statistics, K_n—Continued

N	80	81	82	83	84	85	86
11	1.11631	1.12396	1.13148	1.13889	1.14618	1.15336	1.16043
12	1.05978	1.06764	1.07539	1.08300	1.09050	1.09788	1.10515
13	1.00644	1.01453	1.02249	1.03031	1.03802	1.04560	1.05306
14	0.95584	0.96414	0.97231	0.98034	0.98825	0.99603	1.00369
15	0.90757	0.91609	0.92447	0.93271	0.94082	0.94880	0.95665
16	0.86134	0.87007	0.87867	0.88711	0.89542	0.90360	0.91164
17	0.81687	0.82583	0.83464	0.84329	0.85180	0.86017	0.86841
18	0.77398	0.78315	0.79217	0.80103	0.80975	0.81832	0.82675
19	0.73246	0.74186	0.75109	0.76016	0.76908	0.77785	0.78647
20	0.69217	0.70179	0.71124	0.72053	0.72965	0.73862	0.74744
21	0.65297	0.66282	0.67249	0.68199	0.69133	0.70050	0.70952
22	0.61476	0.62484	0.63473	0.64445	0.65399	0.66337	0.67259
23	0.57742	0.58773	0.59785	0.60779	0.61755	0.62714	0.63656
24	0.54088	0.55143	0.56178	0.57193	0.58191	0.59171	0.60133
25	0.50504	0.51583	0.52641	0.53680	0.54700	0.55701	0.56684
26	0.46985	0.48088	0.49170	0.50232	0.51274	0.52297	0.53301
27	0.43522	0.44651	0.45757	0.46842	0.47907	0.48952	0.49979
28	0.40111	0.41265	0.42397	0.43506	0.44594	0.45662	0.46710
29	0.36747	0.37927	0.39084	0.40218	0.41330	0.42421	0.43491
30	0.33423	0.34630	0.35813	0.36972	0.38108	0.39223	0.40316
31	0.30136	0.31371	0.32580	0.33765	0.34926	0.36065	0.37182
32	0.26881	0.28144	0.29381	0.30592	0.31779	0.32943	0.34084
33	0.23655	0.24947	0.26212	0.27450	0.28664	0.29852	0.31018
34	0.20453	0.21775	0.23069	0.24335	0.25576	0.26790	0.27981
35	0.17272	0.18625	0.19949	0.21244	0.22512	0.23753	0.24970
36	0.14108	0.15493	0.16848	0.18172	0.19469	0.20738	0.21981
37	0.10959	0.12377	0.13763	0.15118	0.16444	0.17741	0.19012
38	0.07820	0.09272	0.10691	0.12078	0.13434	0.14761	0.16059
39	0.04689	0.06177	0.07629	0.09049	0.10436	0.11793	0.13121
40	0.01562	0.03087	0.04575	0.06028	0.07448	0.08836	0.10193
41		0.00000	0.01524	0.03013	0.04466	0.05886	0.07275
42				0.00000	0.01488	0.02942	0.04362
43						0.00000	0.01454

Exhibit 18–2 Expected values of normal order statistics, K_n—Continued

N	87	88	89	90	91	92	93
1	2.45741	2.46156	2.46565	2.46970	2.47370	2.47764	2.48154
2	2.09170	2.09637	2.10099	2.10554	2.11004	2.11448	2.11887
3	1.88560	1.89064	1.89561	1.90052	1.90536	1.91015	1.91487
4	1.73750	1.74283	1.74810	1.75329	1.75842	1.76348	1.76848
5	1.61984	1.62544	1.63096	1.63641	1.64178	1.64709	1.65232
6	1.52110	1.52693	1.53269	1.53836	1.54396	1.54949	1.55494
7	1.43529	1.44135	1.44732	1.45321	1.45903	1.46476	1.47042
8	1.35893	1.36520	1.37138	1.37747	1.38348	1.38941	1.39526
9	1.28976	1.29624	1.30262	1.30891	1.31511	1.32123	1.32726
10	1.22628	1.23295	1.23952	1.24600	1.25239	1.25869	1.26491
11	1.16740	1.17426	1.18102	1.18769	1.19426	1.20073	1.20712
12	1.11231	1.11936	1.12631	1.13316	1.13990	1.14656	1.15311
13	1.06041	1.06765	1.07478	1.08181	1.08873	1.09555	1.10228
14	1.01122	1.01865	1.02596	1.03316	1.04026	1.04726	1.05415
15	0.96437	0.97198	0.97948	0.98686	0.99413	1.00129	1.00835
16	0.91956	0.92735	0.93502	0.94258	0.95002	0.95735	0.96458
17	0.87651	0.88449	0.89234	0.90007	0.90769	0.91519	0.92258
18	0.83504	0.84320	0.85123	0.85914	0.86693	0.87460	0.88215
19	0.79496	0.80330	0.81152	0.81960	0.82756	0.83540	0.84312
20	0.75611	0.76465	0.77304	0.78131	0.78944	0.79745	0.80533
21	0.71838	0.72710	0.73568	0.74412	0.75243	0.76061	0.76866
22	0.68165	0.69056	0.69932	0.70795	0.71643	0.72478	0.73300
23	0.64581	0.65492	0.66387	0.67267	0.68134	0.68986	0.69825
24	0.61079	0.62009	0.62923	0.63822	0.64706	0.65576	0.66432
25	0.57650	0.58600	0.59533	0.60451	0.61353	0.62241	0.63115
26	0.54288	0.55258	0.56210	0.57147	0.58068	0.58974	0.59865
27	0.50986	0.51976	0.52949	0.53905	0.54845	0.55769	0.56678
28	0.47739	0.48750	0.49743	0.50718	0.51677	0.52620	0.53547
29	0.44542	0.45574	0.46587	0.47582	0.48561	0.49522	0.50468
30	0.41389	0.42443	0.43477	0.44493	0.45491	0.46472	0.47436
31	0.38278	0.39353	0.40409	0.41445	0.42463	0.43464	0.44447
32	0.35203	0.36300	0.37378	0.38436	0.39474	0.40495	0.41498
33	0.32161	0.33281	0.34381	0.35461	0.36520	0.37561	0.38584
34	0.29148	0.30292	0.31415	0.32517	0.33598	0.34660	0.35702
35	0.26162	0.27330	0.28476	0.29601	0.30704	0.31787	0.32850

Exhibit 18–2 Expected values of normal order statistics, K_n—Continued

N	87	88	89	90	91	92	93
36	0.23199	0.24392	0.25562	0.26710	0.27835	0.28940	0.30025
37	0.20256	0.21475	0.22669	0.23841	0.24990	0.26117	0.27223
38	0.17330	0.18576	0.19796	0.20991	0.22164	0.23314	0.24443
39	0.14420	0.15692	0.16938	0.18159	0.19356	0.20530	0.21681
40	0.11521	0.12821	0.14094	0.15341	0.16563	0.17761	0.18936
41	0.08633	0.09961	0.11262	0.12536	0.13783	0.15006	0.16205
42	0.05751	0.07110	0.08439	0.09740	0.11014	0.12262	0.13486
43	0.02874	0.04263	0.05622	0.06952	0.08253	0.09528	0.10777
44	0.00000	0.01421	0.02810	0.04169	0.05499	0.06801	0.08076
45			0.00000	0.01389	0.02748	0.04078	0.05381
46					0.00000	0.01359	0.02689
47							0.00000

N	94	95	96	97	98	99	100
1	2.48540	2.48920	2.49297	2.49669	2.50036	2.50400	2.50759
2	2.12321	2.12749	2.13172	2.13590	2.14003	2.14411	2.14814
3	1.91953	1.92414	1.92869	1.93318	1.93763	1.94201	1.94635
4	1.77341	1.77828	1.78309	1.78784	1.79254	1.79718	1.80176
5	1.65749	1.66259	1.66763	1.67261	1.67752	1.68238	1.68718
6	1.56033	1.56564	1.57089	1.57607	1.58118	1.58624	1.59123
7	1.47600	1.48151	1.48695	1.49232	1.49762	1.50286	1.50803
8	1.40103	1.40673	1.41235	1.41790	1.42338	1.42879	1.43414
9	1.33321	1.33909	1.34489	1.35061	1.35626	1.36183	1.36734
10	1.27104	1.27708	1.28305	1.28894	1.29475	1.30049	1.30615
11	1.21342	1.21964	1.22577	1.23182	1.23779	1.24368	1.24950
12	1.15958	1.16596	1.17226	1.17847	1.18459	1.19064	1.19661
13	1.10891	1.11546	1.12191	1.12827	1.13455	1.14075	1.14687
14	1.06095	1.06765	1.07426	1.08078	1.08721	1.09356	1.09982
15	1.01531	1.02217	1.02894	1.03561	1.04219	1.04868	1.05509
16	0.97170	0.97872	0.98564	0.99246	0.99919	1.00583	1.01238
17	0.92986	0.93704	0.94411	0.95109	0.95797	0.96475	0.97145
18	0.88959	0.89693	0.90416	0.91129	0.91831	0.92524	0.93208
19	0.85072	0.85822	0.86560	0.87288	0.88006	0.88713	0.89411
20	0.81310	0.82075	0.82829	0.83572	0.84305	0.85027	0.85739

Exhibit 18–2 Expected values of normal order statistics, K_n—Continued

N	94	95	96	97	98	99	100
21	0.77659	0.78441	0.79210	0.79968	0.80716	0.81452	0.82179
22	0.74110	0.74907	0.75692	0.76466	0.77228	0.77980	0.78720
23	0.70651	0.71464	0.72266	0.73055	0.73832	0.74598	0.75353
24	0.67275	0.68105	0.68922	0.69727	0.70519	0.71301	0.72070
25	0.63974	0.64821	0.65654	0.66474	0.67282	0.68079	0.68863
26	0.60742	0.61605	0.62454	0.63291	0.64115	0.64926	0.65725
27	0.57572	0.58452	0.59318	0.60170	0.61010	0.61837	0.62651
28	0.54459	0.55356	0.56239	0.57108	0.57963	0.58805	0.59635
29	0.51398	0.52312	0.53212	0.54097	0.54969	0.55827	0.56672
30	0.48384	0.49316	0.50233	0.51136	0.52024	0.52898	0.53758
31	0.45414	0.46364	0.47299	0.48218	0.49123	0.50013	0.50890
32	0.42483	0.43452	0.44404	0.45341	0.46263	0.47170	0.48062
33	0.39588	0.40576	0.41547	0.42501	0.43440	0.44364	0.45273
34	0.36727	0.37733	0.38722	0.39695	0.40652	0.41593	0.42518
35	0.33895	0.34921	0.35929	0.36920	0.37895	0.38853	0.39796
36	0.31090	0.32136	0.33163	0.34173	0.35166	0.36142	0.37102
37	0.28309	0.29375	0.30423	0.31452	0.32464	0.33458	0.34436
38	0.25550	0.26637	0.27705	0.28754	0.29785	0.30797	0.31793
39	0.22810	0.23919	0.25008	0.26077	0.27127	0.28159	0.29173
40	0.20088	0.21219	0.22328	0.23418	0.24488	0.25539	0.26572
41	0.17380	0.18533	0.19665	0.20776	0.21866	0.22937	0.23990
42	0.14685	0.15861	0.17015	0.18148	0.19259	0.20351	0.21423
43	0.12001	0.13201	0.14378	0.15533	0.16666	0.17778	0.18870
44	0.09325	0.10550	0.11750	0.12928	0.14083	0.15217	0.16330
45	0.06656	0.07906	0.09131	0.10332	0.11510	0.12666	0.13800
46	0.03992	0.05267	0.06518	0.07743	0.08944	0.10123	0.11279
47	0.01330	0.02633	0.03909	0.05159	0.06385	0.07586	0.08765
48		0.00000	0.01303	0.02579	0.03829	0.05055	0.06257
49				0.00000	0.01276	0.02527	0.03753
50						0.00000	0.01251

Exhibit 18–3 Tables of Percentage Points of the Pearson Type III Distribution

(Exhibit 18–3 is a reprint of the March 1976 revision of Technical Release 38, Tables of Percentage Points of the Pearson Type III Distribution.)

Introduction

Table 1 in exhibit 18–3 was computed on the IBM 7094[1] by Dr. H. Leon Harter, senior scientist (mathematical statistician), Applied Mathematics Research Laboratory, Wright–Patterson Air Force Base, Ohio 45433. Table 2 was computed from table 1 on the IBM 360/40 by the Central Technical Unit of the Soil Conservation Service (now the Natural Resources Conservation Service). Tables 1 and 2 as published in "A Uniform Technique for Determining Flood Flow Frequencies," Bulletin No. 15, Water Resources Council, Washington, DC, December 1967, are an abbreviated form of the new tables. Bulletin No. 17B, "Guidelines for Determining Flood Flow Frequencies," Water Resources Council, Washington, DC, 1982, includes Appendix 3, "Tables of K Values" for skewness of 0 to +9.0 and 31 levels of probability.

Purpose of these tables

Tables 1 and 2 are intended for use in computer applications where 3– or 4–point Lagrangian (parabolic) interpolation will be sufficiently accurate in any case, while linear interpolation is good enough if the value of the coefficient of skewness (G1) is near one of the tabular values or if only three–decimal place accuracy is required.

Manual computations, using tables 1 and 2 of Bulletins 15 and 17B (WRC) with linear interpolation[2] and 4 or 5 place log tables, should in most cases be suitable for preliminary estimates in the field. However, statistics computed in this manner may not be sufficiently accurate for regional studies, etc.

Warning

The presence of high outliers may cause the coefficient of skewness to be close to zero or positive when a plot of the raw data indicates a negative skewness. These are special cases of the log–Pearson Type III distribution for which it may be desirable to submit a plot of the raw data and the calculations of the statistics to the state hydrologist for interpretation and should be reported to the West National Technology Support Center, Water Quality and Quantity Technology Development Team. The variation alone in some data may be responsible for positive skewness, too, when the full period of record without outliers is negatively skewed. (See Bulletins 15 and 17B, pages 12 and 13 and pages 16 and 17, respectively).

1 Trade names mentioned are for specific information and do not constitute a guarantee or warranty of the product by the Department of Agriculture or an endorsement by the Department over other products not mentioned.

2 Since G1 is an inefficient estimator of the skewness, rounding up of G1 to the next tabulated value will usually be sufficient for manual computations.

Table 1 Percentage Points of Pearson Type III Distribution, P(K_p, G1) 1/

For Positive Skewness

G1 = 0.0 to 9.0

The values in the body of the table are the values of K_p, standardized units, that correspond to these values of Gl for P = 0.0001, 0.0005, 0.0010, 0.0020, 0.0050, 0.0100, 0.0200, 0.0250, 0.0400, 0.0500, 0.1000, 0.2000, 0.3000, 0.4000, 0.429624, 0.5000, 0.570376, 0.6000, 0.7000, 0.8000, 0.9000, 0.9500, 0.9600, 0.9750, 0.9800, 0.9900, 0.9950, 0.9980, 0.9990, 0.9995, and 0.9999 cumulative probability equal to or less than a particular discharge in ft^3/s, or other variable being analyzed. Five decimals have been retained. The Return Period (T) is defined as 1/Q, and P + Q = 1.0.

Example: Given G1= +1.0 Find K_p for P = 0.9900 and Q = .0100 or T = 100

G1 = +1.0 K_p = +3.02256

Find K_p for P = Q = 0.5000 or T = 2

G1 = +1.0 K_p = –0.16397

where:

G1 = the coefficient of skewness

P = the cumulative probability equal to or less than a particular discharge in ft^3/s, or other variable being analyzed

Q = the cumulative probability equal to or greater than a particular discharge in ft^3/s, or other variable being analyzed

T = the return period and/or recurrence interval

K_p = the K–value for selected percentage points and skewness

1/ This table was computed on the IBM 7094 by Dr. H. Leon Harter, senior scientist (mathematical statistician), Applied Mathematics Research Laboratory, Wright–Patterson Air Force Base, Ohio, 45433, by special arrangements for the Soil Conservation Service, USDA, Central Technical Unit, 269 Federal Building, Hyattsville, Maryland 20782. This table was published in Technometrics, Vol. 11, No. 1, Feb. 1969, pp 177–187, and Vol. 13, No. 1, Feb. 1971, pp 203–204, “A new table of percentage points of the Pearson type III distribution” and “More percentage points of the Pearson distribution,” respectively.

Table 1 Percentage points of Pearson Type III distribution (positive skewness)

P	G1=0.0	G1=0.1	G1=0.2	G1=0.3	G1=0.4	G1=0.5	G1=0.6	Q	T
0.000100	–3.71902	–3.50703	–3.29921	–3.09631	–2.89907	–2.70836	–2.52507	0.9999	1.0001
0.000500	–3.29053	–3.12767	–2.96698	–2.80889	–2.65390	–2.50257	–2.35549	0.9995	1.0005
0.001000	–3.09023	–2.94834	–2.80786	–2.66915	–2.53261	–2.39867	–2.26780	0.9990	1.0010
0.002000	–2.87816	–2.75706	–2.63672	–2.51741	–2.39942	–2.28311	–2.16884	0.9980	1.0020
0.005000	–2.57583	–2.48187	–2.38795	–2.29423	–2.20092	–2.10825	–2.01644	0.9950	1.0050
0.010000	–2.32635	–2.25258	–2.17840	–2.10394	–2.02933	–1.95472	–1.88029	0.9900	1.0101
0.020000	–2.05375	–1.99973	–1.94499	–1.88959	–1.83361	–1.77716	–1.72033	0.9800	1.0204
0.025000	–1.95996	–1.91219	–1.86360	–1.81427	–1.76427	–1.71366	–1.66253	0.9750	1.0256
0.040000	–1.75069	–1.71580	–1.67999	–1.64329	–1.60574	–1.56740	–1.52830	0.9600	1.0417
0.050000	–1.64485	–1.61594	–1.58607	–1.55527	–1.52357	–1.49101	–1.45762	0.9500	1.0526
0.100000	–1.28155	–1.27037	–1.25824	–1.24516	–1.23114	–1.21618	–1.20028	0.9000	1.1111
0.200000	–0.84162	–0.84611	–0.84986	–0.85285	–0.85508	–0.85653	–0.85718	0.8000	1.2500
0.300000	–0.52440	–0.53624	–0.54757	–0.55839	–0.56867	–0.57840	–0.58757	0.7000	1.4286
0.400000	–0.25335	–0.26882	–0.28403	–0.29897	–0.31362	–0.32796	–0.34198	0.6000	1.6667
0.429624	–0.17733	–0.19339	–0.20925	–0.22492	–0.24037	–0.25558	–0.27047	0.5704	1.7532
0.500000	0.00000	–0.01662	–0.03325	–0.04993	–0.06651	–0.08302	–0.09945	0.5000	2.0000
0.570376	0.17733	0.16111	0.14472	0.12820	0.11154	0.09478	0.07791	0.4296	2.3276
0.600000	0.25335	0.23763	0.22168	0.20552	0.18916	0.17261	0.15589	0.4000	2.5000
0.700000	0.52440	0.51207	0.49927	0.48600	0.47228	0.45812	0.44352	0.3000	3.3333
0.800000	0.84162	0.83639	0.83044	0.82377	0.81638	0.80829	0.79950	0.2000	5.0000
0.900000	1.28155	1.29178	1.30105	1.30936	1.31671	1.32309	1.32850	0.1000	10.000
0.950000	1.64485	1.67279	1.69971	1.72562	1.75048	1.77428	1.79701	0.0500	20.000
0.960000	1.75069	1.78462	1.81756	1.84949	1.88039	1.91022	1.93896	0.0400	25.000
0.975000	1.95996	2.00688	2.05290	2.09795	2.14202	2.18505	2.22702	0.0250	40.000
0.980000	2.05375	2.10697	2.15935	2.21081	2.26133	2.31084	2.35931	0.0200	50.000
0.990000	2.32635	2.39961	2.47226	2.54421	2.61539	2.68572	2.75514	0.0100	100.00
0.995000	2.57583	2.66965	2.76321	2.85636	2.94900	3.04102	3.13232	0.0050	200.00
0.998000	2.87816	2.99978	3.12169	3.24371	3.36566	3.48737	3.60872	0.0020	500.00
0.999000	3.09023	3.23322	3.37703	3.52139	3.66608	3.81090	3.95567	0.0010	1000.0
0.999500	3.29053	3.45513	3.62113	3.78820	3.95605	4.12443	4.29311	0.0005	2000.0
0.999900	3.71902	3.93453	4.15301	4.37394	4.59687	4.82141	5.04718	0.0001	10000.

P	G1=0.7	G1=0.8	G1=0.9	G1=1.0	G1=1.1	G1=1.2	G1=1.3	Q	T
0.000100	–2.35015	–2.18448	–2.02891	–1.88410	–1.75053	–1.62838	–1.51752	0.9999	1.0001
0.000500	–2.21328	–2.07661	–1.94611	–1.82241	–1.70603	–1.59738	–1.49673	0.9995	1.0005
0.001000	–2.14053	–2.01739	–1.89894	–1.78572	–1.67825	–1.57695	–1.48216	0.9990	1.0010
0.002000	–2.05701	–1.94806	–1.84244	–1.74062	–1.64305	–1.55016	–1.46232	0.9980	1.0020
0.005000	–1.92580	–1.83660	–1.74919	–1.66390	–1.58110	–1.50114	–1.42439	0.9950	1.0050
0.010000	–1.80621	–1.73271	–1.66001	–1.58838	–1.51808	–1.44942	–1.38267	0.9900	1.0101
0.020000	–1.66325	–1.60604	–1.54886	–1.49188	–1.43529	–1.37929	–1.32412	0.9800	1.0204
0.025000	–1.61099	–1.55914	–1.50712	–1.45507	–1.40314	–1.35153	–1.30042	0.9750	1.0256
0.040000	–1.48852	–1.44813	–1.40720	–1.36584	–1.32414	–1.28225	–1.24028	0.9600	1.0417
0.050000	–1.42345	–1.38855	–1.35299	–1.31684	–1.28019	–1.24313	–1.20578	0.9500	1.0526
0.100000	–1.18347	–1.16574	–1.14712	–1.12762	–1.10726	–1.08608	–1.06413	0.9000	1.1111
0.200000	–0.85703	–0.85607	–0.85426	–0.85161	–0.84809	–0.84369	–0.83841	0.8000	1.2500
0.300000	–0.59615	–0.60412	–0.61146	–0.61815	–0.62415	–0.62944	–0.63400	0.7000	1.4286
0.400000	–0.35565	–0.36889	–0.38186	–0.39434	–0.40638	–0.41794	–0.42899	0.6000	1.6667
0.429624	–0.28516	–0.29961	–0.31368	–0.32740	–0.34075	–0.35370	–0.36620	0.5704	1.7532
0.500000	–0.11578	–0.13199	–0.14807	–0.16397	–0.17968	–0.19517	–0.21040	0.5000	2.0000
0.570376	0.06097	0.04397	0.02693	0.00987	–0.00719	–0.02421	–0.04116	0.4296	2.3276
0.600000	0.13901	0.12199	0.10486	0.08763	0.07032	0.05297	0.03560	0.4000	2.5000
0.700000	0.42851	0.41309	0.39729	0.38111	0.36458	0.34772	0.33054	0.3000	3.3333
0.800000	0.79002	0.77986	0.76902	0.75752	0.74537	0.73257	0.71915	0.2000	5.0000
0.900000	1.33294	1.33640	1.33889	1.34039	1.34092	1.34047	1.33904	0.1000	10.000
0.950000	1.81864	1.83916	1.85856	1.87683	1.89395	1.90992	1.92472	0.0500	20.000
0.960000	1.96660	1.99311	2.01848	2.04269	2.06573	2.08758	2.10823	0.0400	25.000
0.975000	2.26790	2.30764	2.34623	2.38364	2.41984	2.45482	2.48855	0.0250	40.000
0.980000	2.40670	2.45298	2.49811	2.54206	2.58480	2.62631	2.66657	0.0200	50.000
0.990000	2.82359	2.89101	2.95735	3.02256	3.08660	3.14944	3.21103	0.0100	100.00
0.995000	3.22281	3.31243	3.40109	3.48874	3.57530	3.66073	3.74497	0.0050	200.00
0.998000	3.72957	3.84981	3.96932	4.08802	4.20582	4.32263	4.43839	0.0020	500.00
0.999000	4.10022	4.24439	4.38807	4.53112	4.67344	4.81492	4.95549	0.0010	1000.0
0.999500	4.46189	4.63057	4.79899	4.96701	5.13449	5.30130	5.46735	0.0005	2000.0
0.999900	5.27389	5.50124	5.72899	5.95691	6.18480	6.41249	6.63980	0.0001	10000.

Table 1 Percentage points of Pearson Type III distribution (positive skewness)—Continued

P	G1=1.4	G1=1.5	G1=1.6	G1=1.7	G1=1.8	G1=1.9	G1=2.0	Q	T
0.000100	–1.41753	–1.32774	–1.24728	–1.17520	–1.11054	–1.05239	–0.99990	0.9999	1.0001
0.000500	–1.40413	–1.31944	–1.24235	–1.17240	–1.10901	–1.05159	–0.99950	0.9995	1.0005
0.001000	–1.39408	–1.31275	–1.23805	–1.16974	–1.10743	–1.05068	–0.99900	0.9990	1.0010
0.002000	–1.37981	–1.30279	–1.23132	–1.16534	–1.10465	–1.04898	–0.99800	0.9980	1.0020
0.005000	–1.35114	–1.28167	–1.21618	–1.15477	–1.09749	–1.04427	–0.99499	0.9950	1.0050
0.010000	–1.31815	–1.25611	–1.19680	–1.14042	–1.08711	–1.03695	–0.98995	0.9900	1.0101
0.020000	–1.26999	–1.21716	–1.16584	–1.11628	–1.06864	–1.02311	–0.97980	0.9800	1.0204
0.025000	–1.25004	–1.20059	–1.15229	–1.10537	–1.06001	–1.01640	–0.97468	0.9750	1.0256
0.040000	–1.19842	–1.15682	–1.11566	–1.07513	–1.03543	–0.99672	–0.95918	0.9600	1.0417
0.050000	–1.16827	–1.13075	–1.09338	–1.05631	–1.01973	–0.98381	–0.94871	0.9500	1.0526
0.100000	–1.04144	–1.01810	–0.99418	–0.96977	–0.94496	–0.91988	–0.89464	0.9000	1.1111
0.200000	–0.83223	–0.82516	–0.81720	–0.80837	–0.79868	–0.78816	–0.77686	0.8000	1.2500
0.300000	–0.63779	–0.64080	–0.64300	–0.64436	–0.64488	–0.64453	–0.64333	0.7000	1.4286
0.400000	–0.43949	–0.44942	–0.45873	–0.46739	–0.47538	–0.48265	–0.48917	0.6000	1.6667
0.429624	–0.37824	–0.38977	–0.40075	–0.41116	–0.42095	–0.43008	–0.43854	0.5704	1.7532
0.500000	–0.22535	–0.23996	–0.25422	–0.26808	–0.28150	–0.29443	–0.30685	0.5000	2.0000
0.570376	–0.05803	–0.07476	–0.09132	–0.10769	–0.12381	–0.13964	–0.15516	0.4296	2.3276
0.600000	0.01824	0.00092	–0.01631	–0.03344	–0.05040	–0.06718	–0.08371	0.4000	2.5000
0.700000	0.31307	0.29535	0.27740	0.25925	0.24094	0.22250	0.20397	0.3000	3.3333
0.800000	0.70512	0.69050	0.67532	0.65959	0.64335	0.62662	0.60944	0.2000	5.0000
0.900000	1.33665	1.33330	1.32900	1.32376	1.31760	1.31054	1.30259	0.1000	10.000
0.950000	1.93836	1.95083	1.96213	1.97227	1.98124	1.98906	1.99573	0.0500	20.000
0.960000	2.12768	2.14591	2.16293	2.17873	2.19332	2.20670	2.21888	0.0400	25.000
0.975000	2.52102	2.55222	2.58214	2.61076	2.63810	2.66413	2.68888	0.0250	40.000
0.980000	2.70556	2.74325	2.77964	2.81472	2.84848	2.88091	2.91202	0.0200	50.000
0.990000	3.27134	3.33035	3.38804	3.44438	3.49935	3.55295	3.60517	0.0100	100.00
0.995000	3.82798	3.90973	3.99016	4.06926	4.14700	4.22336	4.29832	0.0050	200.00
0.998000	4.55304	4.66651	4.77875	4.88971	4.99937	5.10768	5.21461	0.0020	500.00
0.999000	5.09505	5.23353	5.37087	5.50701	5.64190	5.77549	5.90776	0.0010	1000.0
0.999500	5.63252	5.79673	5.95990	6.12196	6.28285	6.44251	6.60090	0.0005	2000.0
0.999900	6.86661	7.09277	7.31818	7.54272	7.76632	7.98888	8.21034	0.0001	10000.

P	G1=2.1	G1=2.2	G1=2.3	G1=2.4	G1=2.5	G1=2.6	G1=2.7	Q	T
0.000100	–0.95234	–0.90908	–0.86956	–0.83333	–0.80000	–0.76923	–0.74074	0.9999	1.0001
0.000500	–0.95215	–0.90899	–0.86952	–0.83331	–0.79999	–0.76923	–0.74074	0.9995	1.0005
0.001000	–0.95188	–0.90885	–0.86945	–0.83328	–0.79998	–0.76922	–0.74074	0.9990	1.0010
0.002000	–0.95131	–0.90854	–0.86929	–0.83320	–0.79994	–0.76920	–0.74073	0.9980	1.0020
0.005000	–0.94945	–0.90742	–0.86863	–0.83283	–0.79973	–0.76909	–0.74067	0.9950	1.0050
0.010000	–0.94607	–0.90521	–0.86723	–0.83196	–0.79921	–0.76878	–0.74049	0.9900	1.0101
0.020000	–0.93878	–0.90009	–0.86371	–0.82959	–0.79765	–0.76779	–0.73987	0.9800	1.0204
0.025000	–0.93495	–0.89728	–0.86169	–0.82817	–0.79667	–0.76712	–0.73943	0.9750	1.0256
0.040000	–0.92295	–0.88814	–0.85486	–0.82315	–0.79306	–0.76456	–0.73765	0.9600	1.0417
0.050000	–0.91458	–0.88156	–0.84976	–0.81927	–0.79015	–0.76242	–0.73610	0.9500	1.0526
0.100000	–0.86938	–0.84422	–0.81929	–0.79472	–0.77062	–0.74709	–0.72422	0.9000	1.1111
0.200000	–0.76482	–0.75211	–0.73880	–0.72495	–0.71067	–0.69602	–0.68111	0.8000	1.2500
0.300000	–0.64125	–0.63833	–0.63456	–0.62999	–0.62463	–0.61854	–0.61176	0.7000	1.4286
0.400000	–0.49494	–0.49991	–0.50409	–0.50744	–0.50999	–0.51171	–0.51263	0.6000	1.6667
0.429240	–0.44628	–0.45329	–0.45953	–0.46499	–0.46966	–0.47353	–0.47660	0.5704	1.7320
0.500000	–0.31872	–0.32999	–0.34063	–0.35062	–0.35992	–0.36852	–0.37640	0.5000	2.0000
0.570376	–0.17030	–0.18504	–0.19933	–0.21313	–0.22642	–0.23915	–0.25129	0.4296	2.3276
0.600000	–0.09997	–0.11590	–0.13148	–0.14665	–0.16138	–0.17564	–0.18939	0.4000	2.5000
0.700000	0.18540	0.16682	0.14827	0.12979	0.11143	0.09323	0.07523	0.3000	3.3333
0.800000	0.59183	0.57383	0.55549	0.53683	0.51789	0.49872	0.47934	0.2000	5.0000
0.900000	1.29377	1.28412	1.27365	1.26240	1.25039	1.23766	1.22422	0.1000	10.000
0.950000	2.00128	2.00570	2.00903	2.01128	2.01247	2.01263	2.01177	0.0500	20.000
0.960000	2.22986	2.23967	2.24831	2.25581	2.26217	2.26743	2.27160	0.0400	25.000
0.975000	2.71234	2.73451	2.75541	2.77506	2.79345	2.81062	2.82658	0.0250	40.000
0.980000	2.94181	2.97028	2.99744	3.02330	3.04787	3.07116	3.09320	0.0200	50.000
0.990000	3.65600	3.70543	3.75347	3.80013	3.84540	3.88930	3.93183	0.0100	100.00
0.995000	4.37186	4.44398	4.51467	4.58393	4.65176	4.71815	4.78313	0.0050	200.00
0.998000	5.32014	5.42426	5.52694	5.62818	5.72796	5.82629	5.93160	0.0020	500.00
0.999000	6.03865	6.16816	6.29626	6.42292	6.54814	6.67191	6.79421	0.0010	1000.0
0.999500	6.75798	6.91370	7.06804	7.22098	7.37250	7.52258	7.67121	0.0005	2000.0
0.999900	8.43064	8.64971	8.86753	9.08403	9.29920	9.51301	9.72543	0.0001	10000.

Table 1 Percentage points of Pearson Type III distribution (positive skewness)—Continued

P	G1=2.8	G1=2.9	G1=3.0	G1=3.1	G1=3.2	G1=3.3	G1=3.4	Q	T
0.000100	–0.71429	–0.68966	–0.66667	–0.64516	–0.62500	–0.60606	–0.58824	0.9999	1.0001
0.000500	–0.71429	–0.68966	–0.66667	–0.64516	–0.62500	–0.60606	–0.58824	0.9995	1.0005
0.001000	–0.71428	–0.68965	–0.66667	–0.64516	–0.62500	–0.60606	–0.58824	0.9990	1.0010
0.002000	–0.71428	–0.68965	–0.66667	–0.64516	–0.62500	–0.60606	–0.58824	0.9980	1.0020
0.005000	–0.71425	–0.68964	–0.66666	–0.64516	–0.62500	–0.60606	–0.58824	0.9950	1.0050
0.010000	–0.71415	–0.68959	–0.66663	–0.64514	–0.62499	–0.60606	–0.58823	0.9900	1.0101
0.020000	–0.71377	–0.68935	–0.66649	–0.64507	–0.62495	–0.60603	–0.58822	0.9800	1.0204
0.025000	–0.71348	–0.68917	–0.66638	–0.64500	–0.62491	–0.60601	–0.58821	0.9750	1.0256
0.040000	–0.71227	–0.68836	–0.66585	–0.64465	–0.62469	–0.60587	–0.58812	0.9600	1.0417
0.050000	–0.71116	–0.68759	–0.66532	–0.64429	–0.62445	–0.60572	–0.58802	0.9500	1.0526
0.100000	–0.70209	–0.68075	–0.66023	–0.64056	–0.62175	–0.60379	–0.58666	0.9000	1.1111
0.200000	–0.66603	–0.65086	–0.63569	–0.62060	–0.60567	–0.59096	–0.57652	0.8000	1.2500
0.300000	–0.60434	–0.59634	–0.58783	–0.57887	–0.56953	–0.55989	–0.55000	0.7000	1.4286
0.400000	–0.51276	–0.51212	–0.51073	–0.50863	–0.50585	–0.50244	–0.49844	0.6000	1.6667
0.429624	–0.47888	–0.48037	–0.48109	–0.48107	–0.48033	–0.47890	–0.47682	0.5704	1.7532
0.500000	–0.38353	–0.38991	–0.39554	–0.40041	–0.40454	–0.40792	–0.41058	0.5000	2.0000
0.570376	–0.26292	–0.27372	–0.28395	–0.29351	–0.30238	–0.31055	–0.31802	0.4296	2.3276
0.600000	–0.20259	–0.21523	–0.22726	–0.23868	–0.24946	–0.25958	–0.26904	0.4000	2.5000
0.700000	0.05746	0.03997	0.02279	0.00596	–0.01050	–0.02654	–0.04215	0.3000	3.3333
0.800000	0.45980	0.44015	0.42040	0.40061	0.38081	0.36104	0.34133	0.2000	5.0000
0.900000	1.21013	1.19539	1.18006	1.16416	1.14772	1.13078	1.11337	0.1000	10.000
0.950000	2.00992	2.00710	2.00335	1.99869	1.99314	1.98674	1.97951	0.0500	20.000
0.960000	2.27470	2.27676	2.27780	2.27785	2.27693	2.27506	2.27229	0.0400	25.000
0.975000	2.84134	2.85492	2.86735	2.87865	2.88884	2.89795	2.90599	0.0250	40.000
0.980000	3.11399	3.13356	3.15193	3.16911	3.18512	3.20000	3.21375	0.0200	50.000
0.990000	3.97301	4.01286	4.05138	4.08859	4.12452	4.15917	4.19257	0.0100	100.00
0.995000	4.84669	4.90884	4.96959	5.02897	5.08697	5.14362	5.19892	0.0050	200.00
0.998000	6.01858	6.11254	6.20506	6.29613	6.38578	6.47401	6.56084	0.0020	500.00
0.999000	6.91505	7.03443	7.15235	7.26881	7.38382	7.49739	7.60953	0.0010	1000.0
0.999500	7.81839	7.96411	8.10836	8.25115	8.39248	8.53236	8.67079	0.0005	2000.0
0.999900	9.93643	10.14602	10.35418	10.56090	10.76618	10.97001	11.17239	0.0001	10000.

P	G1=3.5	G1=3.6	G1=3.7	G1=3.8	G1=3.9	G1=4.0	G1=4.1	Q	T
0.000100	–0.57143	–0.55556	–0.54054	–0.52632	–0.51282	–0.50000	–0.48780	0.9999	1.0001
0.000500	–0.57143	–0.55556	–0.54054	–0.52632	–0.51282	–0.50000	–0.48780	0.9995	1.0005
0.001000	–0.57143	–0.55556	–0.54054	–0.52632	–0.51282	–0.50000	–0.48780	0.9990	1.0010
0.002000	–0.57143	–0.55556	–0.54054	–0.52632	–0.51282	–0.50000	–0.48780	0.9980	1.0020
0.005000	–0.57143	–0.55556	–0.54054	–0.52632	–0.51282	–0.50000	–0.48780	0.9950	1.0050
0.010000	–0.57143	–0.55556	–0.54054	–0.52632	–0.51282	–0.50000	–0.48780	0.9900	1.0101
0.020000	–0.57142	–0.55555	–0.54054	–0.52631	–0.51282	–0.50000	–0.48780	0.9800	1.0204
0.025000	–0.57141	–0.55555	–0.54054	–0.52631	–0.51282	–0.50000	–0.48780	0.9750	1.0256
0.040000	–0.57136	–0.55552	–0.54052	–0.52630	–0.51281	–0.50000	–0.48780	0.9600	1.0417
0.050000	–0.57130	–0.55548	–0.54050	–0.52629	–0.51281	–0.49999	–0.48780	0.9500	1.0526
0.100000	–0.57035	–0.55483	–0.54006	–0.52600	–0.51261	–0.49986	–0.48772	0.9000	1.1111
0.200000	–0.56242	–0.54867	–0.53533	–0.52240	–0.50990	–0.49784	–0.48622	0.8000	1.2500
0.300000	–0.53993	–0.52975	–0.51952	–0.50929	–0.49911	–0.48902	–0.47906	0.7000	1.4286
0.400000	–0.49391	–0.48888	–0.48342	–0.47758	–0.47141	–0.46496	–0.45828	0.6000	1.6667
0.429624	–0.47413	–0.47088	–0.46711	–0.46286	–0.45819	–0.45314	–0.44777	0.5704	1.7532
0.500000	–0.41253	–0.41381	–0.41442	–0.41441	–0.41381	–0.41265	–0.41097	0.5000	2.0000
0.570376	–0.32479	–0.33085	–0.33623	–0.34092	–0.34494	–0.34831	–0.35105	0.4296	2.3276
0.600000	–0.27782	–0.28592	–0.29335	–0.30010	–0.30617	–0.31159	–0.31635	0.4000	2.5000
0.700000	–0.05730	–0.07195	–0.08610	–0.09972	–0.11279	–0.12530	–0.13725	0.3000	3.3333
0.800000	0.32171	0.30223	0.28290	0.26376	0.24484	0.22617	0.20777	0.2000	5.0000
0.900000	1.09552	1.07726	1.05863	1.03965	1.02036	1.00079	0.98096	0.1000	10.000
0.950000	1.97147	1.96266	1.95311	1.94283	1.93186	1.92023	1.90796	0.0500	20.000
0.960000	2.26862	2.26409	2.25872	2.25254	2.24558	2.23786	2.22940	0.0400	25.000
0.975000	2.91299	2.91898	2.92397	2.92799	2.93107	2.93324	2.93450	0.0250	40.000
0.980000	3.22641	3.23800	3.24853	3.25803	3.26653	3.27404	3.28060	0.0200	50.000
0.990000	4.22473	4.25569	4.28545	4.31403	4.34147	4.36777	4.39296	0.0100	100.00
0.995000	5.25291	5.30559	5.35698	5.40711	5.45598	5.50362	5.55005	0.0050	200.00
0.998000	6.64627	6.73032	6.81301	6.89435	6.97435	7.05304	7.13043	0.0020	500.00
0.999000	7.72024	7.82954	7.93744	8.04395	8.14910	8.25289	8.35534	0.0010	1000.0
0.999500	8.80779	8.94335	9.07750	9.21023	9.34158	9.47154	9.60013	0.0005	2000.0
0.999900	11.37334	11.57284	11.77092	11.96757	12.16280	12.35663	12.54906	0.0001	10000.

Table 1 Percentage points of Pearson Type III distribution (positive skewness)—Continued

P	G1=4.2	G1=4.3	G1=4.4	G1=4.5	G1=4.6	G1=4.7	G17-=4.8	Q	T
0.000100	–0.47619	–0.46512	–0.45455	–0.44444	–0.43478	–0.42553	–0.41667	0.9999	1.0001
0.000500	–0.47619	–0.46512	–0.45455	–0.44444	–0.43478	–0.42553	–0.41667	0.9995	1.0005
0.001000	–0.47619	–0.46512	–0.45455	–0.44444	–0.43478	–0.42553	–0.41667	0.9990	1.0010
0.002000	–0.47619	–0.46512	–0.45455	–0.44444	–0.43478	–0.42553	–0.41667	0.9980	1.0020
0.005000	–0.47619	–0.46512	–0.45455	–0.44444	–0.43478	–0.42553	–0.41667	0.9950	1.0050
0.010000	–0.47619	–0.46512	–0.45455	–0.44444	–0.43478	–0.42553	–0.41667	0.9900	1.0101
0.020000	–0.47619	–0.46512	–0.45455	–0.44444	–0.43478	–0.42553	–0.41667	0.9800	1.0204
0.025000	–0.47619	–0.46512	–0.45455	–0.44444	–0.43478	–0.42553	–0.41667	0.9750	1.0256
0.040000	–0.47619	–0.46512	–0.45455	–0.44444	–0.43478	–0.42553	–0.41667	0.9600	1.0417
0.050000	–0.47619	–0.46511	–0.45454	–0.44444	–0.43478	–0.42553	–0.41667	0.9500	1.0526
0.100000	–0.47614	–0.46508	–0.45452	–0.44443	–0.43477	–0.42553	–0.41666	0.9000	1.1111
0.200000	–0.47504	–0.46428	–0.45395	–0.44402	–0.43448	–0.42532	–0.41652	0.8000	1.2500
0.300000	–0.46927	–0.45967	–0.45029	–0.44114	–0.43223	–0.42357	–0.41517	0.7000	1.4286
0.400000	–0.45142	–0.44442	–0.43734	–0.43020	–0.42304	–0.41590	–0.40880	0.6000	1.6667
0.429624	–0.44212	–0.43623	–0.43016	–0.42394	–0.41761	–0.41121	–0.40477	0.5704	1.7532
0.500000	–0.40881	–0.40621	–0.40321	–0.39985	–0.39617	–0.39221	–0.38800	0.5000	2.0000
0.570376	–0.35318	–0.35473	–0.35572	–0.35619	–0.35616	–0.35567	–0.35475	0.4296	2.3276
0.600000	–0.32049	–0.32400	–0.32693	–0.32928	–0.33108	–0.33236	–0.33315	0.4000	2.5000
0.700000	–0.14861	–0.15939	–0.16958	–0.17918	–0.18819	–0.19661	–0.20446	0.3000	3.3333
0.800000	0.18967	0.17189	0.15445	0.13737	0.12067	0.10436	0.08847	0.2000	5.0000
0.900000	0.96090	0.94064	0.92022	0.89964	0.87895	0.85817	0.83731	0.1000	10.000
0.950000	1.89508	1.88160	1.86757	1.85300	1.83792	1.82234	1.80631	0.0500	20.000
0.960000	2.22024	2.21039	2.19988	2.18874	2.17699	2.16465	2.15174	0.0400	25.000
0.975000	2.93489	2.93443	2.93314	2.93105	2.92818	2.92455	2.92017	0.0250	40.000
0.980000	3.28622	3.29092	3.29473	3.29767	3.29976	3.30103	3.30149	0.0200	50.000
0.990000	4.41706	4.44009	4.46207	4.48303	4.50297	4.52192	4.53990	0.0100	100.00
0.995000	5.59528	5.63934	5.68224	5.72400	5.76464	5.80418	5.84265	0.0050	200.00
0.998000	7.20654	7.28138	7.35497	7.42733	7.49847	7.56842	7.63718	0.0020	500.00
0.999000	8.45646	8.55627	8.65479	8.75202	8.84800	8.94273	9.03623	0.0010	1000.0
0.999500	9.72737	9.85326	9.97784	10.10110	10.22307	10.34375	10.46318	0.0005	2000.0
0.999900	12.74 010	12.92977	13.11808	13.30504	13.49066	13.67495	13.85794	0.0001	10000.

P	G1=4.9	G1=5.0	G1=5.1	G1=5.2	G1=5.3	G1=5.4	G1=5.5	Q	T
0.000100	–0.40816	–0.40000	–0.39216	–0.38462	–0.37736	–0.37037	–0.36364	0.9999	1.0001
0.000500	–0.40816	–0.40000	–0.39216	–0.38462	–0.37736	–0.37037	–0.36364	0.9995	1.0005
0.001000	–0.40816	–0.40000	–0.39216	–0.38462	–0.37736	–0.37037	–0.36364	0.9990	1.0010
0.002000	–0.40816	–0.40000	–0.39216	–0.38462	–0.37736	–0.37037	–0.36364	0.9980	1.0020
0.005000	–0.40816	–0.40000	–0.39216	–0.38462	–0.37736	–0.37037	–0.36364	0.9950	1.0050
0.010000	–0.40816	–0.40000	–0.39216	–0.38462	–0.37736	–0.37037	–0.36364	0.9900	1.0101
0.020000	–0.40816	–0.40000	–0.39216	–0.38462	–0.37736	–0.37037	–0.36364	0.9800	1.0204
0.025000	–0.40816	–0.40000	–0.39216	–0.38462	–0.37736	–0.37037	–0.36364	0.9750	1.0256
0.040000	–0.40816	–0.40000	–0.39216	–0.38462	–0.37736	–0.37037	–0.36364	0.9600	1.0417
0.050000	–0.40816	–0.40000	–0.39216	–0.38462	–0.37736	–0.37037	– 0.36364	0.9500	1.0526
0.100000	–0.40816	–0.40000	–0.39216	–0.38462	–0.37736	–0.37037	–0.36364	0.9000	1.1111
0.200000	–0.40806	–0.39993	–0.39211	–0.38458	–0.37734	–0.37036	–0.36363	0.8000	1.2500
0.300000	–0.40703	–0.39914	–0.39152	–0.38414	–0.37701	–0.37011	–0.36345	0.7000	1.4286
0.400000	–0.40177	–0.39482	–0.38799	–0.38127	–0.37469	–0.36825	–0.36196	0.6000	1.6667
0.429624	–0.39833	–0.39190	–0.38552	–0.37919	–0.37295	–0.36680	–0.36076	0.5704	1.7532
0.500000	–0.38359	–0.37901	–0.37428	–0.36945	–0.36453	–0.35956	–0.35456	0.5000	2.0000
0.570376	–0.35343	–0.35174	–0.34972	–0.34740	–0.34481	–0.34198	–0.33895	0.4296	2.3276
0.600000	–0.33347	–0.33336	–0.33284	–0.33194	–0.33070	–0.32914	–0.32729	0.4000	2.5000
0.700000	–0.21172	–0.21843	–0.22458	–0.23019	–0.23527	–0.23984	–0.24391	0.3000	3.3333
0.800000	0.07300	0.05798	0.04340	0.02927	0.01561	0.00243	–0.01028	0.2000	5.0000
0.900000	0.81641	0.79548	0.77455	0.75364	0.73277	0.71195	0.69122	0.1000	10.000
0.950000	1.78982	1.77292	1.75563	1.73795	1.71992	1.70155	1.68287	0.0500	20.000
0.960000	2.13829	2.12432	2.10985	2.09490	2.07950	2.06365	2.04739	0.0400	25.000
0.975000	2.91508	2.90930	2.90283	2.89572	2.88796	2.87959	2.87062	0.0250	40.000
0.980000	3.30116	3.30007	3.29823	3.29567	3.29240	3.28844	3.28381	0.0200	50.000
0.990000	4.55694	4.57304	4.58823	4.60252	4.61594	4.62850	4.64022	0.0100	100.00
0.995000	5.88004	5.91639	5.95171	5.98602	6.01934	6.05169	6.08307	0.0050	200.00
0.998000	7.70479	7.77124	7.83657	7.90078	7.96390	8.02594	8.08691	0.0020	500.00
0.999000	9.12852	9.21961	9.30952	9.39827	9.48586	9.57232	9.65766	0.0010	1000.0
0.999500	10.58135	10.69829	10.81401	10.92853	11.04186	11.15402	11.26502	0.0005	2000.0
0.999900	14.03963	14.22004	14.39918	14.57706	14.75370	14.92912	15.10332	0.0001	10000.

Table 1 Percentage points of Pearson Type III distribution (positive skewness)—Continued

P	G1=5.6	G1=5.7	G1=5.8	G1=5.9	G1=6.0	G1=6.1	G1=6.2	Q	T
0.000100	–0.35714	–0.35088	–0.34483	–0.33898	–0.33333	–0.32787	–0.32258	0.9999	1.0001
0.000500	–0.35714	–0.35088	–0.34483	–0.33898	–0.33333	–0.32787	–0.32258	0.9995	1.0005
0.001000	–0.35714	–0.35088	–0.34483	–0.33898	–0.33333	–0.32787	–0.32258	0.9990	1.0010
0.002000	–0.35714	–0.35088	–0.34483	–0.33898	–0.33333	–0.32787	–0.32258	0.9980	1.0020
0.005000	–0.35714	–0.35088	–0.34483	–0.33898	–0.33333	–0.32787	–0.32258	0.9950	1.0050
0.010000	–0.35714	–0.35088	–0.34483	–0.33898	–0.33333	–0.32787	–0.32258	0.9900	1.0101
0.020000	–0.35714	–0.35088	–0.34483	–0.33898	–0.33333	–0.32787	–0.32258	0.9800	1.0204
0.025000	–0.35714	–0.35088	–0.34483	–0.33898	–0.33333	–0.32787	–0.32258	0.9750	1.0256
0.040000	–0.35714	–0.35088	–0.34483	–0.33898	–0.33333	–0.32787	–0.32258	0.9600	1.0417
0.050000	–0.35714	–0.35088	–0.34483	–0.33898	–0.33333	–0.32787	–0.32258	0.9500	1.0526
0.100000	–0.35714	–0.35088	–0.34483	–0.33898	–0.33333	–0.32787	–0.32258	0.9000	1.1111
0.200000	–0.35714	–0.35087	–0.34483	–0.33898	–0.33333	–0.32787	–0.32258	0.8000	1.2500
0.300000	–0.35700	–0.35078	–0.34476	–0.33893	–0.33330	–0.32784	–0.32256	0.7000	1.4286
0.400000	–0.35583	–0.34985	–0.34402	–0.33836	–0.33285	–0.32750	–0.32230	0.6000	1.6667
0.429624	–0.35484	–0.34903	–0.34336	–0.33782	–0.33242	–0.32715	–0.32202	0.5704	1.7532
0.500000	–0.34955	–0.34455	–0.33957	–0.33463	–0.32974	–0.32492	–0.32016	0.5000	2.0000
0.570376	–0.33573	–0.33236	–0.32886	–0.32525	–0.32155	–0.31780	–0.31399	0.4296	2.3276
0.600000	–0.32519	–0.32285	–0.32031	–0.31759	–0.31472	–0.31171	–0.30859	0.4000	2.5000
0.700000	–0.24751	–0.25064	–0.25334	–0.25562	–0.25750	–0.25901	–0.26015	0.3000	3.3333
0.800000	–0.02252	–0.03427	–0.04553	–0.05632	–0.06662	–0.07645	–0.08580	0.2000	5.0000
0.900000	0.67058	0.65006	0.62966	0.60941	0.58933	0.56942	0.54970	0.1000	10.000
0.950000	1.66390	1.64464	1.62513	1.60538	1.58541	1.56524	1.54487	0.0500	20.000
0.960000	2.03073	2.01369	1.99629	1.97855	1.96048	1.94210	1.92343	0.0400	25.000
0.975000	2.86107	2.85096	2.84030	2.82912	2.81743	2.80525	2.79259	0.0250	40.000
0.980000	3.27854	3.27263	3.26610	3.25898	3.25128	3.24301	3.23419	0.0200	50.000
0.990000	4.65111	4.66120	4.67050	4.67903	4.68680	4.69382	4.70013	0.0100	100.00
0.995000	6.11351	6.14302	6.17162	6.19933	6.22616	6.25212	6.27723	0.0050	200.00
0.998000	8.14683	8.20572	8.26359	8.32046	8.37634	8.43125	8.48519	0.0020	500.00
0.999000	9.74190	9.82505	9.90713	9.98815	10.06812	10.14706	10.22499	0.0010	1000.0
0.999500	11.37487	11.48360	11.59122	11.69773	11.80316	11.90752	12.01082	0.0005	2000.0
0.999900	15.27632	15.44813	15.61878	15.78826	15.95660	16.12380	16.28989	0.0001	10000.

P	G1=6.3	G1=6.4	G1=6.5	G1=6.6	G1=6.7	G1=6.8	G1=6.9	Q	T
0.000100	–0.31746	–0.31250	–0.30769	–0.30303	–0.29851	–0.29412	–0.28986	0.9999	1.0001
0.000500	–0.31746	–0.31250	–0.30769	–0.30303	–0.29851	–0.29412	–0.28986	0.9995	1.0005
0.001000	–0.31746	–0.31250	–0.30769	–0.30303	–0.29851	–0.29412	–0.28986	0.9990	1.0010
0.002000	–0.31746	–0.31250	–0.30769	–0.30303	–0.29851	–0.29412	–0.28986	0.9980	1.0020
0.005000	–0.31746	–0.31250	–0.30769	–0.30303	–0.29851	–0.29412	–0.28986	0.9950	1.0050
0.010000	–0.31746	–0.31250	–0.30769	–0.30303	–0.29851	–0.29412	–0.28986	0.9900	1.0101
0.020000	–0.31746	–0.31250	–0.30769	–0.30303	–0.29851	–0.29412	–0.28986	0.9800	1.0204
0.025000	–0.31746	–0.31250	–0.30769	–0.30303	–0.29851	–0.29412	–0.28986	0.9750	1.0256
0.040000	–0.31746	–0.31250	–0.30769	–0.30303	–0.29851	–0.29412	–0.28986	0.9600	1.0417
0.050000	–0.31746	–0.31250	–0.30769	–0.30303	–0.29851	–0.29412	–0.28986	0.9500	1.0526
0.100000	–0.31746	–0.31250	–0.30769	–0.30303	–0.29851	–0.29412	–0.28986	0.9000	1.1111
0.200000	–0.31746	–0.31250	–0.30769	–0.30303	–0.29851	–0.29412	–0.28986	0.8000	1.2500
0.300000	–0.31745	–0.31249	–0.30769	–0.30303	–0.29850	–0.29412	–0.28985	0.7000	1.4286
0.400000	–0.31724	–0.31234	–0.30757	–0.30294	–0.29844	–0.29407	–0.28982	0.6000	1.6667
0.429624	–0.31702	–0.31216	–0.30743	–0.30283	–0.29835	–0.29400	–0.28977	0.5704	1.7532
0.500000	–0.31549	–0.31090	–0.30639	–0.30198	–0.29766	–0.29344	–0.28931	0.5000	2.0000
0.570376	–0.31016	–0.30631	–0.30246	–0.29862	–0.29480	–0.29101	–0.28726	0.4296	2.3276
0.600000	–0.30538	–0.30209	–0.29875	–0.29537	–0.29196	–0.28854	–0.28511	0.4000	2.5000
0.700000	–0.26097	–0.26146	–0.26167	–0.26160	–0.26128	–0.26072	–0.25995	0.3000	3.3333
0.800000	–0.09469	–0.10311	–0.11107	–0.11859	–0.12566	–0.13231	–0.13853	0.2000	5.0000
0.900000	0.53019	0.51089	0.49182	0.47299	0.45440	0.43608	0.41803	0.1000	10.000
0.950000	1.52434	1.50365	1.48281	1.46186	1.44079	1.41963	1.39839	0.0500	20.000
0.960000	1.90449	1.88528	1.86584	1.84616	1.82627	1.80618	1.78591	0.0400	25.000
0.975000	2.77947	2.76591	2.75191	2.73751	2.72270	2.70751	2.69195	0.0250	40.000
0.980000	3.22484	3.21497	3.20460	3.19374	3.18241	3.17062	3.15838	0.0200	50.000
0.990000	4.70571	4.71061	4.71482	4.71836	4.72125	4.72350	4.72512	0.0100	100.00
0.995000	6.30151	6.32497	6.34762	6.36948	6.39055	6.41086	6.43042	0.0050	200.00
0.998000	8.53820	8.59027	8.64142	8.69167	8.74102	8.78950	8.83711	0.0020	500.00
0.999000	10.30192	10.37785	10.45281	10.52681	10.59986	10.67197	10.74316	0.0010	1000.0
0.999500	12.11307	12.21429	12.31450	12.41370	12.51190	12.60913	12.70539	0.0005	2000.0
0.999900	16.45487	16.61875	16.78156	16.94329	17.10397	17.26361	17.42221	0.0001	10000.

Table 1 Percentage points of Pearson Type III distribution (positive skewness)—Continued

P	G1=7.0	G1=7.1	G1=7.2	G1=7.3	G1=7.4	G1=7.5	G1=7.6	Q	T
0.000100	–0.28571	–0.28169	–0.27778	–0.27397	–0.27027	–0.26667	–0.26316	0.9999	1.0001
0.000500	–0.28571	–0.28169	–0.27778	–0.27397	–0.27027	–0.26667	–0.26316	0.9995	1.0005
0.001000	–0.28571	–0.28169	–0.27778	–0.27397	–0.27027	–0.26667	–0.26316	0.9990	1.0010
0.002000	–0.28571	–0.28169	–0.27778	–0.27397	–0.27027	–0.26667	–0.26316	0.9980	1.0020
0.005000	–0.28571	–0.28169	–0.27778	–0.27397	–0.27027	–0.26667	–0.26316	0.9950	1.0050
0.010000	–0.28571	–0.28169	–0.27778	–0.27397	–0.27027	–0.26667	–0.26316	0.9900	1.0101
0.020000	–0.28571	–0.28169	–0.27778	–0.27397	–0.27027	–0.26667	–0.26316	0.9800	1.0204
0.025000	–0.28571	–0.28169	–0.27778	–0.27397	–0.27027	–0.26667	–0.26316	0.9750	1.0256
0.040000	–0.28571	–0.28169	–0.27778	–0.27397	–0.27027	–0.26667	–0.26316	0.9600	1.0417
0.050000	–0.28571	–0.28169	–0.27778	–0.27397	–0.27027	–0.26667	–0.26316	0.9500	1.0526
0.100000	–0.28571	–0.28169	–0.27778	–0.27397	–0.27027	–0.26667	–0.26316	0.9000	1.1111
0.200000	–0.28571	–0.28169	–0.27778	–0.27397	–0.27027	–0.26667	–0.26316	0.8000	1.2500
0.300000	–0.28571	–0.28169	–0.27778	–0.27397	–0.27027	–0.26667	–0.26316	0.7000	1.4286
0.400000	–0.28569	–0.28167	–0.27776	–0.27396	–0.27026	–0.26666	–0.26315	0.6000	1.6667
0.429624	–0.28565	–0.28164	–0.27774	–0.27394	–0.27025	–0.26665	–0.26315	0.5704	1.7532
0.500000	–0.28528	–0.28135	–0.27751	–0.27376	–0.27010	–0.26654	–0.26306	0.5000	2.0000
0.570376	–0.28355	–0.27990	–0.27629	–0.27274	–0.26926	–0.26584	–0.26248	0.4296	2.3276
0.600000	–0.28169	–0.27829	–0.27491	–0.27156	–0.26825	–0.26497	–0.26175	0.4000	2.5000
0.700000	–0.25899	–0.25785	–0.25654	–0.25510	–0.25352	–0.25183	–0.25005	0.3000	3.3333
0.800000	–0.14434	–0.14975	–0.15478	–0.15942	–0.16371	–0.16764	–0.17123	0.2000	5.0000
0.900000	0.40026	0.38277	0.36557	0.34868	0.33209	0.31582	0.29986	0.1000	10.000
0.950000	1.37708	1.35571	1.33430	1.31287	1.29141	1.26995	1.24850	0.0500	20.000
0.960000	1.76547	1.74487	1.72412	1.70325	1.68225	1.66115	1.63995	0.0400	25.000
0.975000	2.67603	2.65977	2.64317	2.62626	2.60905	2.59154	2.57375	0.0250	40.000
0.980000	3.14572	3.13263	3.11914	3.10525	3.09099	3.07636	3.06137	0.0200	50.000
0.990000	4.72613	4.72653	4.72635	4.72559	4.72427	4.72240	4.71998	0.0100	100.00
0.995000	6.44924	6.46733	6.48470	6.50137	6.51735	6.53264	6.54727	0.0050	200.00
0.998000	8.88387	8.92979	8.97488	9.01915	9.06261	9.10528	9.14717	0.0020	500.00
0.999000	10.81343	10.88281	10.95129	11.01890	11.08565	11.15154	11.21658	0.0010	1000.0
0.999500	12.80069	12.89505	12.98848	13.08098	13.17258	13.26328	13.35309	0.0005	2000.0
0.999900	17.57979	17.73636	17.89193	18.04652	18.20013	18.35278	18.50447	0.0001	10000.

P	G1=7.7	G1=7.8	G1=7.9	G1=8.0	G1=8.1	G1=8.2	G1=8.3	Q	T
0.000100	–0.25974	–0.25641	–0.25316	–0.25000	–0.24691	–0.24390	–0.24096	0.9999	1.0001
0.000500	–0.25974	–0.25641	–0.25316	–0.25000	–0.24691	–0.24390	–0.24096	0.9995	1.0005
0.001000	–0.25974	–0.25641	–0.25316	–0.25000	–0.24691	–0.24390	–0.24096	0.9990	1.0010
0.002000	–0.25974	–0.25641	–0.25316	–0.25000	–0.24691	–0.24390	–0.24096	0.9980	1.0020
0.005000	–0.25974	–0.25641	–0.25316	–0.25000	–0.24691	–0.24390	–0.24096	0.9950	1.0050
0.010000	–0.25974	–0.25641	–0.25316	–0.25000	–0.24691	–0.24390	–0.24096	0.9900	1.0101
0.020000	–0.25974	–0.25641	–0.25316	–0.25000	–0.24691	–0.24390	–0.24096	0.9800	1.0204
0.025000	–0.25974	–0.25641	–0.25316	–0.25000	–0.24691	–0.24390	–0.24096	0.9750	1.0256
0.040000	–0.25974	–0.25641	–0.25316	–0.25000	–0.24691	–0.24390	–0.24096	0.9600	1.0417
0.050000	–0.25974	–0.25641	–0.25316	–0.25000	–0.24691	–0.24390	–0.24096	0.9500	1.0526
0.100000	–0.25974	–0.25641	–0.25316	–0.25000	–0.24691	–0.24390	–0.24096	0.9000	1.1111
0.200000	–0.25974	–0.25641	–0.25316	–0.25000	–0.24691	–0.24390	–0.24096	0.8000	1.2500
0.300000	–0.25974	–0.25641	–0.25316	–0.25000	–0.24691	–0.24390	–0.24096	0.7000	1.4286
0.400000	–0.25974	–0.25641	–0.25316	–0.25000	–0.24691	–0.24390	–0.24096	0.6000	1.6667
0.429624	–0.25973	–0.25640	–0.25316	–0.25000	–0.24691	–0.24390	–0.24096	0.5704	1.7532
0.500000	–0.25966	–0.25635	–0.25312	–0.24996	–0.24689	–0.24388	–0.24095	0.5000	2.0000
0.570376	–0.25919	–0.25596	–0.25280	–0.24970	–0.24667	–0.24371	–0.24081	0.4296	2.3276
0.600000	–0.25857	–0.25544	–0.25236	–0.24933	–0.24637	–0.24345	–0.24060	0.4000	2.5000
0.700000	–0.24817	–0.24622	–0.24421	–0.24214	–0.24003	–0.23788	–0.23571	0.3000	3.3333
0.800000	–0.17450	–0.17746	–0.18012	–0.18249	–0.18459	–0.18643	–0.18803	0.2000	5.0000
0.900000	0.28422	0.26892	0.25394	0.23929	0.22498	0.21101	0.19737	0.1000	10.000
0.950000	1.22706	1.20565	1.18427	1.16295	1.14168	1.12048	1.09936	0.0500	20.000
0.960000	1.61867	1.59732	1.57591	1.55444	1.53294	1.51141	1.48985	0.0400	25.000
0.975000	2.55569	2.53737	2.51881	2.50001	2.48099	2.46175	2.44231	0.0250	40.000
0.980000	3.04604	3.03038	3.01439	2.99810	2.98150	2.96462	2.94746	0.0200	50.000
0.990000	4.71704	4.71358	4.70961	4.70514	4.70019	4.69476	4.68887	0.0100	100.00
0.995000	6.56124	6.57456	6.58725	6.59931	6.61075	6.62159	6.63183	0.0050	200.00
0.998000	9.18828	9.22863	9.26823	9.30709	9.34521	9.38262	9.41931	0.0020	500.00
0.999000	11.28080	11.34419	11.40677	11.46855	11.52953	11.58974	11.64917	0.0010	1000.0
0.999500	13.44202	13.53009	13.61730	13.70366	13.78919	13.87389	13.95778	0.0005	2000.0
0.999900	18.65522	18.80504	18.95393	19.10191	19.24898	19.39517	19.54046	0.0001	10000.

Table 1 Percentage points of Pearson Type III distribution (positive skewness)—Continued

P	G1=8.4	G1=8.5	G1=8.6	G1=8.7	G1=8.8	G1=8.9	G1=9.0	Q	T
0.000100	–0.23810	–0.23529	–0.23256	–0.22989	–0.22727	–0.22472	–0.22222	0.9999	1.0001
0.000500	–0.23810	–0.23529	–0.23256	–0.22989	–0.22727	–0.22472	–0.22222	0.9995	1.0005
0.001000	–0.23810	–0.23529	–0.23256	–0.22989	–0.22727	–0.22472	–0.22222	0.9990	1.0010
0.002000	–0.23810	–0.23529	–0.23256	–0.22989	–0.22727	–0.22472	–0.22222	0.9980	1.0020
0.005000	–0.23810	–0.23529	–0.23256	–0.22989	–0.22727	–0.22472	–0.22222	0.9950	1.0050
0.010000	–0.23810	–0.23529	–0.23256	–0.22989	–0.22727	–0.22472	–0.22222	0.9900	1.0101
0.020000	–0.23810	–0.23529	–0.23256	–0.22989	–0.22727	–0.22472	–0.22222	0.9800	1.0204
0.025000	–0.23810	–0.23529	–0.23256	–0.22989	–0.22727	–0.22472	–0.22222	0.9750	1.0256
0.040000	–0.23810	–0.23529	–0.23256	–0.22989	–0.22727	–0.22472	–0.22222	0.9600	1.0417
0.050000	–0.23810	–0.23529	–0.23256	–0.22989	–0.22727	–0.22472	–0.22222	0.9500	1.0526
0.100000	–0.23810	–0.23529	–0.23256	–0.22989	–0.22727	–0.22472	–0.22222	0.9000	1.1111
0.200000	–0.23810	–0.23529	–0.23256	–0.22989	–0.22727	–0.22472	–0.22222	0.8000	1.2500
0.300000	–0.23810	–0.23529	–0.23256	–0.22989	–0.22727	–0.22472	–0.22222	0.7000	1.4286
0.400000	–0.23810	–0.23529	–0.23256	–0.22988	–0.22727	–0.22472	–0.22222	0.6000	1.6667
0.429624	–0.23809	–0.23529	–0.23256	–0.22988	–0.22727	–0.22472	–0.22222	0.5704	1.7532
0.500000	–0.23808	–0.23528	–0.23255	–0.22988	–0.22727	–0.22472	–0.22222	0.5000	2.0000
0.570376	–0.23797	–0.23520	–0.23248	–0.22982	–0.22722	–0.22468	–0.22219	0.4296	2.3276
0.600000	–0.23779	–0.23505	–0.23236	–0.22972	–0.22714	–0.22461	–0.22214	0.4000	2.5000
0.700000	–0.23352	–0.23132	–0.22911	–0.22690	–0.22469	–0.22249	–0.22030	0.3000	3.3333
0.800000	–0.18939	–0.19054	–0.19147	–0.19221	–0.19277	–0.19316	–0.19338	0.2000	5.0000
0.900000	0.18408	0.17113	0.15851	0.14624	0.13431	0.12272	0.11146	0.1000	10.000
0.950000	1.07832	1.05738	1.03654	1.01581	0.99519	0.97471	0.95435	0.0500	20.000
0.960000	1.46829	1.44673	1.42518	1.40364	1.38213	1.36065	1.33922	0.0400	25.000
0.975000	2.42268	2.40287	2.38288	2.36273	2.34242	2.32197	2.30138	0.0250	40.000
0.980000	2.93002	2.91234	2.89440	2.87622	2.85782	2.83919	2.82035	0.0200	50.000
0.990000	4.68252	4.67573	4.66850	4.66085	4.65277	4.64429	4.63541	0.0100	100.00
0.995000	6.64148	6.65056	6.65907	6.66703	6.67443	6.68130	6.68763	0.0050	200.00
0.998000	9.45530	9.49060	9.52521	9.55915	9.59243	9.62504	9.65701	0.0020	500.00
0.999000	11.70785	11.76576	11.82294	11.87938	11.93509	11.99009	12.04437	0.0010	1000.0
0.999500	14.04086	14.12314	14.20463	14.28534	14.36528	14.44446	14.52288	0.0005	2000.0
0.999900	19.68489	19.82845	19.97115	20.11300	20.25402	20.39420	20.53356	0.0001	10000.

Table 2 Percentage Points of Pearson Type III Distribution, P (K_p, G1) 1/

For Negative Skewness
G1 = 0.0 to –9.0

The values in the body of the table are the values of K_p, standardized units, that correspond to these values of Gl for P = 0.0001, 0.0005, 0.0010, 0.0020, 0.0050, 0.0100, 0.0200, 0.0250, 0.0400, 0.0500, 0.1000, 0.2000, 0.3000, 0.4000, 0.429624, 0.5000, 0.570376, 0.6000, 0.7000, 0.8000, 0.9000, 0.9500, 0.9600, 0.9750, 0.9800, 0.9900, 0.9950, 0.9980, 0.9990, 0.9995, and 0.9999 cumulative probability equal to or less than a particular discharge in ft^3/s, or other variable being analyzed.

Five decimals have been retained. The Return Period (T) is defined as l/Q, and P + Q 1.0.

Example:	Given	G1 = – 1.0	Find K_p for P = 0.9900 and Q = .0100 or T = 100
		G1 = – 1.0	K_p = + 1.58838
			Find K_p for P = Q = 0.5000 or T = 2
		G1 = – 1.0	K_p = + 0.16397

where:
G1 = the coefficient of skewness
P = the cumulative probability equal to or less than a particular discharge in ft^3/s, or other variable being analyzed
Q = the cumulative probability equal to or greater than a particular discharge in ft^3/s, or other variable being analyzed
T = the return period and/or recurrence interval
K_p = value for selected percentage points and skewness

1/ This table was produced on the IBM 360/140 by the Central Technical Unit, SCS. The K–values for negative coefficient of skewness were obtained by inverting the K–values in table 1 and changing the signs of the K–values.

Table 2 Percentage points of Pearson Type III distribution (negative skewness)

P	G1=0.0	G1=–0.1	G1=–0.2	G1=–0.3	G1=–0.4	G1=–0.5	G1=–0.6	Q	T
0.000100	–3.71902	–3.93453	–4.15301	–4.37394	–4.59687	–4.82141	–5.04718	0.9999	1.0001
0.000500	–3.29053	–3.45513	–3.62113	–3.78820	–3.95605	–4.12443	–4.29311	0.9995	1.0005
0.001000	–3.09023	–3.23322	–3.37703	–3.52139	–3.66608	–3.81090	–3.95567	0.9990	1.0010
0.002000	–2.87816	–2.99978	–3.12169	–3.24371	–3.36566	–3.48137	–3.60872	0.9980	1.0020
0.005000	–2.57583	–2.66965	–2.76321	–2.85636	–2.94900	–3.04102	–3.13232	0.9950	1.0050
0.010000	–2.32635	–2.39961	–2.47226	–2.54421	–2.61539	–2.68572	–2.75514	0.9900	1.0101
0.020000	–2.05375	–2.10697	–2.15935	–2.21081	–2.26133	–2.31084	–2.35931	0.9800	1.0204
0.025000	–1.95996	–2.00688	–2.05290	–2.09795	–2.14202	–2.18505	–2.22702	0.9750	1.0256
0.040000	–1.75069	–1.78462	–1.81756	–1.84949	–1.88039	–1.91022	–1.93896	0.9600	1.0417
0.050000	–1.64485	–1.67279	–1.69971	–1.72562	–1.75048	–1.77428	–1.79701	0.9500	1.0526
0.100000	–1.28155	–1.29178	–1.30105	–1.30936	–1.31671	–1.32309	–1.32850	0.9000	1.1111
0.200000	–0.84162	–0.83639	–0.83044	–0.82377	–0.81638	–0.80829	–0.79950	0.8000	1.2500
0.300000	–0.52440	–0.51207	–0.49927	–0.48600	–0.47228	–0.45812	–0.44352	0.7000	1.4286
0.400000	–0.25335	–0.23763	–0.22168	–0.20552	–0.18916	–0.17261	–0.15589	0.6000	1.6667
0.429624	–0.17733	–0.16111	–0.14472	–0.12820	–0.11154	–0.09478	–0.07791	0.5704	1.7532
0.500000	0.00000	0.01662	0.03325	0.04993	0.06651	0.08302	0.09945	0.5000	2.0000
0.570376	0.17733	0.19339	0.20925	0.22492	0.24037	0.25558	0.27047	0.4296	2.3276
0.600000	0.25335	0.26882	0.28403	0.29897	0.31362	0.32796	0.34198	0.4000	2.5000
0.700000	0.52440	0.53624	0.54757	0.55839	0.56867	0.57840	0.58757	0.3000	3.3333
0.800000	0.84162	0.84611	0.84986	0.85285	0.85508	0.85653	0.85718	0.2000	5.0000
0.900000	1.28155	1.27037	1.25824	1.24516	1.23114	1.21618	1.20028	0.1000	10.000
0.950000	1.64485	1.61594	1.58607	1.55527	1.52357	1.49101	1.45762	0.0500	20.000
0.960000	1.75069	1.71580	1.67999	1.64329	1.60574	1.56740	1.52830	0.0400	25.000
0.975000	1.95996	1.91219	1.86360	1.81427	1.76427	1.71366	1.66253	0.0250	40.000
0.980000	2.05375	1.99973	1.94499	1.88959	1.83361	1.77716	1.72033	0.0200	50.000
0.990000	2.32635	2.25258	2.17840	2.10394	2.02933	1.95472	1.88029	0.0100	100.00
0.995000	2.57583	2.48187	2.38795	2.29423	2.20092	2.10825	2.01644	0.0050	200.00
0.998000	2.87816	2.75706	2.63672	2.51741	2.39942	2.28311	2.16884	0.0020	500.00
0.999000	3.09023	2.94834	2.80786	2.66915	2.53261	2.39867	2.26780	0.0010	1000.0
0.999500	3.29053	3.12767	2.96698	2.80889	2.65390	2.50257	2.35549	0.0005	2000.0
0.999900	3.71902	3.50703	3.29921	3.09631	2.89907	2.70836	2.52507	0.0001	10000.

P	G1=–0.7	G1=–0.8	G1=–0.9	G1=–1.0	G1=–1.1	G1=–1.2	G1=–1.3	Q	T
0.000100	–5.27389	–5.50124	–5.72899	–5.95691	–6.18480	–6.41249	–6.63980	0.9999	1.0001
0.000500	–4.46189	–4.63057	–4.79899	–4.96701	–5.13449	–5.30130	–5.46735	0.9995	1.0005
0.001000	–4.10022	–4.24439	–4.38807	–4.53112	–4.67344	–4.81492	–4.95549	0.9990	1.0010
0.002000	–3.72957	–3.84981	–3.96932	–4.08802	–4.20582	–4.32263	–4.43839	0.9980	1.0020
0.005000	–3.22281	–3.31243	–3.40109	–3.48874	–3.57530	–3.66073	–3.74497	0.9950	1.0050
0.010000	–2.82359	–2.89101	–2.95735	–3.02256	–3.08660	–3.14944	–3.21103	0.9900	1.0101
0.020000	–2.40670	–2.45298	–2.49811	–2.54206	–2.58480	–2.62631	–2.66657	0.9800	1.0204
0.025000	–2.26790	–2.30764	–2.34623	–2.38364	–2.41984	–2.45482	–2.48855	0.9750	1.0256
0.040000	–1.96660	–1.99311	–2.01848	–2.04269	–2.06573	–2.08758	–2.10823	0.9600	1.0417
0.050000	–1.81864	–1.83916	–1.85856	–1.87683	–1.89395	–1.90992	–1.92472	0.9500	1.0526
0.100000	–1.33294	–1.33640	–1.33889	–1.34039	–1.34092	–1.34047	–1.33904	0.9000	1.1111
0.200000	–0.79002	–0.77986	–0.76902	–0.75752	–0.74537	–0.73257	–0.71915	0.8000	1.2500
0.300000	–0.42851	–0.41309	–0.39729	–0.38111	–0.36458	–0.34772	–0.33054	0.7000	1.4286
0.400000	–0.13901	–0.12199	–0.10486	–0.08763	–0.07032	–0.05297	–0.03560	0.6000	1.6667
0.429624	–0.06097	–0.04397	–0.02693	–0.00987	0.00719	0.02421	0.04116	0.5704	1.7532
0.500000	0.11578	0.13199	0.14807	0.16397	0.17968	0.19517	0.21040	0.5000	2.0000
0.570376	0.28516	0.29961	0.31368	0.32740	0.34075	0.35370	0.36620	0.4296	2.3276
0.600000	0.35565	0.36889	0.38186	0.39434	0.40638	0.41794	0.42899	0.4000	2.5000
0.700000	0.59615	0.60412	0.61146	0.61815	0.62415	0.62944	0.63400	0.3000	3.3333
0.800000	0.85703	0.85607	0.85426	0.85161	0.84809	0.84369	0.83841	0.2000	5.0000
0.900000	1.18347	1.16574	1.14712	1.12762	1.10726	1.08608	1.06413	0.1000	10.000
0.950000	1.42345	1.38855	1.35299	1.31684	1.28019	1.24313	1.20578	0.0500	20.000
0.960000	1.48852	1.44813	1.40720	1.36584	1.32414	1.28225	1.24028	0.0400	25.000
0.975000	1.61099	1.55914	1.50712	1.45507	1.40314	1.35153	1.30042	0.0250	40.000
0.980000	1.66325	1.60604	1.54886	1.49188	1.43529	1.37929	1.32412	0.0200	50.000
0.990000	1.80621	1.73271	1.66001	1.58838	1.51808	1.44942	1.38267	0.0100	100.00
0.995000	1.92580	1.83660	1.74919	1.66390	1.58110	1.50114	1.42439	0.0050	200.00
0.998000	2.05701	1.94806	1.84244	1.74062	1.64305	1.55016	1.46232	0.0020	500.00
0.999000	2.14053	2.01739	1.89894	1.78572	1.67825	1.57695	1.48216	0.0010	1000.0
0.999500	2.21328	2.07661	1.94611	1.82241	1.70603	1.59738	1.49673	0.0005	2000.0
0.999900	2.35015	2.18448	2.02891	1.88410	1.75053	1.62838	1.51752	0.0001	10000.

Table 2 Percentage points of Pearson Type III distribution (negative skewness)—Continued

P	G1=–1.4	G1=–1.5	G1=–1.6	G1=–1.7	G1=–1.8	G1=–1.9	G1=–2.0	Q	T
0.000100	–6.86661	–7.09277	–7.31818	–7.54272	–7.76632	–7.98888	–8.21034	0.9999	1.0001
0.000500	–5.63252	–5.79673	–5.95990	–6.12196	–6.28285	–6.44251	–6.60090	0.9995	1.0005
0.001000	–5.09505	–5.23353	–5.37087	–5.50701	–5.64190	–5.77549	–5.90776	0.9990	1.0010
0.002000	–4.55304	–4.66651	–4.77875	–4.88971	–4.99937	–5.10768	–5.21461	0.9980	1.0020
0.005000	–3.82798	–3.90973	–3.99016	–4.06926	–4.14700	–4.22336	–4.29832	0.9950	1.0050
0.010000	–3.27134	–3.33035	–3.38804	–3.44438	–3.49935	–3.55295	–3.60517	0.9900	1.0101
0.020000	–2.70556	–2.74325	–2.77964	–2.81472	–2.84848	–2.88091	–2.91202	0.9800	1.0204
0.025000	–2.52102	–2.55222	–2.58214	–2.61076	–2.63810	–2.66413	–2.68888	0.9750	1.0256
0.040000	–2.12768	–2.14591	–2.16293	–2.17873	–2.19332	–2.20670	–2.21888	0.9600	1.0417
0.050000	–1.93836	–1.95083	–1.96213	–1.97227	–1.98124	–1.98906	–1.99573	0.9500	1.0526
0.100000	–1.33665	–1.33330	–1.32900	–1.32376	–1.31760	–1.31054	–1.30259	0.9000	1.1111
0.200000	–0.70512	–0.69050	–0.67532	–0.65959	–0.64335	–0.62662	–0.60944	0.8000	1.2500
0.300000	–0.31307	–0.29535	–0.27740	–0.25925	–0.24094	–0.22250	–0.20397	0.7000	1.4286
0.400000	–0.01824	–0.00092	0.01631	0.03344	0.05040	0.06718	0.08371	0.6000	1.6667
0.429624	0.05803	0.07476	0.09132	0.10769	0.12381	0.13964	0.15516	0.5704	1.7532
0.500000	0.22535	0.23996	0.25422	0.26808	0.28150	0.29443	0.30685	0.5000	2.0000
0.570376	0.37824	0.38977	0.40075	0.41116	0.42095	0.43008	0.43854	0.4296	2.3276
0.600000	0.43949	0.44942	0.45873	0.46739	0.47538	0.48265	0.48917	0.4000	2.5000
0.700000	0.63779	0.64080	0.64300	0.64436	0.64488	0.64453	0.64333	0.3000	3.3333
0.800000	0.83223	0.82516	0.81720	0.80837	0.79868	0.78816	0.77686	0.2000	5.0000
0.900000	1.04144	1.01810	0.99418	0.96977	0.94496	0.91988	0.89464	0.1000	10.000
0.950000	1.16827	1.13075	1.09338	1.05631	1.01973	0.98381	0.94871	0.0500	20.000
0.960000	1.19842	1.15682	1.11566	1.07513	1.03543	0.99672	0.95918	0.0400	25.000
0.975000	1.25004	1.20059	1.15229	1.10537	1.06001	1.01640	0.97468	0.0250	40.000
0.980000	1.26999	1.21716	1.16584	1.11628	1.06864	1.02311	0.97980	0.0200	50.000
0.990000	1.31815	1.25611	1.19680	1.14042	1.08711	1.03695	0.98995	0.0100	100.00
0.995000	1.35114	1.28167	1.21618	1.15477	1.09749	1.04427	0.99499	0.0050	200.00
0.998000	1.37981	1.30279	1.23132	1.16534	1.10465	1.04898	0.99800	0.0020	500.00
0.999000	1.39408	1.31275	1.23805	1.16974	1.10743	1.05068	0.99900	0.0010	1000.0
0.999500	1.40413	1.31944	1.24235	1.17240	1.10901	1.05159	0.99950	0.0005	2000.0
0.999900	1.41753	1.32774	1.24728	1.17520	1.11054	1.05239	0.99990	0.0001	10000.

P	G1=–2.1	G1=–2.2	G1=–2.3	G1=–2.4	G1=–2.5	G1=–2.6	G1=–2.7	Q	T
0.000100	–8.43064	–8.64971	–8.86753	–9.08403	–9.29920	–9.51301	–9.72543	0.9999	1.0001
0.000500	–6.75798	–6.91370	–7.06804	–7.22098	–7.37250	–7.52258	–7.67121	0.9995	1.0005
0.001000	–6.03865	–6.16816	–6.29626	–6.42292	–6.54814	–6.67191	–6.79421	0.9990	1.0010
0.002000	–5.32014	–5.42426	–5.52694	–5.62818	–5.72796	–5.82629	–5.92316	0.9980	1.0020
0.005000	–4.37186	–4.44398	–4.51467	–4.58393	–4.65176	–4.71815	–4.78313	0.9950	1.0050
0.010000	–3.65600	–3.70543	–3.75347	–3.80013	–3.84540	–3.88930	–3.93183	0.9900	1.0101
0.020000	–2.94181	–2.97028	–2.99744	–3.02330	–3.04787	–3.07116	–3.09320	0.9800	1.0204
0.025000	–2.71234	–2.73451	–2.75541	–2.77506	–2.79345	–2.81062	–2.82658	0.9750	1.0256
0.040000	–2.22986	–2.23967	–2.24831	–2.25581	–2.26217	–2.26743	–2.27160	0.9600	1.0417
0.050000	–2.00128	–2.00570	–2.00903	–2.01128	–2.01247	–2.01263	–2.01177	0.9500	1.0526
0.100000	–1.29377	–1.28412	–1.27365	–1.26240	–1.25039	–1.23766	–1.22422	0.9000	1.1111
0.200000	–0.59183	–0.57383	–0.55549	–0.53683	–0.51789	–0.49872	–0.47934	0.8000	1.2500
0.300000	–0.18540	–0.16682	–0.14827	–0.12979	–0.11143	–0.09323	–0.07523	0.7000	1.4286
0.400000	0.09997	0.11590	0.13148	0.14665	0.16138	0.17564	0.18939	0.6000	1.6667
0.429624	0.17030	0.18504	0.19933	0.21313	0.22642	0.23915	0.25129	0.5704	1.7532
0.500000	0.31872	0.32999	0.34063	0.35062	0.35992	0.36852	0.37640	0.5000	2.0000
0.570376	0.44628	0.45329	0.45953	0.46499	0.46966	0.47353	0.47660	0.4296	2.3276
0.600000	0.49494	0.49991	0.50409	0.50744	0.50999	0.51171	0.51263	0.4000	2.5000
0.700000	0.64125	0.63833	0.63456	0.62999	0.62463	0.61854	0.61176	0.3000	3.3333
0.800000	0.76482	0.75211	0.73880	0.72495	0.71067	0.69602	0.68111	0.2000	5.0000
0.900000	0.86938	0.84422	0.81929	0.79472	0.77062	0.74709	0.72422	0.1000	10.000
0.950000	0.91458	0.88156	0.84976	0.81927	0.79015	0.76242	0.73610	0.0500	20.000
0.960000	0.92295	0.88814	0.85486	0.82315	0.79306	0.76456	0.73765	0.0400	25.000
0.975000	0.93495	0.89728	0.86169	0.82817	0.79667	0.76712	0.73943	0.0250	40.000
0.980000	0.93878	0.90009	0.86371	0.82959	0.79765	0.76779	0.73987	0.0200	50.000
0.990000	0.94607	0.90521	0.86723	0.83196	0.79921	0.76878	0.74049	0.0100	100.00
0.995000	0.94945	0.90742	0.86863	0.83283	0.79973	0.76909	0.74067	0.0050	200.00
0.998000	0.95131	0.90854	0.86929	0.83320	0.79994	0.76920	0.74073	0.0020	500.00
0.999000	0.95188	0.90885	0.86945	0.83328	0.79998	0.76922	0.74074	0.0010	1000.0
0.999500	0.95215	0.90899	0.86952	0.83331	0.79999	0.76923	0.74074	0.0005	2000.0
0.999900	0.95234	0.90908	0.86956	0.83333	0.80000	0.76923	0.74074	0.0001	10000.

Table 2 Percentage points of Pearson Type III distribution (negative skewness)—Continued

P	G1=–2.8	G1=–2.9	G1=–3.0	G1=–3.1	G1=–3.2	G1=–3.3	G1=–3.4	Q	T
0.000100	–9.93643	–10.14602	–10.35418	–10.56090	–10.76618	–10.97001	–11.17239	0.9999	1.0001
0.000500	–7.81839	–7.96411	–8.10836	–8.25115	–8.39248	–8.53236	–8.67079	0.9995	1.0005
0.001000	–6.91505	–7.03443	–7.15235	–7.26881	–7.38382	–7.49739	–7.60953	0.9990	1.0010
0.002000	–6.01858	–6.11254	–6.20506	–6.29613	–6.38578	–6.47401	–6.56084	0.9980	1.0020
0.005000	–4.84669	–4.90884	–4.96959	–5.02897	–5.08697	–5.14362	–5.19892	0.9950	1.0050
0.010000	–3.97301	–4.01286	–4.05138	–4.08859	–4.12452	–4.15917	–4.19257	0.9900	1.0101
0.020000	–3.11399	–3.13356	–3.15193	–3.16911	–3.18512	–3.20000	–3.21375	0.9800	1.0204
0.025000	–2.84134	–2.85492	–2.86735	–2.87865	–2.88884	–2.89795	–2.90599	0.9750	1.0256
0.040000	–2.27470	–2.27676	–2.27780	–2.27785	–2.27693	–2.27506	–2.27229	0.9600	1.0417
0.050000	–2.00992	–2.00710	–2.00335	–1.99869	–1.99314	–1.98674	–1.97951	0.9500	1.0526
0.100000	–1.21013	–1.19539	–1.18006	–1.16416	–1.14772	–1.13078	–1.11337	0.9000	1.1111
0.200000	–0.45980	–0.44015	–0.42040	–0.40061	–0.38081	–0.36104	–0.34133	0.8000	1.2500
0.300000	–0.05746	–0.03997	–0.02279	–0.00596	0.01050	0.02654	0.04215	0.7000	1.4286
0.400000	0.20259	0.21523	0.22726	0.23868	0.24946	0.25958	0.26904	0.6000	1.6667
0.429624	0.26282	0.27372	0.28395	0.29351	0.30238	0.31055	0.31802	0.5704	1.7532
0.500000	0.38353	0.38991	0.39554	0.40041	0.40454	0.40792	0.41058	0.5000	2.0000
0.570376	0.47888	0.48037	0.48109	0.48107	0.48033	0.47890	0.47682	0.4296	2.3276
0.600000	0.51276	0.51212	0.51073	0.50863	0.50585	0.50244	0.49844	0.4000	2.5000
0.700000	0.60434	0.59634	0.58783	0.57887	0.56953	0.55989	0.55000	0.3000	3.3333
0.800000	0.66603	0.65086	0.63569	0.62060	0.60567	0.59096	0.57652	0.2000	5.0000
0.900000	0.70209	0.68075	0.66023	0.64056	0.62175	0.60379	0.58666	0.1000	10.000
0.950000	0.71116	0.68759	0.66532	0.64429	0.62445	0.60572	0.58802	0.0500	20.000
0.960000	0.71227	0.68336	0.66585	0.64465	0.62469	0.60587	0.58812	0.0400	25.000
0.975000	0.71348	0.68917	0.66638	0.64500	0.62491	0.60601	0.58821	0.0250	40.000
0.980000	0.71377	0.68935	0.66649	0.64507	0.62495	0.60603	0.58822	0.0200	50.000
0.990000	0.71415	0.68959	0.66663	0.64514	0.62499	0.60606	0.58823	0.0100	100.00
0.995000	0.71425	0.68964	0.66666	0.64516	0.62500	0.60606	0.58824	0.0050	200.00
0.998000	0.71428	0.68965	0.66667	0.64516	0.62500	0.60606	0.58824	0.0020	500.00
0.999000	0.71428	0.68965	0.66667	0.64516	0.62500	0.60606	0.58824	0.0010	1000.0
0.999500	0.71429	0.68966	0.66667	0.64516	0.62500	0.60606	0.58824	0.0005	2000.0
0.999900	0.71429	0.68966	0.66667	0.64516	0.62500	0.60606	0.58824	0.0001	10000.

P	G1=–3.5	G1=–3.6	G1=–3.7	G1=–3.8	G1=–3.9	G1=–4.0	G1=–4.1	Q	T
0.000100	–11.37334	–11.57284	–11.77092	–11.96757	–12.16280	–12.35663	–12.54906	0.9999	1.0001
0.000500	–8.80779	–8.94335	–9.07750	–9.21023	–9.34158	–9.47154	–9.60013	0.9995	1.0005
0.001000	–7.72024	–7.82954	–7.93744	–8.04395	–8.14910	–8.25289	–8.35534	0.9990	1.0010
0.002000	–6.64627	–6.73032	–6.81301	–6.89435	–6.97435	–7.05304	–7.13043	0.9980	1.0020
0.005000	–5.25291	–5.30559	–5.35698	–5.40711	–5.45598	–5.50362	–5.55005	0.9950	1.0050
0.010000	–4.22473	–4.25569	–4.28545	–4.31403	–4.34147	–4.36777	–4.39296	0.9900	1.0101
0.020000	–3.22641	–3.23800	–3.24853	–3.25803	–3.26653	–3.27404	–3.28060	0.9800	1.0204
0.025000	–2.91299	–2.91898	–2.92397	–2.92799	–2.93107	–2.93324	–2.93450	0.9750	1.0256
0.040000	–2.26862	–2.26409	–2.25872	–2.25254	–2.24558	–2.23786	–2.22940	0.9600	1.0417
0.050000	–1.97147	–1.96266	–1.95311	–1.94283	–1.93186	–1.92023	–1.90796	0.9500	1.0526
0.100000	–1.09552	–1.07726	–1.05863	–1.03965	–1.02036	–1.00079	–0.98096	0.9000	1.1111
0.200000	–0.32171	–0.30223	–0.28290	–0.26376	–0.24484	–0.22617	–0.20777	0.8000	1.2500
0.300000	0.05730	0.07195	0.08610	0.09972	0.11279	0.12530	0.13725	0.7000	1.4286
0.400000	0.27782	0.28592	0.29335	0.30010	0.30617	0.31159	0.31635	0.6000	1.6667
0.429624	0.32479	0.33085	0.33623	0.34092	0.34494	0.34831	0.35105	0.5704	1.7532
0.500000	0.41253	0.41381	0.41442	0.41441	0.41381	0.41265	0.41097	0.5000	2.0000
0.570376	0.47413	0.47088	0.46711	0.46286	0.45819	0.45314	0.44777	0.4296	2.3276
0.600000	0.49391	0.48888	0.48342	0.47758	0.47141	0.46496	0.45828	0.4000	2.5000
0.700000	0.53993	0.52975	0.51952	0.50929	0.49911	0.48902	0.47906	0.3000	3.3333
0.800000	0.56242	0.54867	0.53533	0.52240	0.50990	0.49784	0.48622	0.2000	5.0000
0.900000	0.57035	0.55483	0.54006	0.52600	0.51261	0.49986	0.48772	0.1000	10.000
0.950000	0.57130	0.55548	0.54050	0.52629	0.51281	0.49999	0.48780	0.0500	20.000
0.960000	0.57136	0.55552	0.54052	0.52630	0.51281	0.50000	0.48780	0.0400	25.000
0.975000	0.57141	0.55555	0.54054	0.52631	0.51282	0.50000	0.48780	0.0250	40.000
0.980000	0.57142	0.55555	0.54054	0.52631	0.51282	0.50000	0.48780	0.0200	50.000
0.990000	0.57143	0.55556	0.54054	0.52632	0.51282	0.50000	0.48780	0.0100	100.00
0.995000	0.57143	0.55556	0.54054	0.52632	0.51282	0.50000	0.48780	0.0050	200.00
0.998000	0.57143	0.55556	0.54054	0.52632	0.51282	0.50000	0.48780	0.0020	500.00
0.999000	0.57143	0.55556	0.54054	0.52632	0.51282	0.50000	0.48780	0.0010	1000.0
0.999500	0.57143	0.55556	0.54054	0.52632	0.51282	0.50000	0.48780	0.0005	2000.0
0.999900	0.57143	0.55556	0.54054	0.52632	0.51282	0.50000	0.48780	0.0001	10000.

Table 2 Percentage points of Pearson Type III distribution (negative skewness)—Continued

P	G1=–4.2	G1=–4.3	G1=–4.4	G1=–4.5	G1=–4.6	G1=–4.7	G1=–4.8	Q	T
0.000100	–12.74010	–12.92977	–13.11808	–13.30504	–13.49066	–13.67495	–13.85794	0.9999	1.0001
0.000500	–9.72737	–9.85326	–9.97784	–10.10110	–10.22307	–10.34375	–10.46318	0.9995	1.0005
0.001000	–8.45646	–8.55627	–8.65479	–8.75202	–8.84800	–8.94273	–9.03623	0.9990	1.0010
0.002000	–7.20654	–7.28138	–7.35497	–7.42733	–7.49847	–7.56842	–7.63718	0.9980	1.0020
0.005000	–5.59528	–5.63934	–5.68224	–5.72400	–5.76464	–5.80418	–5.84265	0.9950	1.0050
0.010000	–4.41706	–4.44009	–4.46207	–4.48303	–4.50297	–4.52192	–4.53990	0.9900	1.0101
0.020000	–3.28622	–3.29092	–3.29473	–3.29767	–3.29976	–3.30103	–3.30149	0.9800	1.0204
0.025000	–2.93489	–2.93443	–2.93314	–2.93105	–2.92818	–2.92455	–2.92017	0.9750	1.0256
0.040000	–2.22024	–2.21039	–2.19988	–2.18874	–2.17699	–2.16465	–2.15174	0.9600	1.0417
0.050000	–1.89508	–1.88160	–1.86757	–1.85300	–1.83792	–1.82234	–1.80631	0.9500	1.0526
0.100000	–0.96090	–0.94064	–0.92022	–0.89964	–0.87895	–0.85817	–0.83731	0.9000	1.1111
0.200000	–0.18967	–0.17189	–0.15445	–0.13737	–0.12067	–0.10436	–0.08847	0.8000	1.2500
0.300000	0.14861	0.15939	0.16958	0.17918	0.18819	0.19661	0.20446	0.7000	1.4286
0.400000	0.32049	0.32400	0.32693	0.32928	0.33108	0.33236	0.33315	0.6000	1.6667
0.429624	0.35318	0.35473	0.35572	0.35619	0.35616	0.35567	0.35475	0.5704	1.7532
0.500000	0.40881	0.40621	0.40321	0.39985	0.39617	0.39221	0.38800	0.5000	2.0000
0.570376	0.44212	0.43623	0.43016	0.42394	0.41761	0.41121	0.40477	0.4296	2.3276
0.600000	0.45142	0.44442	0.43734	0.43020	0.42304	0.41590	0.40880	0.4000	2.5000
0.700000	0.46927	0.45967	0.45029	0.44114	0.43223	0.42357	0.41517	0.3000	3.3333
0.800000	0.47504	0.46428	0.45395	0.44402	0.43448	0.42532	0.41652	0.2000	5.0000
0.900000	0.47614	0.46508	0.45452	0.44443	0.43477	0.42553	0.41666	0.1000	10.000
0.950000	0.47619	0.46511	0.45454	0.44444	0.43478	0.42553	0.41667	0.0500	20.000
0.960000	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667	0.0400	25.000
0.975000	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667	0.0250	40.000
0.980000	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667	0.0200	50.000
0.990000	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667	0.0100	100.00
0.995000	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667	0.0050	200.00
0.998000	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667	0.0020	500.00
0.999000	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667	0.0010	1000.0
0.999500	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667	0.0005	2000.0
0.999900	0.47619	0.46512	0.45455	0.44444	0.43478	0.42553	0.41667	0.0001	10000.

P	G1=–4.9	G1=–5.0	G1=–5.1	G1=–5.2	G1=–5.3	G1=–5.4	G1=–5.5	Q	T
0.000100	–14.03963	–14.22004	–14.39918	–14.57706	–14.75370	–14.92912	–15.10332	0.9999	1.0001
0.000500	–10.58135	–10.69829	–10.81401	–10.92853	–11.04186	–11.15402	–11.26502	0.9995	1.0005
0.001000	–9.12852	–9.21961	–9.30952	–9.39827	–9.48586	–9.57232	–9.65766	0.9990	1.0010
0.002000	–7.70479	–7.77124	–7.83657	–7.90078	–7.96390	–8.02594	–8.08691	0.9980	1.0020
0.005000	–5.88004	–5.91639	–5.95171	–5.98602	–6.01934	–6.05169	–6.08307	0.9950	1.0050
0.010000	–4.55694	–4.57304	–4.58823	–4.60252	–4.61594	–4.62850	–4.64022	0.9900	1.0101
0.020000	–3.30116	–3.30007	–3.29823	–3.29567	–3.29240	–3.28844	–3.28381	0.9800	1.0204
0.025000	–2.91508	–2.90930	–2.90283	–2.89572	–2.88796	–2.87959	–2.87062	0.9750	1.0256
0.040000	–2.13829	–2.12432	–2.10985	–2.09490	–2.07950	–2.06365	–2.04739	0.9600	1.0417
0.050000	–1.78982	–1.77292	–1.75563	–1.73795	–1.71992	–1.70155	–1.68287	0.9500	1.0526
0.100000	–0.81641	–0.79548	–0.77455	–0.75364	–0.73277	–0.71195	–0.69122	0.9000	1.1111
0.200000	–0.07300	–0.05798	–0.04340	–0.02927	–0.01561	–0.00243	0.01028	0.8000	1.2500
0.300000	0.21172	0.21843	0.22458	0.23019	0.23527	0.23984	0.24391	0.7000	1.4286
0.400000	0.33347	0.33336	0.33284	0.33194	0.33070	0.32914	0.32729	0.6000	1.6667
0.429624	0.35343	0.35174	0.34972	0.34740	0.34481	0.34198	0.33895	0.5704	1.7532
0.500000	0.38359	0.37901	0.37428	0.36945	0.36453	0.35956	0.35456	0.5000	2.0000
0.570376	0.39833	0.39190	0.38552	0.37919	0.37295	0.36680	0.36076	0.4296	2.3276
0.600000	0.40177	0.39482	0.38799	0.38127	0.37469	0.36825	0.36196	0.4000	2.5000
0.700000	0.40703	0.39914	0.39152	0.38414	0.37701	0.37011	0.36345	0.3000	3.3333
0.800000	0.40806	0.39993	0.39211	0.38458	0.37734	0.37036	0.36363	0.2000	5.0000
0.900000	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364	0.1000	10.000
0.950000	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364	0.0500	20.000
0.960000	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364	0.0400	25.000
0.975000	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364	0.0250	40.000
0.980000	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364	0.0200	50.000
0.990000	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364	0.0100	100.00
0.995000	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364	0.0050	200.00
0.998000	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364	0.0020	500.00
0.999000	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364	0.0010	1000.0
0.999500	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364	0.0005	2000.0
0.999900	0.40816	0.40000	0.39216	0.38462	0.37736	0.37037	0.36364	0.0001	10000.

Table 2 Percentage points of Pearson Type III distribution (negative skewness)—Continued

P	G1=–5.6	G1=–5.7	G1=–5.8	G1=–5.9	G1=–6.0	G1=–6.1	G1=–6.2	Q	T
0.000100	–15.27632	–15.44813	–15.61878	–15.78826	–15.95660	–16.12380	–16.28989	0.9999	1.0001
0.000500	–11.37487	–11.48360	–11.59122	–11.69773	–11.80316	–11.90752	–12.01082	0.9995	1.0005
0.001000	–9.74190	–9.82505	–9.90713	–9.98815	–10.06812	–10.14706	–10.22499	0.9990	1.0010
0.002000	–8.14683	–8.20572	–8.26359	–8.32046	–8.37634	–8.43125	–8.48519	0.9980	1.0020
0.005000	–6.11351	–6.14302	–6.17162	–6.19933	–6.22616	–6.25212	–6.27723	0.9950	1.0050
0.010000	–4.65111	–4.66120	–4.67050	–4.67903	–4.68680	–4.69382	–4.70013	0.9900	1.0101
0.020000	–3.27854	–3.27263	–3.26610	–3.25898	–3.25128	–3.24301	–3.23419	0.9800	1.0204
0.025000	–2.86107	–2.85096	–2.84030	–2.82912	–2.81743	–2.80525	–2.79259	0.9750	1.0256
0.040000	–2.03073	–2.01369	–1.99629	–1.97855	–1.96048	–1.94210	–1.92343	0.9600	1.0417
0.050000	–1.66390	–1.64464	–1.62513	–1.60538	–1.58541	–1.56524	–1.54487	0.9500	1.0526
0.100000	–0.67058	–0.65006	–0.62966	–0.60941	–0.58933	–0.56942	–0.54970	0.9000	1.1111
0.200000	0.02252	0.03427	0.04553	0.05632	0.06662	0.07645	0.08580	0.8000	1.2500
0.300000	0.24751	0.25064	0.25334	0.25562	0.25750	0.25901	0.26015	0.7000	1.4286
0.400000	0.32519	0.32285	0.32031	0.31759	0.31472	0.31171	0.30859	0.6000	1.6667
0.429624	0.33573	0.33236	0.32886	0.32525	0.32155	0.31780	0.31399	0.5704	1.7532
0.500000	0.34955	0.34455	0.33957	0.33463	0.32974	0.32492	0.32016	0.5000	2.0000
0.570376	0.35484	0.34903	0.34336	0.33782	0.33242	0.32715	0.32202	0.4296	2.3216
0.600000	0.35583	0.34985	0.34402	0.33836	0.33285	0.32750	0.32230	0.4000	2.5000
0.700000	0.35700	0.35078	0.34476	0.33893	0.33330	0.32784	0.32256	0.3000	3.3333
0.800000	0.35714	0.35087	0.34483	0.33898	0.33333	0.32787	0.32258	0.2000	5.0000
0.900000	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258	0.1000	10.000
0.950000	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258	0.0500	20.000
0.960000	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258	0.0400	25.000
0.975000	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258	0.0250	40.000
0.980000	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258	0.0200	50.000
0.990000	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258	0.0100	100.00
0.995000	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258	0.0050	200.00
0.998000	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258	0.0020	500.00
0.999000	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258	0.0010	1000.0
0.999500	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258	0.0005	2000.0
0.999900	0.35714	0.35088	0.34483	0.33898	0.33333	0.32787	0.32258	0.0001	10000.

P	G1=–6.3	G1=–6.4	G1=–6.5	G1=–6.6	G1=–6.7	G1=–6.8	G1=–6.9	Q	T
0.000100	–16.45487	–16.61875	–16.78156	–16.94329	–17.10397	–17.26361	–17.42221	0.9999	1.0001
0.000500	–12.11307	–12.21429	–12.31450	–12.41370	–12.51190	–12.60913	–12.70539	0.9995	1.0005
0.001000	–10.30192	–10.37785	–10.45281	–10.52681	–10.59986	–10.67197	–10.74316	0.9990	1.0010
0.002000	–8.53820	–8.59027	–8.64142	–8.69167	–8.74102	–8.78950	–8.83711	0.9980	1.0020
0.005000	–6.30151	–6.32497	–6.34762	–6.36948	–6.39055	–6.41086	–6.43042	0.9950	1.0050
0.010000	–4.70571	–4.71061	–4.71482	–4.71836	–4.72125	–4.72350	–4.72512	0.9900	1.0101
0.020000	–3.22484	–3.21497	–3.20460	–3.19374	–3.18241	–3.17062	–3.15838	0.9800	1.0204
0.025000	–2.77947	–2.76591	–2.75191	–2.73751	–2.72270	–2.70751	–2.69195	0.9750	1.0256
0.040000	–1.90449	–1.88528	–1.86584	–1.84616	–1.82627	–1.80618	–1.78591	0.9600	1.0417
0.050000	–1.52434	–1.50365	–1.48281	–1.46186	–1.44079	–1.41963	–1.39839	0.9500	1.0526
0.100000	–0.53019	–0.51089	–0.49182	–0.47299	–0.45440	–0.43608	–0.41803	0.9000	1.1111
0.200000	0.09469	0.10311	0.11107	0.11859	0.12566	0.13231	0.13853	0.8000	1.2500
0.300000	0.26097	0.26146	0.26167	0.26160	0.26128	0.26072	0.25995	0.7000	1.4286
0.400000	0.30538	0.30209	0.29875	0.29537	0.29196	0.28854	0.28511	0.6000	1.6667
0.429624	0.31016	0.30631	0.30246	0.29862	0.29480	0.29101	0.28726	0.5704	1.7532
0.500000	0.31549	0.31090	0.30639	0.30198	0.29766	0.29344	0.28931	0.5000	2.0000
0.570376	0.31702	0.31216	0.30743	0.30283	0.29835	0.29400	0.28977	0.4296	2.3276
0.600000	0.31724	0.31234	0.30757	0.30294	0.29844	0.29407	0.28982	0.4000	2.5000
0.700000	0.31745	0.31249	0.30769	0.30303	0.29850	0.29412	0.28985	0.3000	3.3333
0.800000	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986	0.2000	5.0000
0.900000	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986	0.1000	10.000
0.950000	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986	0.0500	20.000
0.960000	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986	0.0400	25.000
0.975000	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986	0.0250	40.000
0.980000	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986	0.0200	50.000
0.990000	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986	0.0100	100.00
0.995000	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986	0.0050	200.00
0.998000	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986	0.0020	500.00
0.999000	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986	0.0010	1000.0
0.999500	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986	0.0005	2000.0
0.999900	0.31746	0.31250	0.30769	0.30303	0.29851	0.29412	0.28986	0.0001	10000.

Table 2 Percentage points of Pearson Type III distribution (negative skewness)—Continued

P	G1=–7.0	G1=–7.1	G1=–7.2	G1=–7.3	G1=–7.4	G1=–7.5	G1=–7.6	Q	T
0.000100	–17.57979	–17.73636	–17.89193	–18.04652	–18.20013	–18.35278	–18.50447	0.9999	1.0001
0.000500	–12.80069	–12.89505	–12.98848	–13.08098	–13.17258	–13.26328	–13.35309	0.9995	1.0005
0.001000	–10.81343	–10.88281	–10.95129	–11.01890	–11.08565	–11.15154	–11.21658	0.9990	1.0010
0.002000	–8.88387	–8.92979	–8.97488	–9.01915	–9.06261	–9.10528	–9.14717	0.9980	1.0020
0.005000	–6.44924	–6.46733	–6.48470	–6.50137	–6.51735	–6.53264	–6.54727	0.9950	1.0050
0.010000	–4.72613	–4.72653	–4.72635	–4.72559	–4.72427	–4.72240	–4.71998	0.9900	1.0101
0.020000	–3.14572	–3.13263	–3.11914	–3.10525	–3.09099	–3.07636	–3.06137	0.9800	1.0204
0.025000	–2.67603	–2.65977	–2.64317	–2.62626	–2.60905	–2.59154	–2.57375	0.9750	1.0256
0.040000	–1.76547	–1.74487	–1.72412	–1.70325	–1.68225	–1.66115	–1.63995	0.9600	1.0417
0.050000	–1.37708	–1.35571	–1.33430	–1.31287	–1.29141	–1.26995	–1.24850	0.9500	1.0526
0.100000	–0.40026	–0.38277	–0.36557	–0.34868	–0.33209	–0.31582	–0.29986	0.9000	1.1111
0.200000	0.14434	0.14975	0.15478	0.15942	0.16371	0.16764	0.17123	0.8000	1.2500
0.300000	0.25899	0.25785	0.25654	0.25510	0.25352	0.25183	0.25005	0.7000	1.4286
0.400000	0.28169	0.27829	0.27491	0.27156	0.26825	0.26497	0.26175	0.6000	1.6667
0.429624	0.28355	0.27990	0.27629	0.27274	0.26926	0.26584	0.26248	0.5704	1.7532
0.500000	0.28528	0.28135	0.27751	0.27376	0.27010	0.26654	0.26306	0.5000	2.0000
0.570376	0.28565	0.28164	0.27774	0.27394	0.27025	0.26665	0.26315	0.4296	2.3276
0.600000	0.28569	0.28167	0.27776	0.27396	0.27026	0.26666	0.26315	0.4000	2.5000
0.700000	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316	0.3000	3.3333
0.800000	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316	0.2000	5.0000
0.900000	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316	0.1000	10.000
0.950000	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316	0.0500	20.000
0.960000	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316	0.0400	25.000
0.975000	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316	0.0250	40.000
0.980000	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316	0.0200	50.000
0.990000	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316	0.0100	100.00
0.995000	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316	0.0050	200.00
0.998000	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316	0.0020	500.00
0.999000	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316	0.0010	1000.0
0.999500	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316	0.0005	2000.0
0.999900	0.28571	0.28169	0.27778	0.27397	0.27027	0.26667	0.26316	0.0001	10000.

P	G1=–7.7	G1=–7.8	G1=–7.9	G1=–8.0	G1=–8.1	G1=–8.2	G1=–8.3	Q	T
0.000100	–18.65522	–18.80504	–18.95393	–19.10191	–19.24898	–19.39517	–19.54046	0.9999	1.0001
0.000500	–13.44202	–13.53009	–13.61730	–13.70366	–13.78919	–13.87389	–13.95778	0.9995	1.0005
0.001000	–11.28080	–11.34419	–11.40677	–11.46855	–11.52953	–11.58974	–11.64917	0.9990	1.0010
0.002000	–9.18828	–9.22863	–9.26823	–9.30709	–9.34521	–9.38262	–9.41931	0.9980	1.0020
0.005000	–6.56124	–6.57456	–6.58725	–6.59931	–6.61075	–6.62159	–6.63183	0.9950	1.0050
0.010000	–4.71704	–4.71358	–4.70961	–4.70514	–4.70019	–4.69476	–4.68887	0.9900	1.0101
0.020000	–3.04604	–3.03038	–3.01439	–2.99810	–2.98150	–2.96462	–2.94746	0.9800	1.0204
0.025000	–2.55569	–2.53737	–2.51881	–2.50001	–2.48099	–2.46175	–2.44231	0.9750	1.0256
0.040000	–1.61867	–1.59732	–1.57591	–1.55444	–1.53294	–1.51141	–1.48985	0.9600	1.0417
0.050000	–1.22706	–1.20565	–1.18427	–1.16295	–1.14168	–1.12048	–1.09936	0.9500	1.0526
0.100000	–0.28422	–0.26892	–0.25394	–0.23929	–0.22498	–0.21101	–0.19737	0.9000	1.1111
0.200000	0.17450	0.17746	0.18012	0.18249	0.18459	0.18643	0.18803	0.8000	1.2500
0.300000	0.24817	0.24622	0.24421	0.24214	0.24003	0.23788	0.23571	0.7000	1.4286
0.400000	0.25857	0.25544	0.25236	0.24933	0.24637	0.24345	0.24060	0.6000	1.6667
0.429624	0.25919	0.25596	0.25280	0.24970	0.24667	0.24371	0.24081	0.5704	1.7532
0.500000	0.25966	0.25635	0.25312	0.24996	0.24689	0.24388	0.24095	0.5000	2.0000
0.570376	0.25973	0.25640	0.25316	0.25000	0.24691	0.24390	0.24096	0.4296	2.3276
0.600000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096	0.4000	2.5000
0.700000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096	0.3000	3.3333
0.800000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096	0.2000	5.0000
0.900000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096	0.1000	10.000
0.950000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096	0.0500	20.000
0.960000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096	0.0400	25.000
0.975000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096	0.0250	40.000
0.980000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096	0.0200	50.000
0.990000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096	0.0100	100.00
0.995000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096	0.0050	200.00
0.998000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096	0.0020	500.00
0.999000	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096	0.0010	1000.0
0.999500	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096	0.0005	2000.0
0.999900	0.25974	0.25641	0.25316	0.25000	0.24691	0.24390	0.24096	0.0001	10000.

Table 2 Percentage points of Pearson Type III distribution (negative skewness)—Continued

P	G1=–8.4	G1=–8.5	G1=–8.6	G1=–8.7	G1=–8.8	G1=–8.9	G1=–9.0	Q	T
0.000100	–19.68489	–19.82845	–19.97115	–20.11300	–20.25402	–20.39420	–20.53356	0.9999	1.0001
0.000500	–14.04086	–14.12314	–14.20463	–14.28534	–14.36528	–14.44446	–14.52288	0.9995	1.0005
0.001000	–11.70785	–11.76576	–11.82294	–11.87938	–11.93509	–11.99009	–12.04437	0.9990	1.0010
0.002000	–9.45530	–9.49060	–9.52521	–9.55915	–9.59243	–9.62504	–9.65701	0.9980	1.0020
0.005000	–6.64148	–6.65056	–6.65907	–6.66703	–6.67443	–6.68130	–6.68763	0.9950	1.0050
0.010000	–4.68252	–4.67573	–4.66850	–4.66085	–4.65277	–4.64429	–4.63541	0.9900	1.0101
0.020000	–2.93002	–2.91234	–2.89440	–2.87622	–2.85782	–2.83919	–2.82035	0.9800	1.0204
0.025000	–2.42268	–2.40287	–2.38288	–2.36273	–2.34242	–2.32197	–2.30138	0.9750	1.0256
0.040000	–1.46829	–1.44673	–1.42518	–1.40364	–1.38213	–1.36065	–1.33922	0.9600	1.0417
0.050000	–1.07832	–1.05738	–1.03654	–1.01581	–0.99519	–0.97471	–0.95435	0.9500	1.0526
0.100000	–0.18408	–0.17113	–0.15851	–0.14624	–0.13431	–0.12272	–0.11146	0.9000	1.1111
0.200000	0.18939	0.19054	0.19147	0.19221	0.19277	0.19316	0.19338	0.8000	1.2500
0.300000	0.23352	0.23132	0.22911	0.22690	0.22469	0.22249	0.22030	0.7000	1.4286
0.400000	0.23779	0.23505	0.23236	0.22972	0.22714	0.22461	0.22214	0.6000	1.6667
0.429624	0.23797	0.23520	0.23248	0.22982	0.22722	0.22468	0.22219	0.5704	1.7532
0.500000	0.23808	0.23528	0.23255	0.22988	0.22727	0.22472	0.22222	0.5000	2.0000
0.570376	0.23809	0.23529	0.23256	0.22988	0.22727	0.22472	0.22222	0.4296	2.3276
0.600000	0.23810	0.23529	0.23256	0.22988	0.22727	0.22472	0.22222	0.4000	2.5000
0.700000	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222	0.3000	3.3333
0.800000	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222	0.2000	5.0000
0.900000	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222	0.1000	10.000
0.950000	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222	0.0500	20.000
0.960000	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222	0.0400	25.000
0.975000	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222	0.0250	40.000
0.980000	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222	0.0200	50.000
0.990000	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222	0.0100	100.00
0.995000	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222	0.0050	200.00
0.998000	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222	0.0020	500.00
0.999000	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222	0.0010	1000.0
0.999500	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222	0.0005	2000.0
0.999900	0.23810	0.23529	0.23256	0.22989	0.22727	0.22472	0.22222	0.0001	10000.