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CLASSIFICATION SYSTEM FOR VARIED FLOW IN PRISMATIC CHANNELS

PREFACE

This technical release describes two systems for classifying water surface profile(s), WSP, of steady and equal flows in prismatic channels. When flow is non-uniform, the WSP is termed as varied flow since the depths of flow gradually vary from section to section in the channel. The relation between the two classification systems is given in ES-191.

The definition of subnormal and supernormal flows has been restated and differs from that given in Supplement A of the Hydraulic Section (NEH-5). The restatement of the definition was made to encompass channels of any bottom slope. This definition is consistent with the definition given in Supplement A for channels of positive bottom slopes.

The technical release discusses, in a qualitative manner, the variation of depth, velocity, and specific energy head for each of the classifications of flow. It also gives the direction in which the WSP computations are to be made and the conditions for the termination of these WSP by a hydraulic jump.

This technical release was prepared by Mr. Paul D. Doubt, Head of the Design Unit, with assistance from Mr. H. J. Goon, Civil Engineer, Design Unit, Design Branch, at Hyattsville, Maryland.

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Engineering Standard Drawings

Drawing No.	Title
ES-191	HYDRAULICS: RELATION BETWEEN THE TWO CLASSIFICATION SYSTEMS OF VARIED FLOW
ES-192	HYDRAULICS: TERMINATION OF WSP BY A HYDRAULIC JUMP
ES-193	HYDRAULICS: WSP IN PRISMATIC CHANNELS OF SUFFICIENT LENGTH TO PRODUCE ESSENTIALLY UNIFORM FLOW

NOMENCLATURE

a ≡ Flow area corresponding to d - ft^2

a_c ≡ Critical area corresponding to $d_{c,Q}$ - ft^2

b ≡ Bottom width of trapezoidal channel - ft

d ≡ Actual depth of flow - ft

$d_{c,Q}$ ≡ Critical depth of flow corresponding to Q - ft

$d_{n,Q}$ ≡ Normal depth of flow corresponding to Q - ft

g ≡ Gravitation constant = 32.16 ft/sec^2

h_f ≡ Friction head loss - ft-lb/lb of water

ℓ ≡ Length of channel in downstream direction - ft

n ≡ Manning's coefficient of roughness

p ≡ Wetted perimeter corresponding to d - ft

p_c ≡ Critical wetted perimeter corresponding to $d_{c,Q}$ - ft

Q ≡ Actual discharge - cfs

$Q_{c,d}$ ≡ Critical discharge corresponding to d - cfs

$Q_{n,d}$ ≡ Normal discharge corresponding to d - cfs

r ≡ Hydraulic radius corresponding to d; $r = \frac{a}{p}$ - ft

r_c ≡ Critical hydraulic radius corresponding to $d_{c,Q}$ - ft

s_o ≡ Bottom slope of channel

s_c ≡ Critical bottom slope corresponding to Q

$s_{c,Q}$ ≡ s_c

s_f ≡ Instantaneous rate of friction head loss with respect to channel length ≡ slope of energy gradient of flow at a channel section

s_1 ≡ Bottom slope of channel upstream from a break in grade

s_2 ≡ Bottom slope of channel downstream from a break in grade

T ≡ Top width of water surface corresponding to d - ft

z ≡ Side slopes of trapezoidal channel

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CLASSIFICATION SYSTEM FOR VARIED FLOW IN PRISMATIC CHANNELS

Introduction

Varied flow occurs in open channels having steady and equal flow in which the depth of flow gradually varies from section to section. Steady and equal flow in the channel means the discharge does not change at any section in the channel with respect to time and the discharges at each section in the channel are equal. Prismatic channels are those channels having a straight alignment and constant bottom slope, Manning's coefficient of roughness and cross section.

Varied flows in a prismatic channel are classified to give some perception of the water surface profile (WSP). From a classification of flow, for a specified depth of flow and discharge, one is able to ascertain;

1. The type of flow, that is, whether it is accelerated or retarded flow. Depth of flow, in a downstream direction, decreases or increases according to whether flow is accelerated or retarded.
2. Whether WSP computations are to be carried in an upstream or downstream direction from a known starting depth.
3. Whether depth of the WSP diverges from or converges to $d_{c,Q}$ in the downstream direction.
4. Whether the specific energy head increases or decreases in the downstream direction.

In addition, if the relative values of $d_{n,Q}$ and $d_{c,Q}$ are known or if the critical slope corresponding to the discharge, Q, is computed, one is able to ascertain;

1. The depth of flow towards which the depths of the WSP approach in the direction of computations.
2. The possibility of the occurrence of a hydraulic jump.

Classification for varied flow of a specified discharge, Q, is made for a prismatic channel at an actual depth (or starting depth) of flow before the computations of the profile are commenced. The classification is not concerned with the evaluation of the starting depth of flow nor the manner a starting depth was generated. Examples of specific ways various depths of flow are generated are given in ES-192 and 193 which illustrate prismatic channels that are separated by break(s) in grade.

SCS Classification System

Varied flow is classified at a depth of flow into one of four classifications according to whether flow is subnormal, supernormal, subcritical, or supercritical. The complete system of classifying flow as well as the type of flow is given in Table 1.

Table 1. Classifications and types of varied flow

Classification of flow	Type of flow
Subnormal and Subcritical	Retarded
Subnormal and Supercritical	Accelerated
Supernormal and Subcritical	Accelerated
Supernormal and Supercritical	Retarded

The type of flow for classifications having like prefixes of "sub" or "super" is retarded flow while the type of flow for classifications of unlike prefixes is accelerated flow. Retarded flows are those for which the velocity decreases and depth increases in the direction of flow. The velocity of flow increases and depth decreases in the direction of flow for accelerated flows.

Subnormal, Normal, and Supernormal Flows

Flow at a given depth is supernormal, normal, or subnormal according to whether the rate of friction head loss is greater than, equal to, or less than the bottom slope of the channel, respectively. The rate of friction head loss, $s_f \equiv \frac{dh_f}{d\ell}$, for a given depth of flow in a prismatic channel is given by Manning's formula

$$s_f \equiv \frac{dh_f}{d\ell} = \left[\frac{n\ Q}{1.486\ ar^{2/3}}\right]^2$$

where a and r are evaluated at the given depth of flow.

In other words, subnormal flow is the condition in which the channel bottom slope is greater than the friction head loss and the specific energy head increases in the direction of flow. When the channel bottom slope is less than the friction head loss, then the condition of supernormal flow exists and the specific energy head decreases in the direction of flow. Therefore, in a downstream direction, the depth of flow diverges from critical depth for subnormal flows and converges towards critical depth for supernormal flows. See Figure 1.

The rate of friction head loss per foot length of channel (an instantaneous rate), $\frac{dh_f}{d\ell}$, is always positive. Since this rate is always greater than zero, all flows in channels having horizontal or adverse slopes are classified as supernormal.

At a given depth the rate of friction head loss with respect to channel length may be written

$$s_f \equiv \frac{dh_f}{d\ell} = s_o \left[\frac{Q}{Q_{n,d}}\right]^2 \qquad \text{where}$$

$$\frac{s_o}{Q_{n,d}^2} = \left[\frac{n}{1.486\ ar^{2/3}}\right]^2$$

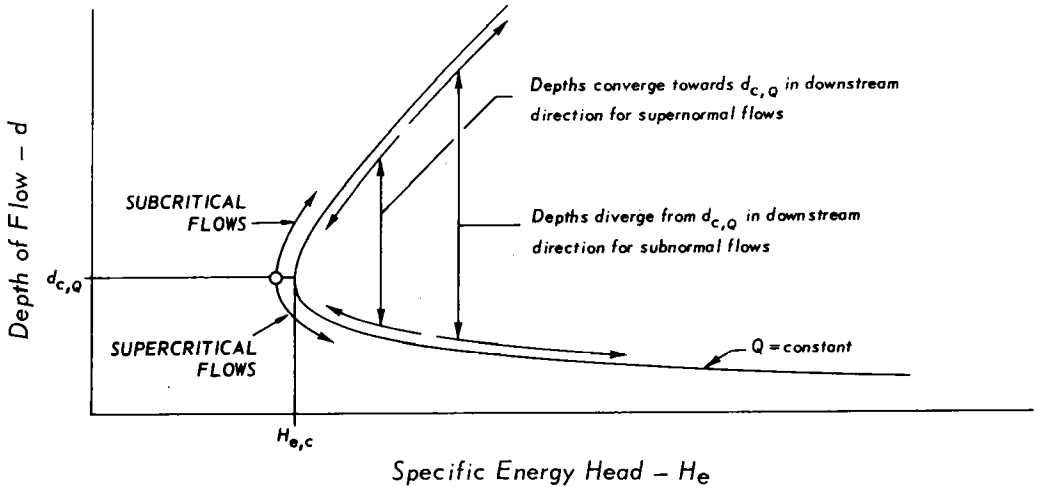


Figure 1. Specific energy head vs depth diagram showing the convergence to and divergence from $d_{c,Q}$ for supernormal and subnormal flows

The quantity $\frac{s_o}{Q_{n,d}^2}$, on observing the right hand side of the previous expression, is always positive.

Flows are subnormal at a given depth when

$$s_o \left[\frac{Q}{Q_{n,d}}\right]^2 < s_o \qquad \text{i.e.} \qquad \left[\frac{Q}{Q_{n,d}}\right]^2 < 1 \text{ (if } s_o > 0)$$

This condition can exist only in channels of positive slope.
Flows are supernormal when

$$s_o \left[\frac{Q}{Q_{n,d}}\right]^2 > s_o \qquad \text{i.e.} \qquad \left[\frac{Q}{Q_{n,d}}\right]^2 > 1 \text{ (if } s_o > 0)$$

$$\left[\frac{Q}{Q_{n,d}}\right]^2 < 1 \text{ (if } s_o < 0)$$

Observe that the quantity $\left[\frac{Q}{Q_{n,d}}\right]^2$ for an adverse slope is negative, since $Q_{n,d}^2 < 0$, and flow is always supernormal.

Again, flows for all depths in horizontal and adversely sloped channels are supernormal.

<u>Subcritical, Critical, and Supercritical Flows</u>
Flow at a given depth is subcritical, critical, or supercritical according to whether the actual discharge is less than, equal to, or greater than the critical discharge corresponding to that depth. The critical discharge corresponding to the depth of flow is given by the relation

$$Q_{c,d}^2 = \frac{g\ a^3}{T}$$

where a and T are evaluated at the given depth of flow.

Flows are subcritical when

$$\left[\frac{Q}{Q_{c,d}}\right]^2 < 1$$

and are supercritical when

$$\left[\frac{Q}{Q_{c,d}}\right]^2 > 1$$

Critical Slope, s_c

The critical slope of a channel corresponding to the discharge, Q, is that bottom slope of the channel which would cause the normal depth, $d_{n,Q}$, to be equal to the critical depth, $d_{c,Q}$. When $d_{n,Q} > d_{c,Q}$ the bottom slope is less than critical slope and is known as a mild slope. When $d_{n,Q} < d_{c,Q}$ the bottom slope is greater than critical slope and is known as a steep slope. See Figure 2.

Control Section

If the tailwater depth at a break in grade is equal to or less than $d_{c,Q}$, the channel section at the break in grade is a control section if $s_1 \leq s_c$ and $s_2 \geq s_c$ where s_1 and s_2 are the bottom slopes of the channel reaches upstream and downstream of the break in grade respectively. The depth of flow at the control section is $d_{c,Q}$. When a control section exists, the depth, $d_{c,Q}$, is taken as the starting depth for WSP computations. The WSP computations are made in an upstream and downstream direction from the control section.

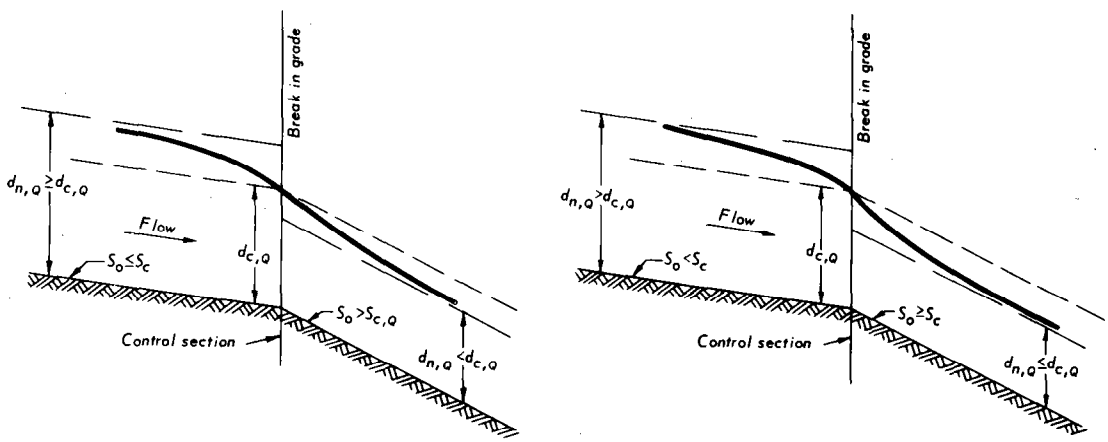


Figure 2. Drawings illustrating control sections

Common Classification System

Another system of classifying varied flow, one which is commonly discussed in literature, consists of classifications, which are designated by a letter prefix and a zone number. The letter prefix designates the magnitude of the bottom slope of the prismatic channel according to one of five conditions given in Table 2.

Table 2. Definition of the letter prefixes used in classifying varied flow

Letter Prefix	Type of channel bottom slope Range of bottom slope of channel
S	Steep bottom slope $s_o > s_c$
C	Critical bottom slope $s_o = s_c$
M	Mild bottom slope $0 < s_o < s_c$
H	Horizontal bottom slope $s_o = 0$
A	Adverse bottom slope $s_o < 0$

The prismatic channel is regarded as being divided into zones by a line representing critical depth and, when normal depth exists, a line representing normal depth. (See Figure 3).

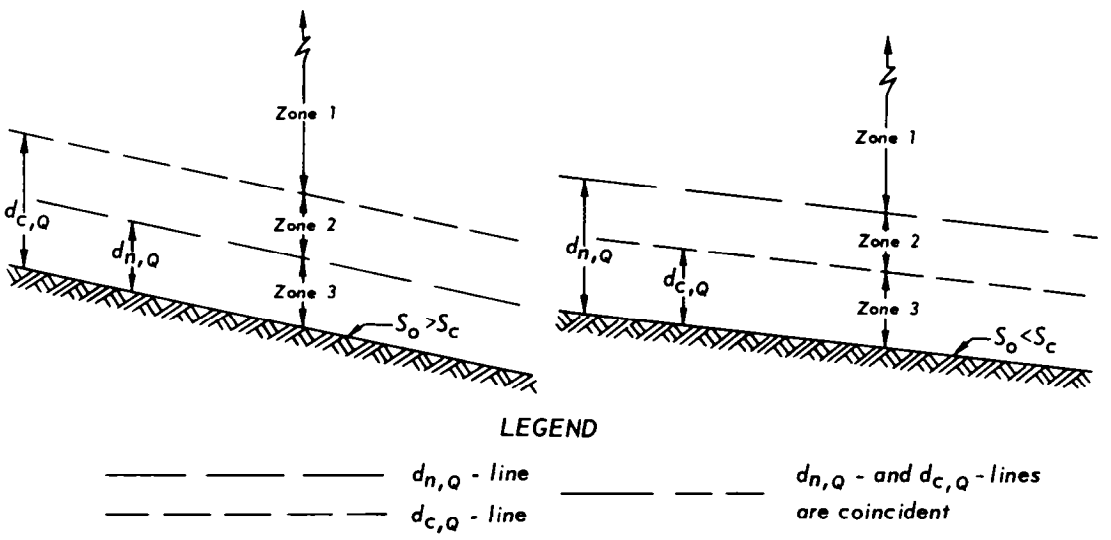


Figure 3. Division of channels into three zones for purposes of classification

When normal depth exists, the zones are numbered from 1 to 3 from the upper zone downward and zone 2 is considered non-existent when the normal and critical lines coincide. Thus, there is no classification with the designation C2.

Since normal depth is non-existent for channels with horizontal and adverse slopes, these channels are divided into two zones numbered 2 and 3. There are no classifications with the designations H1 and A1. Some authors view the normal depth for horizontally sloped channels as infinity, above which no zone exists.

Mild and steep sloped channels are the only channels having three zones.

The common classification system of varied flow consists of twelve classifications, namely

S1, C1, M1, --, --

S2, --, M2, H2, A2

S3, C3, M3, H3, A3

These classifications are referred to as the S1-curve, C1-curve, etc. This classification system depends on the magnitude of the bottom slope as well as the actual depth of flow as related to the normal depth when it exists and to the critical depth.

Relation Between Classification Systems

ES-191 gives the relation between the two classification systems of varied flow. For subcritical flows this drawing is intended to show the WSP upstream of a given depth of flow. For supercritical flows the drawing is intended to show the WSP downstream of a given depth of flow. No attempt is made to illustrate on this drawing, how a starting depth of flow was generated. All classifications of zone 1, namely S1, C1, and M1-curves, are subnormal and subcritical flows and are retarded flows. All five classifications of zone 3 are supernormal and supercritical flows and are also retarded flows. All classifications of zone 2 are accelerated flows; the S2 curve is subnormal and supercritical while the M2, H2, and A2-curves are supernormal and subcritical.

The blocks which are bordered with crosshatching pertain to those classifications which are non-existent in the common classification system, that is, there are no H1-, A1-, and C2-curves. To show the relation between the two classification systems for zone 2, it was necessary to repeat zone 2 since subnormal and supercritical is another classification also in zone 2. The stippled blocks pertain to those classifications which are non-existent in the SCS classification system. The letter prefix and zone number associated with each stippled block are incompatible with the SCS classification system.

The shapes of the profiles as shown on ES-191 are idealized. Consider for instance the S1-curve. For a given set of values for $d_{c,Q}$ and $d_{n,Q}$, the shape of the S1-curve will be above or below or may even cross that shown, according to the cross sectional shape of the prismatic channel. Changing either $d_{c,Q}$ or $d_{n,Q}$ or both will also vary the shape of the S1-curve. Theoretically, at $d_{c,Q}$ on the S1-curve, the rate of change of depth of flow with respect to the length of channel is infinite, i.e., perpendicular to the $d_{c,Q}$-line. The depth of flow and specific energy heads increase in a downstream direction. By varying $d_{c,Q}$ to a value equal to $d_{n,Q}$, the S1-curve varies in shape to that of the C1-curve. Mathematically,

on the C1-curve, at $d = d_{c,Q}$, the rate of change of depth of flow with respect to the length of channel becomes an indeterminate expression (i.e., 0/0) which can be evaluated by L'Hospital's rule. The shape of the C1-curve can vary considerably from that shown; however, the depth of flow and specific energy head will still increase in the direction of flow. An additional decrease in the value of $d_{c,Q}$ leads to the M1-curve. The WSP for the M1-curve is asymptotic to the $d_{n,Q}$-line, for the rate of change of depth with respect to the length of the channel at $d_{n,Q}$ is zero.

Direction of WSP Computations

Computations for WSP are made from a section at which the depth is known to an upstream section when flow is subcritical. WSP computations are carried in a downstream direction from a known depth when flow is supercritical.

Depths of Flow Approach $d_{n,Q}$ or Infinity or $d_{c,Q}$

In the following two statements it is safe and convenient to view the normal depth for horizontally and adversely sloped channels as infinitely large. For any classification of flow, the depth of flow in the direction of computation is seeking the normal depth of flow. If the critical depth lies between the actual depth of flow and normal depth, the WSP approaches the $d_{c,Q}$-line perpendicularly, otherwise the WSP approaches the $d_{n,Q}$-line asymptotically. When $d_{n,Q}$ and $d_{c,Q}$ are nearly equal, the WSP at depths nearly equal to $d_{c,Q}$ is unpredictable since the depths of the WSP fluctuate.

Hydraulic Jump Changes Classification

The classification of flow for the various depths along a WSP in a prismatic channel cannot change to another classification except by the formation of a hydraulic jump.

Inherent Hydraulic Jumps (WSP Approach $d_{c,Q}$)

A WSP of any classification may be terminated, in the direction of computations, by a hydraulic jump. Retarded flows in which depths approach $d_{c,Q}$ in the direction of computations will be terminated by a hydraulic jump provided $d_{c,Q} \neq d_{n,Q}$ and a break in grade does not occur prior to the formation of the jump. WSP of these retarded flows are inherently associated with hydraulic jumps. In each figure of ES-192 the WSP for the classifications given on the left side of the drawing is designated by a heavy line. Each of the Figures 1, 2, and 3 (ES-192) show the termination of a WSP associated with subnormal and subcritical (retarded) flow by a hydraulic jump. The WSP of this classification, which approaches $d_{c,Q}$ in an upstream direction, must necessarily terminate either at a break in grade or a hydraulic jump before the depth, $d_{c,Q}$, is reached if $d_{c,Q} \neq d_{n,Q}$. Figures 4, 5, 6a, 6b, and 6c (ES-192) show the terminations of WSP associated with supernormal and supercritical (retarded) flow by a hydraulic jump. The WSP of this classification, which approaches $d_{c,Q}$ in a downstream direction, must terminate either at a break in grade or a hydraulic jump before the depth, $d_{c,Q}$, is reached if $d_{c,Q} \neq d_{n,Q}$. Unless either the initial depth (Figure 2

of ES-192) or the sequent depth (Figure 5 of ES-192) is $d_{n,Q}$, the WSP will need to be computed upstream towards the jump and downstream towards the jump to ascertain the location at which the initial depth and sequent depth are in correspondence. This is the approximate location of the hydraulic jump. The interrelation of location of jump, the bottom slope, type of jump, length of jump, and the correspondence of initial and sequent depths are not considered in this technical release. For certain conditions the hydraulic jump will oscillate over a considerable distance upstream and downstream from this location.

Induced Hydraulic Jumps (WSP Approach $d_{n,Q}$)

A proper tailwater condition will cause the WSP associated with supercritical flow which approach $d_{n,Q}$ to terminate at a hydraulic jump. These jumps are not inherent but are induced by proper tailwater conditions. Thus, the WSP associated with any subcritical tailwater flow may be terminated by a hydraulic jump in an upstream direction if upstream flows are supercritical. See Figures 7 - 11 (ES-192).

With proper tailwater conditions, see Figures 12 - 14 (ES-192), the WSP associated with supercritical flows will be terminated in a downstream direction by a jump.

WSP in Channels of Sufficient Length to Produce Uniform Flow

ES-193 shows all combinations of two long, positively sloped prismatic channels joined at a break in grade. Uniform flow can occur only in channels having positive slopes. Therefore, channels with horizontal and adverse slopes are not considered since flow would be impossible from or into infinitely long channels with these slopes. The two channels of positive slope must be visualized as greatly condensed horizontally. They have the same cross-sectional shape. The flow conditions are described for each figure.

Observe that all sections at breaks in grade which are control sections have a flow depth of $d_{c,Q}$. The depth, $d_{c,Q}$, is used as the starting depth for computing WSP upstream and downstream from the break in grade.

All sections at breaks in grade which are not control sections have a flow depth of $d_{n,Q}$ associated with either the upstream or downstream channel. If the upstream channel slope is either critical or mild, the depth of flow at the break in grade is $d_{n,Q}$ associated with the downstream channel. If the upstream channel slope is steep, the depth of flow at the break in grade is $d_{n,Q}$ associated with the upstream channel except when a hydraulic jump occurs in the upstream channel, then the depth of flow is $d_{n,Q}$ associated with the downstream channel.

When the section at the break in grade is not a control section, the depth, $d_{n,Q}$, may be used as the starting depth for computing WSP upstream or downstream. The depth, $d_{n,Q}$, should be used as the starting depth only if the prismatic channel is known to be sufficiently long to produce essentially this depth of flow at the break in grade. This determination is made by WSP computations towards the break in grade.

Summary

The SCS classification system is concerned with the establishment of the relative values of d vs $d_{c,Q}$ and, when $d_{n,Q}$ exists, the relative values of d vs $d_{n,Q}$. These relative values establish;

1. whether the depth, velocity, and specific energy head increase or decrease in the direction of flow
2. the direction of WSP computations
3. whether the depths of the WSP converge to or diverge from $d_{c,Q}$ in the direction of flow.

The depth of $d_{n,Q}$, or ∞, or $d_{c,Q}$ to which the depths of the WSP approach is established by the relative values of d, $d_{n,Q}$, and $d_{c,Q}$. For accelerated flows it is known $d_{c,Q}$ never lies between d and $d_{n,Q}$ and the relative value of d, $d_{n,Q}$, and $d_{c,Q}$ can be deduced for all accelerated flows from the classification of flow in channels of any bottom slope. Also, the relative values of d, $d_{n,Q}$, and $d_{c,Q}$ is established by the classification when flow is retarded in channels with horizontal and adverse slopes since $d_{n,Q}$ is taken as infinitely large. However, the relative values of d, $d_{n,Q}$, and $d_{c,Q}$ for retarded flows in channels with positive slopes cannot be established by the classification. Hence, the values of $d_{c,Q}$ and $d_{n,Q}$ would normally be evaluated to establish the relative values of $d_{c,Q}$ and $d_{n,Q}$.

The two quantities $s_o \left[\frac{Q}{Q_{n,d}}\right]^2$ and $\left[\frac{Q}{Q_{c,d}}\right]^2$ are used to classify the flow and are quantities used in the varied flow equation for solving the WSP, see Eq. A-8 of Supplement A of the Hydraulic Section (NEH-5). The procedure for computing WSP is outlined in Supplement A. A recapitulation of those characteristics and properties associated with the WSP which have been discussed in this technical release is given in Table 3.

Table 3. Recapitulation

CLASSIFICATION OF FLOW → / TYPE OF FLOW ↓	SUBNORMAL; $s_o \left[\frac{Q}{Q_{n,d}}\right]^2 < s_o$ $d > d_{n,Q}$ Depths of WSP are diverging from $d_{c,Q}$ in the downstream direction H_e is increasing in the downstream direction	SUPERNORMAL; $s_o \left[\frac{Q}{Q_{n,d}}\right]^2 > s_o$ $d < d_{n,Q}$ Depths of WSP are converging to $d_{c,Q}$ in the downstream direction H_e is decreasing in the downstream direction
SUBCRITICAL; $\left[\frac{Q}{Q_{c,d}}\right]^2 < 1$ Compute WSP upstream	RETARDED; $\frac{dd}{d\ell} > 0$ $0 < s_o < \infty$ Depths of WSP are increasing in the downstream direction Velocities are decreasing in the downstream direction $d > d_{n,Q}$ and $d > d_{c,Q}$ If $d_{c,Q}$ does not lie between d and $d_{n,Q}$ [1] i.e., if $d_{c,Q} \leq d_{n,Q}$ or if $s_o \leq s_c$ Jump can be induced [3] If $d_{c,Q}$ lies between d and $d_{n,Q}$ [2] i.e., if $d_{c,Q} > d_{n,Q}$ or if $s_o > s_c$ Jump is inherent [4]	ACCELERATED; $\frac{dd}{d\ell} < 0$ $-\infty < s_o < s_c$ Depths of WSP are decreasing in the downstream direction Velocities are increasing in the downstream direction $d_{n,Q} > d > d_{c,Q}$ Depths of WSP approach $d_{n,Q}$ in the direction of WSP computations [1] Jump can be induced [3]
SUPERCRITICAL; $\left[\frac{Q}{Q_{c,d}}\right]^2 > 1$ Compute WSP downstream	ACCELERATED; $\frac{dd}{d\ell} < 0$ $s_c < s_o < \infty$ Depths of WSP are decreasing in the downstream direction Velocities are increasing in the downstream direction $d_{n,Q} < d < d_{c,Q}$ Depths of WSP approach $d_{n,Q}$ in the direction of WSP computations [1] Jump can be induced [3]	RETARDED; $\frac{dd}{d\ell} > 0$ $-\infty < s_o < \infty$ Depths of WSP are increasing in the downstream direction Velocities are decreasing in the downstream direction $d < d_{n,Q}$ and $d < d_{c,Q}$ If $d_{c,Q}$ does not lie between d and $d_{n,Q}$ [1] i.e., if $d_{c,Q} \geq d_{n,Q}$ or if $s_o \geq s_c$ Jump can be induced [3] If $d_{c,Q}$ lies between d and $d_{n,Q}$ [2] i.e., if $d_{c,Q} < d_{n,Q}$ or if $s_o < s_c$ Jump is inherent [4]

[1] Depths of WSP approach $d_{n,Q}$, asymptotically if $d_{n,Q} \neq \infty$ or $d_{n,Q} \neq d_{c,Q}$, in the direction of WSP computations
[2] Depths of WSP approach $d_{c,Q}$ in the direction of WSP computation and a hydraulic jump is inherent
[3] Depths of WSP approaching $d_{n,Q}$ can be terminated by an induced jump
[4] Depths of WSP approaching $d_{c,Q}$ will terminate at a hydraulic jump or at a break in grade before $d_{c,Q}$ is reached

Example 1.

Given: A trapezoidal prismatic channel

$n = 0.01$
$b = 10$ ft
$z = 2$
$s_o = 0.02$
$Q = 1000$ cfs
$d = 3.0$ ft

Determine:

1. Classification of flow at the depth $d = 3.0$ ft and the given discharge
2. Type of flow at depth $d = 3.0$ ft
3. Direction of computations of the WSP from $d = 3.0$ ft
4. Whether the depths of the WSP approach $d_{n,Q}$, ∞, or $d_{c,Q}$ in the direction of computations
5. The possibility of the WSP; in the direction of computations, terminating at a hydraulic jump
6. The instantaneous rate of friction head loss at $d = 3.0$ ft
7. a. How the specific energy heads of the WSP are changing
   b. How the velocities of the WSP are changing
   c. How the depths of the WSP are changing
   d. Whether depths of the WSP are converging or diverging from the critical depth
   e. Type of bottom slope

Solution: 1. Determining classification of flow at $d = 3.0$ ft

a. Evaluating a, p, T, r, $Q^2_{n,d}$, and $Q^2_{c,d}$ at $d = 3.0$ ft

$$a = (b + zd)(d) = [10 + 2(3)]3 = 48.0 \text{ ft}^2$$

$$p = (b + 2d\sqrt{z^2 + 1}) = 10 + 2(3)(\sqrt{5}) = 23.42 \text{ ft}$$

$$T = (b + 2zd) = 10 + 2(2)(3) = 22 \text{ ft}$$

$$r = \frac{a}{p} = \frac{48}{23.42} = 2.05 \text{ ft}$$

$$Q^2_{n,d} = \left[\frac{1.486}{n} ar^{2/3}\right]^2 s_o = \left[\frac{1.486}{0.01}(48)(2.05)^{2/3}\right]^2(0.02)$$

$$= 2.656(10^6)$$

$$Q^2_{c,d} = \frac{g\,a^3}{T} = \frac{32.16(48)^3}{22} = 1.617(10^5)$$

b. Determining whether flow is supernormal or subnormal

$$\left[\frac{Q}{Q_{n,d}}\right]^2 = \frac{10^6}{2.656(10^6)} < 1 \qquad (s_o > 0)$$

Flow is subnormal.

c. Determining whether flow is supercritical or subcritical

$$\left[\frac{Q}{Q_{c,d}}\right]^2 = \frac{10^6}{1.617(10^5)} > 1$$

Flow is supercritical

The classification of varied flow at d = 3.0 ft and discharge Q = 1000 cfs is subnormal and supercritical.

2. Determining type of flow
The prefixes of the classification are unlike; hence, flow is accelerated at d = 3.0 ft.

3. Determining direction of computations for the WSP from d = 3.0 ft
Computations for the WSP are to be made in a downstream direction from d = 3.0 ft because flow is supercritical.

4. Determining whether the depths of the WSP approach $d_{n,Q}$, ∞, or $d_{c,Q}$ in the direction of the WSP computations

a. Since $s_o > 0$; $d_{n,Q}$ exists and depths do not approach infinity.

b. *i.* Since flow is subnormal at d = 3.0 ft
$d > d_{n,Q}$

ii. Since flow is supercritical at d = 3.0 ft
$d < d_{c,Q}$

iii. Thus, $d_{n,Q} < d < d_{c,Q}$ therefore $d_{c,Q}$ does not lie between d and $d_{n,Q}$. Depths of the WSP approach $d_{n,Q}$ in the direction of computations (i.e. downstream).

5. Determining the possibility of a hydraulic jump terminating this WSP
Since the depths of the WSP approach $d_{n,Q}$, a hydraulic jump will not terminate this WSP unless a sufficiently high tail water downstream induces the formation of a jump in this channel.

6. Determining the instantaneous rate of friction loss at d = 3.0 ft

$$s_f \equiv \frac{dh_f}{d\ell} = s_o \left[\frac{Q}{Q_{n,d}}\right]^2 = 0.02\left[\frac{10^6}{2.656(10^6)}\right] = 0.00753$$

ft-lb/ft length of channel for each pound of water. This rate of friction loss is less than $s_o = 0.02$ which is to be expected for all subnormal flows.

7. a. Since flow is subnormal the specific energy head is increasing in the downstream direction. This is consistent with the fact that $s_f < s_o$.

b. The velocity is increasing in the downstream direction since flow is accelerated.

c. The depth of flow is decreasing in the downstream direction since flow is accelerated.

d. The depth of flow is diverging from $d_{c,Q}$ in the downstream direction since flow is subnormal.

e. Type of bottom slope of the channel is steep since $d_{n,Q} < d_{c,Q}$.

Example 2.

Given: The same channel as Example 1. The channel is conveying $Q = 100$ cfs and the given depth is $d = 3.0$ ft.

Determine: Same items as listed in Example 1.

Solution: 1. a. The quantities, a, p, T, r, $Q^2_{n,d}$ and $Q^2_{c,d}$ at $d = 3.0$ ft remain the same as were evaluated in Example 1.

b. Determining whether flow is supernormal or subnormal

$$\left[\frac{Q}{Q_{n,d}}\right]^2 = \frac{10^4}{2.656(10^6)} < 1 \qquad (s_o > 0)$$

Flow is subnormal.

c. Determining whether flow is supercritical or subcritical

$$\left[\frac{Q}{Q_{c,d}}\right]^2 = \frac{10^4}{1.617(10^5)} < 1$$

Flow is subcritical.
The classification of varied flow at $d = 3.0$ ft and discharge $Q = 100$ cfs is subnormal and subcritical.

2. Determining type of flow
The prefixes of the classification are alike; hence, flow is retarded at $d = 3.0$ ft.

3. Determining the direction of computations for the WSP from $d = 3.0$ ft
Computations for the WSP are to be made in an upstream direction from $d = 3.0$ ft because flow is subcritical.

4. Determining whether the depths of the WSP approach $d_{n,Q}$, ∞, $d_{c,Q}$ in the direction of the WSP computations.

a. Since $s_o > 0$; $d_{n,Q}$ exists and depths do not approach infinity.

b. Since flow is subnormal and subcritical, i.e., $d > d_{n,Q}$ and $d > d_{c,Q}$ the relative value of $d_{c,Q}$ and $d_{n,Q}$ are unknown. To ascertain the depths towards which the WSP approaches one must know whether $d_{c,Q}$ lies between d and $d_{n,Q}$. Thus, the value of $d_{c,Q}$ and $d_{n,Q}$ are required or the value of $s_{c,Q}$ is required to ascertain if $d_{c,Q}$ lies between d and $d_{n,Q}$.

i. Determining $d_{c,Q}$. From ES-24 for $\frac{Q}{b} = \frac{100}{10} = 10$ and $\frac{z}{b} = \frac{2}{10} = 0.2$ read $d_{c,Q} = 1.33$ ft.

ii. Determining $d_{n,Q}$. From ES-55 for

$$\frac{nQ}{b^{8/3}\ s_o^{1/2}} = \frac{(0.01)(100)}{(10)^{8/3}(0.02)^{1/2}} = 0.01523$$

and $z = 2$ read $\frac{d_{n,Q}}{b} = 0.063$

$d_{n,Q} = 0.63$ ft.

Thus, $d_{c,Q} < d > d_{n,Q}$ and flow depths are approaching $d_{c,Q} = 1.33$ ft in an upstream direction.

5. Determining the possibility of a hydraulic jump terminating this WSP
Since the depths of the WSP approach $d_{c,Q}$, a hydraulic jump will terminate this WSP in the direction of computations unless a break in grade occurs before the sequent depth of the jump is attained.

6. The instantaneous friction head loss of $d = 3.0$ ft is

$$s_f \equiv \frac{dh_f}{d\ell} = s_o \left[\frac{Q}{Q_{n,d}}\right]^2 = 0.02\left[\frac{10^4}{2.656(10^6)}\right] = 0.0000753$$

ft-lbs/ft length of channel per pound of water.

This rate of friction loss is less than $s_o = 0.02$ which is to be expected for all subnormal flows.

7. a. Since flow is subnormal, the specific energy head is increasing in the downstream direction. This is consistent with the fact that

$$s_f < s_o$$

b. The velocity is decreasing in the downstream direction since flow is retarded.

c. The depth of flow is increasing in the downstream direction since flow is retarded.

d. The depth of flow is diverging from $d_{c,Q}$ in the downstream direction since flow is subnormal.

e. Type of bottom slope of the channel is steep since

$$d_{c,Q} > d_{n,Q}$$

<u>Example 3</u>.

<u>Given</u>: The same channel as Example 1 except $s_o = 0.005$. The channel is conveying $Q = 1000$ cfs and the given depth is $d = 3.0$ ft.

<u>Determine</u>: Same items as listed in Example 1.

Solution: 1. a. The quantities a, p, T, r and $Q^2_{c,d}$ at d = 3.0 ft remain the same as were evaluated in Example 1.

$$Q^2_{n,d} = \left[\frac{1.486}{n}\, ar^{2/3}\right]^2 s_o$$

$$= \left[\frac{1.486}{0.01}(48)(2.05)^{2/3}\right]^2 (0.005)$$

$$Q^2_{n,d} = 6.64 \times 10^5$$

b. Determining whether flow is supernormal or subnormal

$$\left[\frac{Q}{Q_{n,d}}\right]^2 = \frac{10^6}{6.64(10^5)} > 1 \qquad (s_o > 0)$$

Flow is supernormal.

c. Determining whether flow is supercritical or subcritical

$$\left[\frac{Q}{Q_{c,d}}\right]^2 = \frac{10^6}{1.617(10^5)} > 1$$

Flow is supercritical.

The classification of varied flow at d = 3.0 ft and discharge Q = 1000 cfs is supernormal and supercritical.

2. Determining type of flow
The prefixes of the classification are alike; hence, flow is retarded at d = 3.0 ft.

3. Determining the direction of computations for the WSP from d = 3.0 ft
Computations for the WSP are to be made in a downstream direction from d = 3.0 ft because flow is supercritical.

4. Determining whether the depths of the WSP approach $d_{n,Q}$, ∞, or $d_{c,Q}$ in the direction of the WSP computations

a. Since $s_o > 0$; $d_{n,Q}$ exists and depths do not approach infinity.

b. Since flow is supernormal and supercritical, i.e., $d < d_{n,Q}$ and $d < d_{c,Q}$, either the values of $d_{n,Q}$ and $d_{c,Q}$ are required or the values of s_c corresponding to the discharge Q is required to ascertain if $d_{c,Q}$ lies between d and $d_{n,Q}$.

Determining the critical slope s_c corresponding to the discharge Q.

From ES-24 for $\frac{Q}{b} = \frac{1000}{10} = 100$ and $\frac{z}{b} = \frac{2}{10}$

read $d_{c,Q} = 4.915$ ft.

Substituting into the relation

$$s_c = \left[\frac{n\ Q}{1.486\ a_c\ r_c^{2/3}}\right]^2 \text{ where}$$

$$a_c = (b + zd_{c,Q})\ d_{c,Q} = \left[10 + 2(4.915)\right] 4.915 = 97.46 \text{ ft}^2$$

$$p_c = b + 2\, d_{c,Q} \sqrt{z^2 + 1} = 10 + 2(4.915)(\sqrt{5})$$
$$= 31.98 \text{ ft}$$

$$r_c = \frac{a_r}{p_c} = \frac{97.46}{31.98} = 3.05 \text{ ft}$$

$$s_c = \left[\frac{0.01(1000)}{1.486(97.46)(3.05)^{2/3}}\right]^2 = 0.001079$$

Thus, since $s_o = 0.005 > s_c = 0.001079$ the slope is steep and $d_{n,Q} < d_{c,Q}$.

Since flow is supernormal and supercritical and on a steep slope

$$d < d_{n,Q} < d_{c,Q}$$

The value of $d_{n,Q}$ can be determined, if desired, from ES-55

$$\frac{n\ Q}{b^{8/3}\ s_o^{1/2}} = \frac{(0.01)(1000)}{(10)^{8/3}(0.005)^{1/2}} = 0.3047$$

Read, from ES-55, the value $\frac{d_{n,Q}}{b} = 0.334$

or $d_{n,Q} = 3.34$ ft

which confirms $d < d_{n,Q} = 3.34 \text{ ft} < d_{c,Q} = 4.915$ ft.

Flow depths of the WSP are approaching $d_{n,Q}$ asymptotically.

5. Determining the possibility of a hydraulic jump terminating this WSP
   Since the depths of the WSP approach $d_{n,Q}$, a hydraulic jump will not terminate this WSP unless a sufficiently high tailwater induces the formation of a jump in this channel.

6. Determining the instantaneous rate of friction loss at $d = 3.0$ ft

$$s_f \equiv \frac{dh_f}{d\ell} = s_o\left[\frac{Q}{Q_{n,d}}\right]^2 = 0.005\left(\frac{10^6}{6.64 \times 10^5}\right) = 0.00753$$

ft lbs/ft length of channel for each pound of water.

This rate is greater than $s_o = 0.005$ which is to be expected for all supernormal flows.

Observe this rate of friction loss is the same as was obtained in Example 1, which is to be expected for the channel cross sections n, Q and d are the same in the two examples.

7. a. Since flow is supernormal, the specific energy head is decreasing in the downstream direction. This is consistent with the fact that $s_f > s_o$.

b. The velocity is decreasing in the downstream direction since flow is retarded.

c. The depth of flow is increasing in the downstream direction since flow is retarded.

d. The flow is converging towards $d_{c,Q}$ in the downstream direction since flow is supernormal.

e. The bottom slope of the channel is steep since $s_o > s_c$.

Example 4.

Given: The same channel as Example 1 except $s_o = 0.0005$. The channel is conveying $Q = 1000$ cfs and the given depth is $d = 3.0$ ft.

Determine: Same items as listed in Example 1.

Solution: 1. a. The quantities a, p, T, r and $Q^2_{c,d}$ at $d = 3.0$ ft remain the same as were evaluated in Example 1.

$$Q^2_{n,d} = \left[\frac{1.486}{n} ar^{2/3}\right]^2 s_o = \left[\frac{1.486}{0.01}(48)(2.05)^{2/3}\right]^2 (0.0005)$$

$$Q^2_{n,d} = 6.64 \times 10^4$$

b. Determining whether flow is supernormal or subnormal

$$\left[\frac{Q}{Q_{n,d}}\right]^2 = \frac{10^3}{6.64 \times 10^4} > 1 \qquad (s_o > 0)$$

Flow is supernormal.

c. Determining whether flow is supercritical or subcritical

$$\left[\frac{Q}{Q_{c,d}}\right]^2 = \frac{10^6}{1.617(10^5)} > 1$$

Flow is supercritical.

The classification of varied flow at $d = 3.0$ ft and discharge 1000 cfs is supernormal and supercritical.

2. Determining type of flow
The prefixes of the classification are alike; hence, flow is retarded at $d = 3.0$ ft

3. Determining the direction of computation for the WSP from $d = 3.0$ ft
Computations for the WSP are to be made in a downstream direction from $d = 3.0$ ft because flow is supercritical.

4. Determining whether the depths of the WSP approach $d_{n,Q}$, ∞, or $d_{c,Q}$ in the direction of the WSP computations.

a. Since $s_o > 0$; $d_{n,Q}$ exists and depths do not approach infinity.

b. Since flow is supernormal and supercritical, i.e, $d < d_{n,Q}$ and $d < d_{c,Q}$, either the values of $d_{n,Q}$ and $d_{c,Q}$ are required or the values of s_c corresponding to the discharge Q is required to ascertain if $d_{c,Q}$ lies between d and $d_{n,Q}$.

The value of the critical slope s_c corresponding to the discharge Q = 1000 cfs was evaluated in problem 3 as

$$s_c = 0.001079$$

Since $s_o = 0.0005 < s_c = 0.001079$ the slope is mild and $d_{c,Q} < d_{n,Q}$. Hence, since $d < d_{c,Q}$ the critical depth $d_{c,Q}$ lies between d and $d_{n,Q}$. Therefore, flow depths are approaching $d_{c,Q}$.

5. Determining the possibility of a hydraulic jump terminating this WSP
Since the depths of the WSP approach $d_{c,Q}$, a hydraulic jump will terminate this WSP unless a break in grade occurs before the initial depth of the jump is attained.

6. Determining the instantaneous rate of friction loss at d = 3.0 ft

$$s_f \equiv \frac{dh_f}{d\ell} = s_o\left[\frac{Q}{Q_{n,d}}\right]^2 = 0.0005\left(\frac{10^6}{6.65 \times 10^4}\right) = 0.00753$$

ft-lbs/ft length of channel for each pound of water.

Observe this rate of friction loss is the same as was obtained in Examples 1 and 3. This is to be expected since the channel cross section, n, Q and d are the same in the three examples.

7. a. Since flow is supernormal, the specific energy head is decreasing in the downstream direction. This is consistent with the fact that $s_f > s_o$.

b. The velocity is decreasing in the downstream direction since flow is retarded.

c. The depth of flow is increasing in the downstream direction since flow is retarded.

d. The flow is converging towards $d_{c,Q}$ in the downstream direction since flow is supernormal.

e. The bottom slope of the channel is mild since $s_o < s_c$.

Example 5.

Given: The same channel as Example 1 except $s_o = 0$. The channel is conveying Q = 100 cfs and the given depth is d = 3.0 ft.

Determine: Same items as listed in Example 1.

Solution: 1. a. The quantities a, p, T, r and $Q^2_{c,d}$ at d = 3.0 ft remain the same as were evaluated in Example 1.

The value of $Q_{n,d}$ is zero, however the value of

$$\frac{Q^2_{n,d}}{s_o} = \left[\frac{1.486}{n} ar^{2/3}\right]^2 = \left[\frac{1.486}{0.01}(48)(2.05)^{2/3}\right]^2 = 1.328(10^8)$$

b. Determining whether flow is supernormal or subnormal

Since $s_o = 0$, flow is supernormal.

c. Determining whether flow is supercritical or subcritical

$$\left[\frac{Q}{Q_{c,d}}\right]^2 = \frac{10^4}{1.617(10^5)} < 1$$

Flow is subcritical.

The classification of varied flow at d = 3.0 ft and discharge Q = 100 cfs is supernormal and subcritical.

2. Determining type of flow
The prefixes of the classification are unlike; hence flow is accelerated at d = 3.0 ft.

3. Determining the direction of computations for the WSP from d = 3.0 ft.
Computations for the WSP are to be made in an upstream direction from d = 3.0 ft because flow is subcritical.

4. Determining whether the depths of the WSP approach $d_{n,Q}$, ∞, or $d_{c,Q}$ in the direction of the WSP computations.

a. Since $s_o = 0$; $d_{n,Q}$ does not exist and depths approach $d_{c,Q}$ or infinity.

Flow is subcritical thus

$$d_{c,Q} < d < \text{infinity}$$

and $d_{c,Q}$ does not lie between d and infinity. Therefore depths of the WSP are approaching infinity in an upstream direction.

5. Determining the possibility of a hydraulic jump terminating this WSP
Since the depth of the WSP does not approach $d_{c,Q}$, a hydraulic jump will not terminate this WSP in the direction of WSP computations unless a jump was induced by some condition causing supercritical flow in the upstream portion of this channel.

6. Determining the instantaneous rate of friction loss at d = 3.0 ft

$$s_f \equiv \frac{dh_f}{d\ell} = s_o\left[\frac{Q}{Q_{n,d}}\right]^2 = \left[\frac{s_o}{Q^2_{n,d}}\right] Q^2 = \frac{10^4}{1.328(10^8)} =$$

0.0000753 ft-lbs/ft length of channel per pound of water.

Observe this rate of friction loss is the same as was obtained in Example 2. This is to be expected since the channel cross section, n, Q and d are the same in the two examples.

7. a. Since flow is supernormal the specific energy head is decreasing in the downstream direction. This is consistent with the fact $s_f > s_o$.

   b. The velocity is increasing in the downstream direction since flow is accelerated.

   c. The depth of flow is decreasing in the downstream direction since flow is accelerated.

   d. The flow is converging towards $d_{c,Q}$ in the downstream direction since flow is supernormal.

   e. The bottom slope of the channel is horizontal.

Example 6.

Given: The same channel as Example 1 except $s_o = -0.005$. The channel is conveying Q = 1000 cfs and the given depth is d = 3.0 ft.

Determine: Same items as listed in Example 1.

Solution: 1. a. The quantities a, p, T, r and $Q^2_{c,d}$ at d = 3.0 ft remain the same as evaluated in Example 1.

$$Q^2_{n,d} = \left[\frac{1.486}{n} ar^{2/3}\right]^2 s_o = \left[\frac{1.486}{0.01}(48)(2.05)^{2/3}\right]^2(-0.005)$$

$$Q^2_{n,d} = -6.64 \times 10^5$$

   b. Determining whether flow is supernormal or subnormal
   Since $s_o < 0$, flow is supernormal.

   c. Determining whether flow is supercritical or subcritical

$$\left[\frac{Q}{Q_{c,d}}\right]^2 = \frac{10^6}{1.617(10^5)} > 1$$

   Flow is supercritical.

   The classification of varied flow at d = 3.0 ft and the discharge Q = 1000 cfs is supernormal and supercritical.

2. Determining type of flow
   The prefixes of the classification are alike; hence, flow is retarded.

3. Determining direction of computations for the WSP from d = 3.0 ft
   Computations for the WSP are to be made in a downstream direction from d = 3.0 ft because flow is supercritical.

4. Determining whether the depths of the WSP approach $d_{n,Q}$, ∞, or $d_{c,Q}$ in the direction of the WSP computations

   a. Since $s_o < 0$; $d_{n,Q}$ does not exist and depths approach infinity or $d_{c,Q}$

Flow is supercritical thus

$$d < d_{c,Q} < \text{infinity}$$

and $d_{c,Q}$ lies between d and infinity. Therefore, depths of the WSP are approaching $d_{c,Q}$ in a downstream direction.

5. Determining the possibility of a hydraulic jump terminating this WSP

   Since the depths of the WSP approach $d_{c,Q}$, a hydraulic jump will terminate this WSP in the direction of WSP computations unless a break in grade occurs before the WSP attains the initial depth of the hydraulic jump.

6. Determining the instantaneous rate of friction loss at $d = 3.0$ ft

$$s_f \equiv \frac{dh_f}{d\ell} = s_o \left[\frac{Q}{Q_{n,d}}\right]^2 = (-0.005)\ \frac{10^6}{-6.64 \times 10^5} = 0.00753$$

   ft-lbs/ft length of channel per pound of water.

   Observe this rate of friction loss is the same as was obtained in Examples 1, 3, and 4. This is to be expected since the channel cross section, n, Q and d are the same in the four examples.

7. a. Since flow is supernormal, the specific energy head is decreasing in the downstream direction. This is consistent with the fact that $s_f > s_o$.

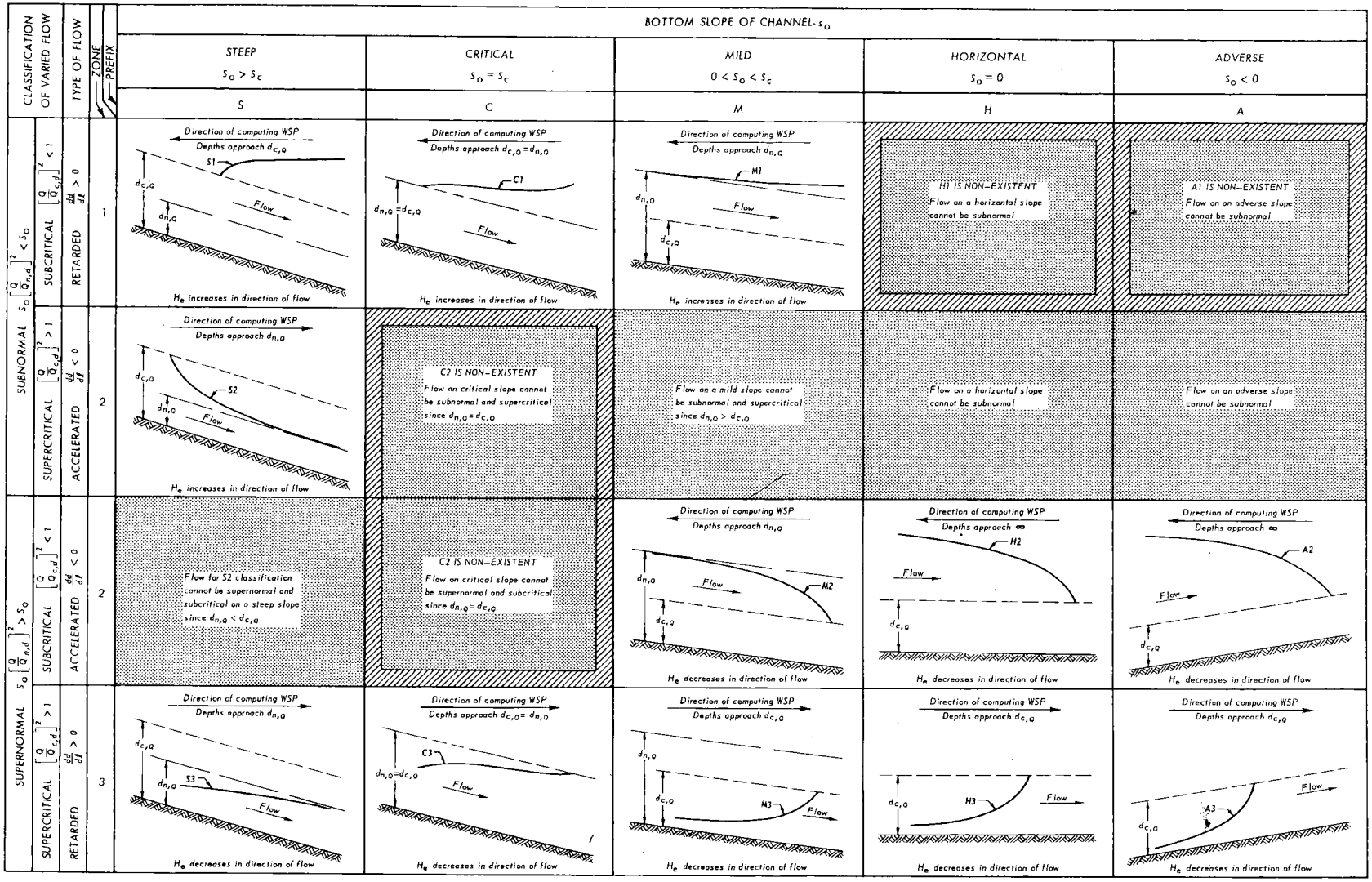
   b. The velocity is decreasing in the downstream direction since flow is retarded.

   c. The depth of flow is increasing in the downstream direction since flow is retarded.

   d. The flow is converging towards $d_{c,Q}$ in the downstream direction since flow is supernormal.

   e. The bottom slope of the channel is adversely sloped.

HYDRAULICS: RELATION BETWEEN THE TWO CLASSIFICATION SYSTEMS OF VARIED FLOW



REFERENCE

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SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN UNIT

STANDARD DWG. NO. ES-191
SHEET 1 OF 1
DATE 10-1-70

HYDRAULICS: TERMINATION OF WSP BY A HYDRAULIC JUMP

In each figure the WSP for the classification given on the left side is designated by a heavy line

If $d_{c,Q} \neq d_{n,Q}$, all WSP of retarded flow which approach $d_{c,Q}$ in the direction of computations will terminate at a hydraulic jump unless terminated by a break in grade prior to the jump. Hydraulic jumps are inherently associated with these WSP.

RETARDED FLOW

SUBNORMAL, SUBCRITICAL — Compute WSP upstream towards $d_{c,Q}$ to the sequent depth of the hydraulic jump

The initial depth of the hydraulic jump lies on a WSP associated with supercritical flow and either

Subnormal flow

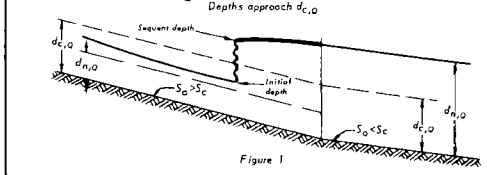

Figure 1

Normal flow

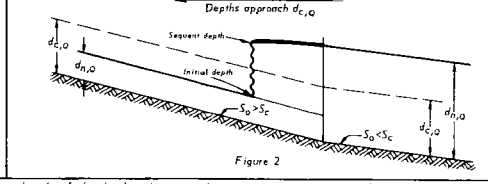

Figure 2

Supernormal flow

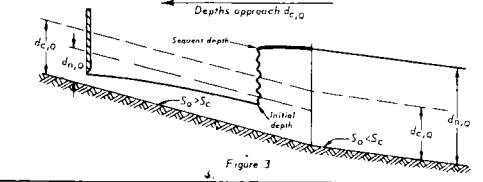

Figure 3

SUPERNORMAL, SUPERCRITICAL — Compute WSP downstream towards $d_{c,Q}$ to the initial depth of the hydraulic jump

The sequent depth of the hydraulic jump lies on a WSP associated with subcritical flow and either

Subnormal flow

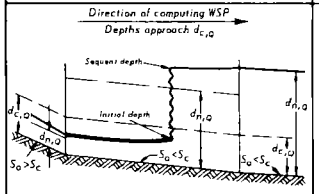

Figure 4

Normal flow

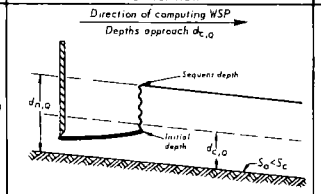

Figure 5

Supernormal flow

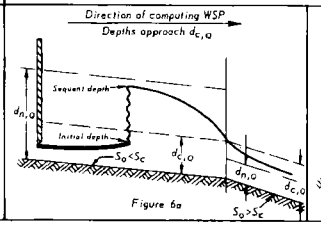

Figure 6a

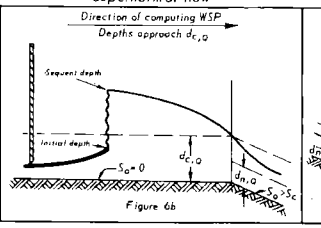

Figure 6b

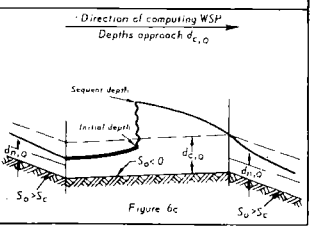

Figure 6c

SUBCRITICAL FLOW

Hydraulic jumps may terminate, in the direction of computations, WSP of any classification. The following examples show the termination of WSP, in the direction of computations, by induced hydraulic jumps.

Compute WSP upstream towards $d_{n,Q}$ or ∞ to the sequent depth of the hydraulic jump

The initial depth of the hydraulic jump lies on a WSP associated with supernormal, supercritical; retarded flow

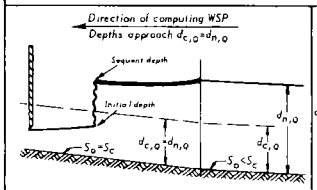

Figure 7

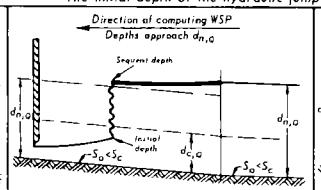

Figure 8

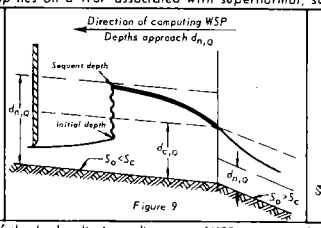

Figure 9

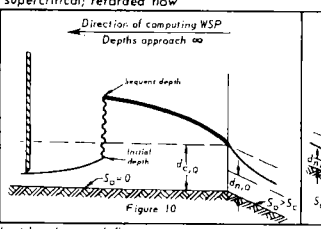

Figure 10

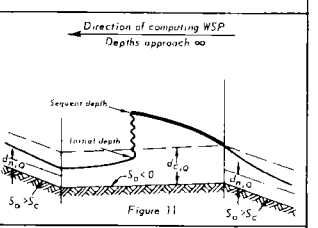

Figure 11

SUPERCRITICAL FLOW — Compute WSP downstream towards $d_{n,Q}$ to the initial depth of the hydraulic jump

The sequent depth of the hydraulic jump lies on a WSP associated with subcritical flow

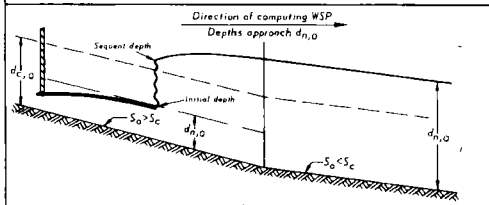

Figure 12

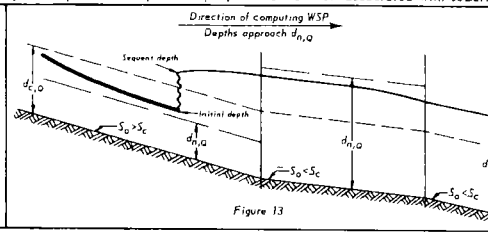

Figure 13

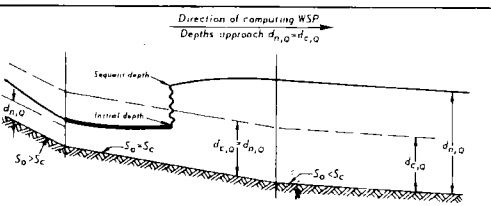

Figure 14

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN UNIT

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HYDRAULICS: WSP IN PRISMATIC CHANNELS OF SUFFICIENT LENGTH TO PRODUCE ESSENTIALLY UNIFORM FLOW

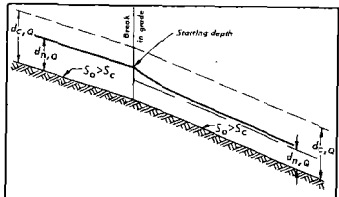

Figure 1

Bottom slope of the downstream channel is steeper than the bottom slope of the upstream channel.

Flow is uniform in the upstream channel.

Flow is subnormal and supercritical immediately downstream from the break in grade. Determine the WSP by computing downstream from the break in grade using $d_{n,o}$ associated with the upstream channel as the starting depth. Flow is accelerated and asymptotically approaches $d_{n,o}$ associated with the downstream channel.

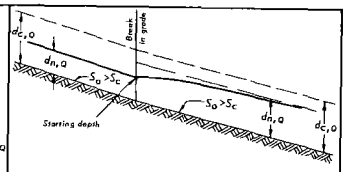

Figure 2

Bottom slope of the upstream channel is steeper than the bottom slope of the downstream channel.

Flow is uniform in the upstream channel.

Flow is supernormal and supercritical immediately downstream from the break in grade. Determine the WSP by computing downstream from the break in grade using $d_{n,o}$ associated with the upstream channel as the starting depth. Flow is retarded and asymptotically approaches $d_{n,o}$ associated with the downstream channel.

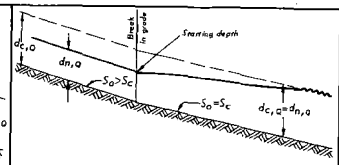

Figure 3

Flow is uniform in the upstream channel.

Flow is supernormal and supercritical immediately downstream from the break in grade. Determine the WSP by computing downstream from the break in grade using $d_{n,o}$ associated with the upstream channel as the starting depth. Flow is retarded and approaches $d_{c,o}$. WSP is unstable (depth of flow fluctuates) as the depth of flow approaches $d_{c,o}$.

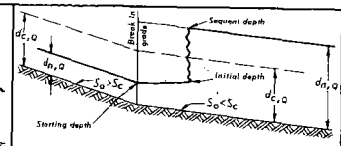

Figure 4a

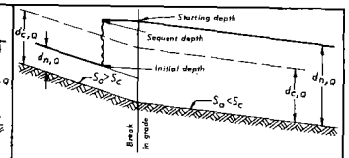

Figure 4b

The determination as to whether the hydraulic jump occurs upstream or downstream of the break in grade can be ascertained as follows. Assume the initial depth equal to $d_{n,o}$ associated with the upstream channel. If the sequent depth corresponding to the assumed initial depth is less than, equal to, or greater than $d_{n,o}$ associated with the downstream channel, the hydraulic jump occurs upstream from, at, or downstream from the break in grade, respectively.

Flow is uniform in the upstream channel. Flow is uniform downstream from the hydraulic jump and the sequent depth is $d_{n,o}$ associated with the downstream channel.

Flow is supernormal and supercritical from the break in grade downstream to the hydraulic jump. Determine the WSP by computing downstream from the break in grade using $d_{n,o}$ associated with the upstream channel as the starting depth. Flow is retarded and approaches $d_{c,o}$. The WSP is terminated at the depth equal to the initial depth corresponding to the sequent depth.

Flow is uniform upstream from the hydraulic jump and the initial depth is $d_{n,o}$ associated with the upstream channel. Flow is uniform in the downstream channel.

Flow is subnormal and subcritical from the break in grade upstream to the hydraulic jump. Determine the WSP by computing upstream from the break in grade using $d_{n,o}$ associated with the downstream channel as the starting depth. Flow is retarded and approaches $d_{c,o}$. The WSP is terminated at the depth equal to the sequent depth corresponding to the initial depth.

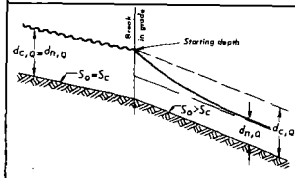

Figure 5

The break in grade is a control section.

Flow is unstable and uniform in the upstream channel.

Flow is subnormal and supercritical immediately downstream from the break in grade Determine the WSP by computing downstream from the break in grade using $d_{c,o}$ as the starting depth. Flow is accelerated and asymptotically approaches $d_{n,o}$ associated with the downstream channel.

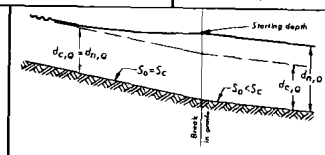

Figure 6

Flow is uniform in the downstream channel.

Flow is subnormal and subcritical immediately upstream from the break in grade. Determine the WSP by computing upstream from the break in grade using $d_{n,o}$ associated with the downstream channel as the starting depth. Flow is retarded and approaches $d_{c,o}$. WSP is unstable (depth of flow fluctuates) as the depth of flow approaches $d_{c,o}$.

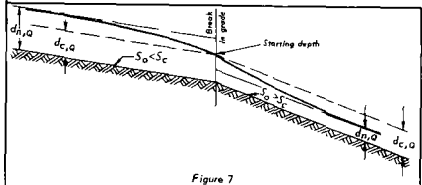

Figure 7

The break in grade is a control section.

Flow is supernormal and subcritical immediately upstream from the break in grade.

Determine the WSP by computing upstream from the break in grade using $d_{c,o}$ as the starting depth. Flow is accelerated and asymptotically approaches $d_{n,o}$ associated with the upstream channel.

Flow is subnormal and supercritical immediately downstream from the break in grade.

Determine the WSP by computing downstream from the break in grade using $d_{c,o}$ as the starting depth. Flow is accelerated and asymptotically approaches $d_{n,o}$ associated with the downstream channel.

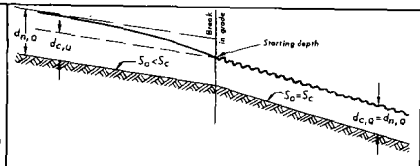

Figure 8

The break in grade is a control section.

Flow is unstable and uniform in the downstream channel.

Flow is supernormal and subcritical immediately upstream from the break in grade. Determine the WSP by computing upstream from the break in grade using $d_{c,o}$ as the starting depth. Flow is accelerated and asymptotically approaches $d_{n,o}$ associated with the upstream channel.

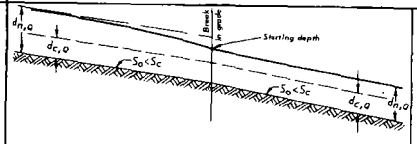

Figure 9

Bottom slope of the downstream channel is steeper than the bottom slope of the upstream channel.

Flow is uniform in the downstream channel.

Flow is supernormal and subcritical immediately upstream from the break in grade. Determine the WSP by computing upstream from the break in grade using $d_{n,o}$ associated with the downstream channel as the starting depth. Flow is accelerated and asymptotically approaches $d_{n,o}$ associated with the upstream channel.

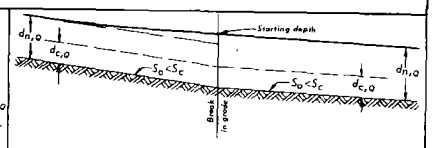

Figure 10

Bottom slope of the upstream channel is steeper than the bottom slope of the downstream channel.

Flow is uniform in the downstream channel.

Flow is subnormal and subcritical immediately upstream from the break in grade. Determine the WSP by computing upstream from the break in grade using $d_{n,o}$ associated with the downstream channel as the starting depth. Flow is accelerated and asymptotically approaches $d_{n,o}$ associated with the upstream channel.

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
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